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Probability distribution of rainfall in the Bia watershed: contribution of Markov chains

MELEDJE N'DIAYE HERMANN[1], KOUASSI KOUAKOU LAZARE[1], NGO YAO ALEXIS[2] & SAVANE ISSIAKA[2]

1 Centre de Recherche en Ecologie, Laboratoire de Géologie Marine, Sédimentologie et Environnement, 08 BP 109 Abidjan 08, Côte d'Ivoire
meledjendiay@yahoo.fr

2 Université Nangui Abrogoua , Laboratoire Géosciences et Environnement, Côte d'Ivoire

Abstract This study aims to highlight the occurrence of dry and wet years in the basin of the Bia by modelling using a Markov chain for the period 1901–2009. From the rainfall data from six stations (Ayame, Bianouan, Agnibilekro, Enchi, Buakuc and Sunyani), the test of sequential patterns of standardized drought index rainfall ISSP has confirmed the existence of three major periods during the last century. An increase of precipitation from 1945 to 1946 that followed a relatively dry phase and a decrease in precipitation since the 1970s. Using Markov chains, some maps are given to illustrate the irregularity of the time and spatial distribution of rainfall occurrence probability.

Key words Markov chain; occurrences probability of dry/wet years; time/spatial distribution

INTRODUCTION

The phenomena of drought and desertification that hit many African countries south of the Sahara have not spared the Ivory Coast, especially since the 1970s. The drought of 1980 highlighted the limits of hydroelectric power production and Ivory Coast was confronted with interruptions in electricity supply. These outages were severely felt in 2010. The drought was followed by high precipitation in October–November which caused flooding downstream of some dams. However, the lack of studies on climate trends and hydrological forecasts has not allowed managers of dams to anticipate environmental problems. The operation of dams becomes increasingly difficult because of fluctuations affecting hydro resources (Kouassi, 2007; Kouassi *et al*., 2013). At the two dams on the Ayame, the oldest hydroelectric dams in the Ivory Coast, floods recorded in 2010 and 2011 destroyed many plantations downstream of the dams. In this context, it is important to understand the irregularity and distribution of rainfall. According to Lana & Burguen (1998) and Doudja *et al*. (2007), droughts, whose appearance and occurrence in a given year represent a case of irregular rainfall, are important climatic phenomena which deserve to be studied in order to understand and adopt adaptation and mitigation measures. In this study we aim to model precipitation occurrence and distribution of wet and dry periods using Markov chain models (James, 1963; Sung-Euii, 1994; Pablo & Javier, 2011). The study area is the Bia transboundary watershed in Ivory Coast and Ghana.

MATERIALS AND METHODS

Data

The Bia catchment includes parts of the Ivory Coast and Ghana (Fig. 1). The basin has an area of approximately 1500 km^2. The Bia River lakes include hydroelectric dams Ayamé 1 and 2, the oldest dams in the Ivory Coast, built in 1959 and 1965, respectively. The Bia catchment has a humid equatorial climate, characterized by the importance of the first rainy season (March to July) with a maximum of precipitation in June, and a second wet season from September to November (Durand & Guiral, 1994).

According to the work of Adjanohoun & Guillaumet (1971) and Girard *et al*. (1971), the Bia River flowed entirely in dense forest with an average flow of 82 m^3/s. The data from the two agencies responsible for the rainfall network namely SODEXAM in Ivory Coast and the Department of Meteorological Services (Meteorological Services Department) in Ghana. The study area is covered by six stations (see Fig. 1).

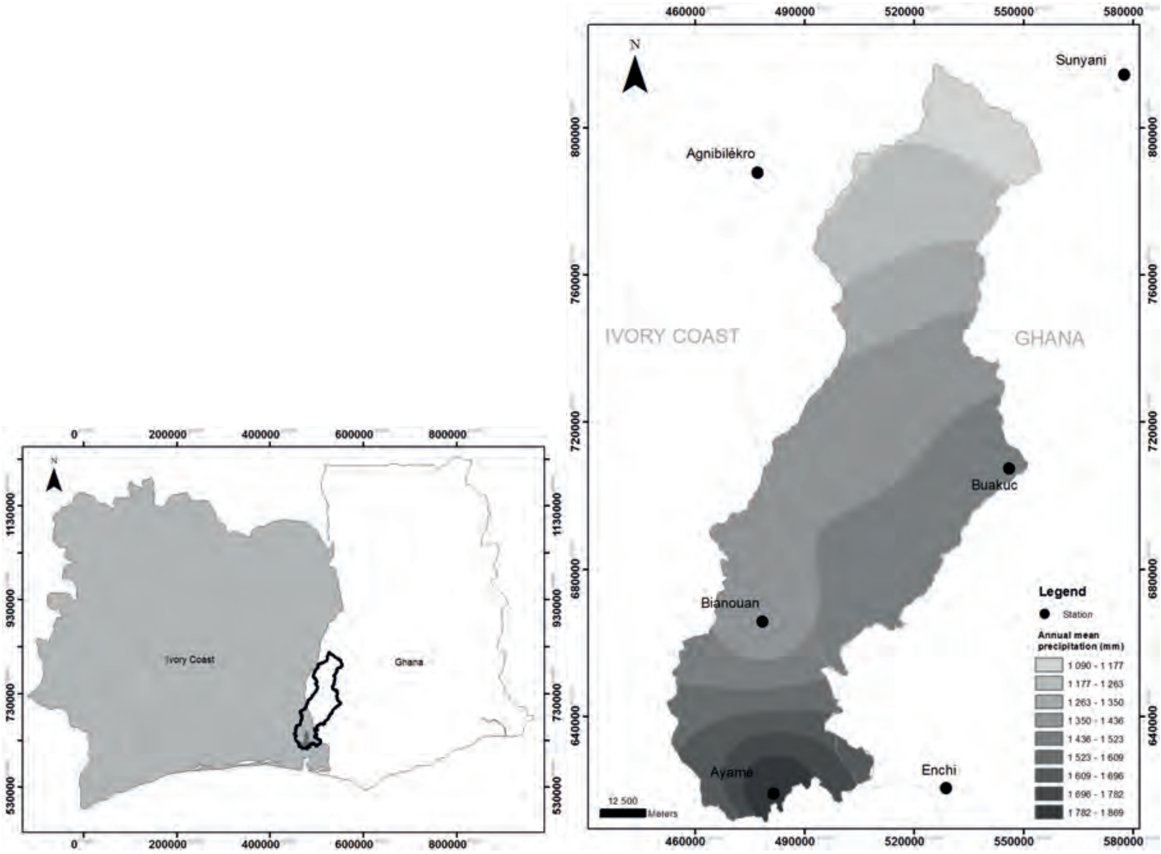

Fig. 1 Location of meteorological stations and spatial distribution of annual mean precipitation during 1901–2009.

Analysis and critical of rainfall data

The rainfall data from six stations used in this study cover the period from 1901 to 2009. The data set included filling gaps which was done using Mouhous developed by Laborde (1998).

Drought index

The standardized index of drought rainfall (ISSP) identifies the various droughts existing in the series from the accumulation and subsequent standardization of monthly rainfall anomaly dry spells. The methodology was developed by Pita (2000).

In the first step, the monthly rainfall anomalies in the series are calculated from the expression:

$$AP_i = P_i - P_{MED} \tag{1}$$

where AP_i = rainfall anomaly of each year i; P_i = the value of the precipitation of the year I; P_{MED} = median rainfall series corresponding to the year of the series studied.

In the second step, the monthly rainfall anomalies are accumulated in the first month of the series.

$$APA_i = \sum AP_i \tag{2}$$

since $i = AP$ negative to positive APA. APA_i = cumulative rainfall anomaly of the month i. Finally in the third step we obtain the index from the standardized cumulative data through their conversion into z value anomalies.

$$ISSP_i = ZAPA_i = (APA_i - APA)/\sigma APA \tag{3}$$

where $ISSP_i$ = Standardized index of drought rainfall in month i; $ZAPA_i$ = Standardized cumulative rainfall anomaly of the month i; APA = Average rainfall anomalies cumulative monthly series; σAPA = Standard deviation of rainfall anomalies cumulative monthly series.

The persistence of drought rainfall by the method of Markov chain

To determine the level of dryness required for the methodology of Markov chains, we opted for annual precipitation anomalies AP_i described above. Negative AP_i = dry year, positive AP_i = wet year.

Presentation of the methodology of Markov chains

The first Markov model is based on the concept of conditioned likelihood or transition since the probability of a dry day occurring will depend on the conditions of the preceding day, which may be dry (0) or wet (1).

$$\Pr\left\{X_t = j \middle| X_{t-1} = i_{t-1}\right\} = P_{ij} \quad \text{where} \qquad P_{ij} = N_{ij} / N_i \tag{4}$$

The value P_{ij} represents the probability that the process will, when in state i, next make a transition into state j.

Let P denote the matrix of one-step transition probabilities P_{ij}, so that

$$P = \begin{bmatrix} P_{00} & P_{01}\cdots & \\ P_{10} & P_{11}\cdots & \\ \cdots\cdots & \cdots\cdots & . \\ P_{i1} & P_{i1} & \end{bmatrix} \tag{5}$$

In the case of a first-order Markov chain, the likelihood of a dry spell $W1_d(n)$ lasting n days (00…n…01) will be given by the following formula:

$$W1_d(n) = P_{00}^{n-1} P_{01} \tag{6}$$

where P_{00} is the probability of a dry year occurring after a dry year and P_{01} is the probability of a wet year occurring after a dry one.

Finally, the expected return period for a rainy and for a dry year can be expressed, respectively, by:

$$T(r) = (P_{01} + P_{10}) / P_{01} \tag{7}$$

$$T(d) = (P_{01} + P_{10}) / P_{10} \tag{8}$$

P_{10} is the probability of a dry year occurring after a wet one.

RESULTS

Application of Drought Index (ISSP) at Bia basin

The application of standardized drought index showed three phases of rainfall, namely (Fig. 2):

- a phase where rainfall is 4% higher than the average, from 1901 to 1945;
- a normal phase from 1946 to 1970 when the rainfall is approximately equal to the average; and
- a dry phase which began in late 1970 (but ended with some wet years).

This historical series of drought index guided statistical studies to assess the probabilities of occurrence of annual droughts by Markov chains.

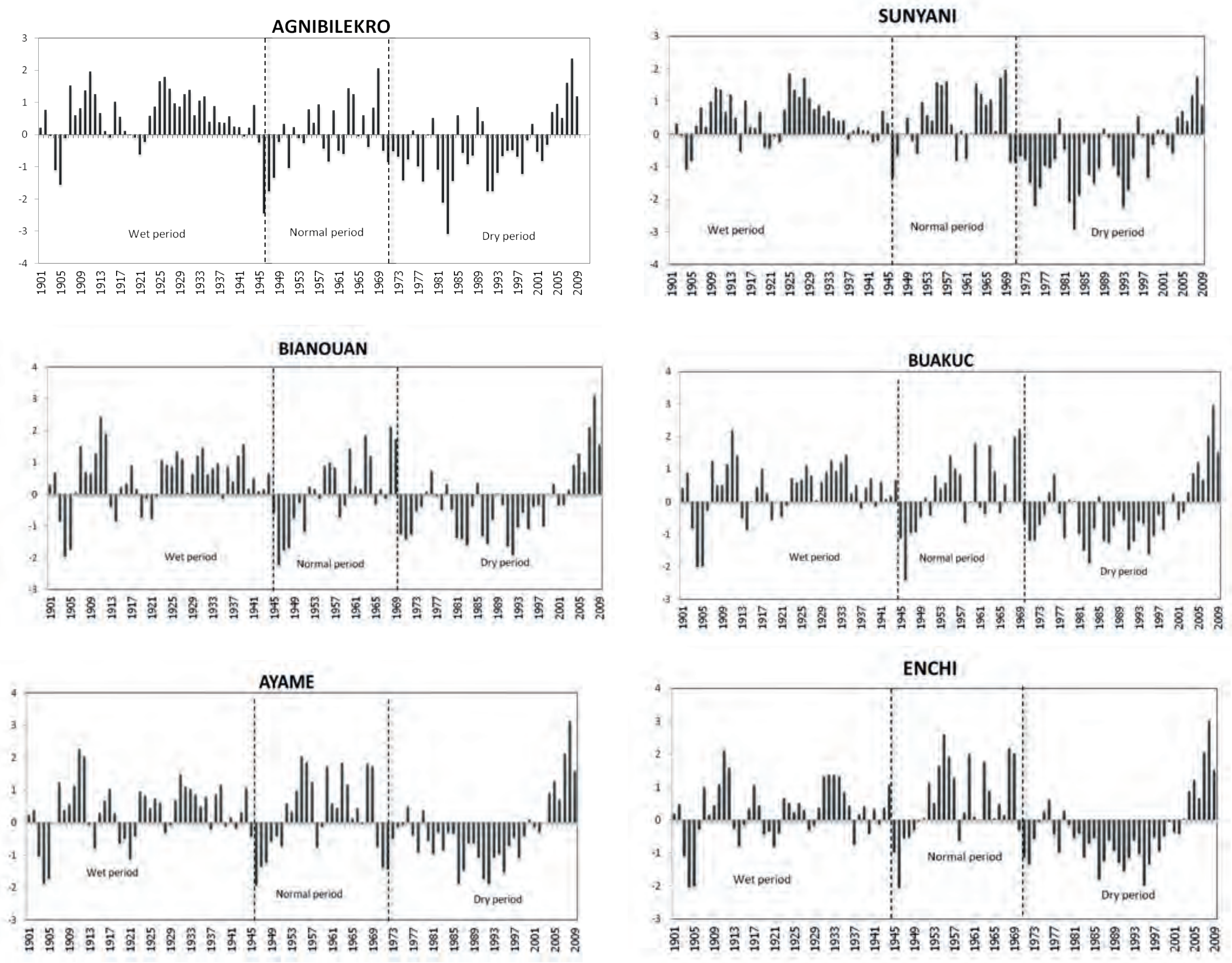

Fig. 2 Evolution of standardized drought to the different stations in the basin of River Bia from 1901 to 2009.

Analysis of the spatial variability of drought

The likelihood of a dry spell WI_d (2) lasting 2 years is mapped through the inverse distance weighted (IDW) technique using Arcgis10. A dry spell WI_d (2) is evaluated empirically and computed by equation (6).

The evolution of drought conditions is shown in Fig. 3, which shows the geographic distribution of drought in the basin of the Bia. For the first two periods (1901–1945, 1946–1970) we see a few more dry about 1/5 of the total area (Fig. 3). Annual totals are organized into wetter and drier restricted areas and the organization is very heterogeneous. If one year remains dry, the probability that the following year is dry is higher in Agnibilekro (50%), Enchi (50%) and Sunyani (57%) areas during 1946–1970. In the Bia catchment, the probability of having two successive dry years is greater than 62%, considering the period 1971–2009. Table 1 summarizes expected lengths deduced empirically and computed by means of equations (7) and (8). We can observe that expected return periods for dry episodes are almost constant, ranging from 1.45 to 1.78 years. In accordance with this we should conclude that no significant differences could be established among the different stations of the basin. The short return period (1.19 years), reflecting the state of having a new dry episode, is observed in the Sunyani region during the period 1971–2009 (see Table 1).

However, the number of years to return to a new rainy episode and become longer, ranging from 2.84 to 4.56 years during the last 30 years (see Table 1).

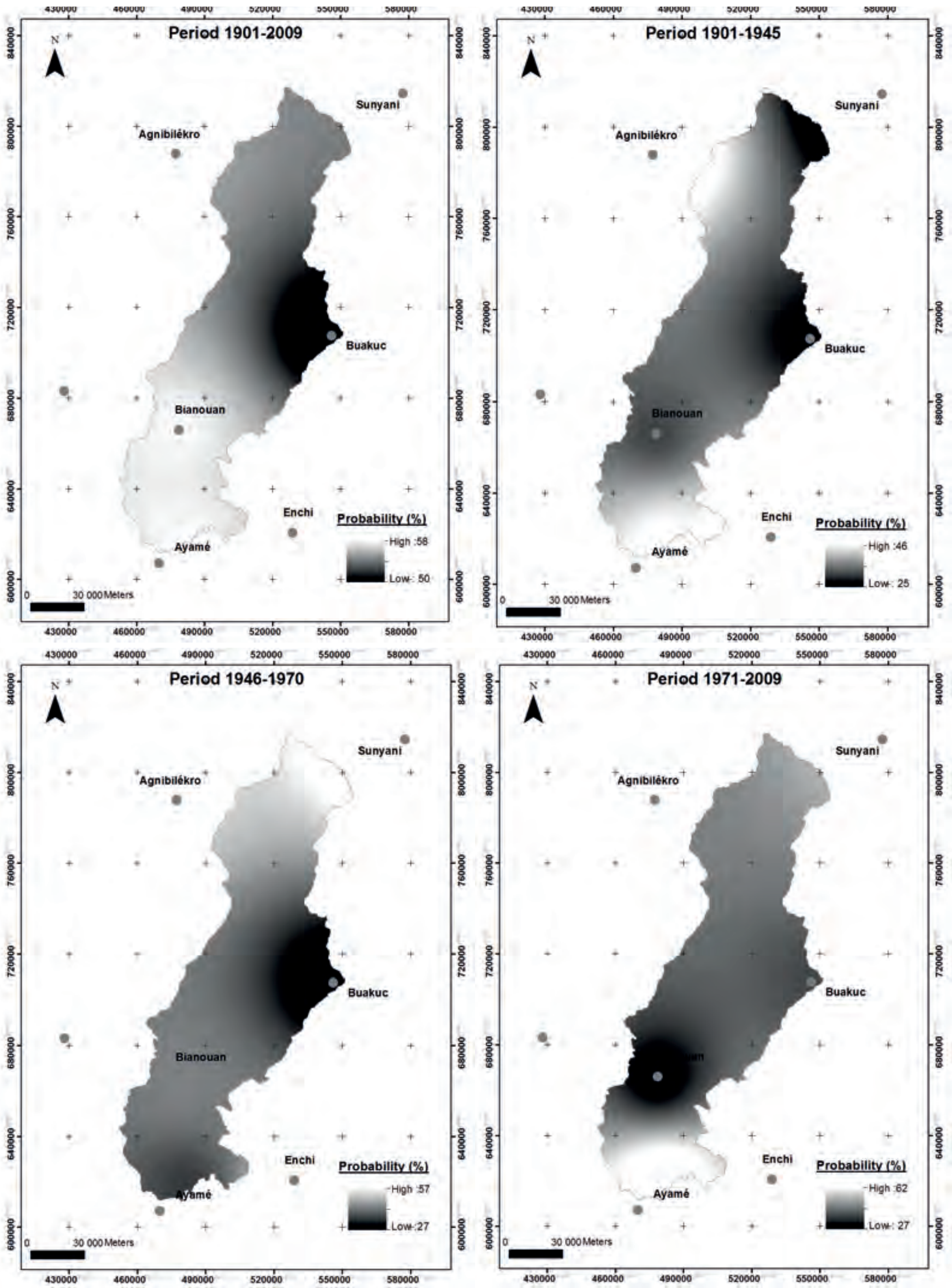

Fig. 3 Spatial distribution of the likelihood of a dry spell lasting 2 years.

Table 1 Return period, computed according to Markov chains of first order and given in year, to a new dry episode (Dry) and to a new wet episode (Wet)

	Period (year)					
	From 1901 to 1945		From 1946 to 1970		From 1971 to 2009	
	Dry	Wet	Dry	Wet	Dry	Wet
Agnibilékro	1.45	1.41	1.73	1.51	1.26	2.84
Enchi	1.45	1.41	1.50	1.08	1.26	2.84
Sunyani	1.73	1.24	1.78	1.27	1.19	4.56
Ayamé	1.56	1.06	1.64	1.42	1.24	3.82
Buakuc	1.70	1.11	1.73	1.51	1.25	3.25
Bianouan	1.67	1.16	1.58	1.58	1.25	3.25

DISCUSSION

By applying the standardized drought index rainfall (ISSP), based on the work of Pita (2000), the rainfall series of Bia watershed from 1901 to 2009 shows a break in 1970 at all stations and the persistence of a dry period from 1970 to 2009. The largest reduction was recorded in Agnibilekro and Ayame with more than a 29% decrease in annual rainfall after the date of termination, and the station Sunyani, with 32.7%. At Enchi station the decline was 23%, and about 18% at Bianouan station. These results confirm those of many researchers which indicate a break in the sub-Saharan area around 1970, with a drying trend after the break (Mahé *et al*, 1995; Boyer *et al*, 1998. Servat *et al*., 1998; Pita, 2000; Goula *et al*., 2006; Bodian, 2011; Noufé *et al*., 2011) These results confirm the occurrence of a rainfall deficit from 1970 and it continued during 1980–2000. According to Pita (2000), the main contribution of the standardized drought index rainfall is a very precise establishment of the duration of the drought and of its intensity.

In all the stations, the probability of a year of deficit increased remarkably during the third sub-period. Regarding the internal structure of stochastic dependencies, we note from the review of conditional probabilities, that the succession of dry conditions increased during 1971–2009 compared to the previous sub-period, with a dominance for the Agnibilekro stations northwest and Sunyani in the northeast and South Ayame. The spatial distribution of the drought shows that the areas of Bianouan (southern part of the basin) and Sunyani and Agnibilekro (the northern part of the basin) are more affected by the drought. (The persistent dry spells could be the result of climate change on the one hand and the degradation of plant cover on the other. The earlier droughts south of the basin could be explained by the diagonal of drought-related upwelling effects that affect the coastal regions. Ultimately, the succession of two dry years is more persistent in the basin of the River Bia. This sequence appears to be more pronounced in Ivory Coast and Ghana. Return periods of two successive dry years are short and are found during 1971–2009 (1.25 years to 1.78 years). Therefore, the return period here certainly take account of the occurrence, but also the intensity of the drought.

CONCLUSION

The application of standardized drought index rainfall has distinguished three climatic trends from 1901 to 2009: a wet period from 1901 to 1945 followed by a normal period 1946 to 1970 and a dry period from 1971 to 2009. The most significant fluctuation is observed around 1970 and marked around 1980, during which there is generally quite a significant decrease in annual rainfall. This deficit period has been characterized ever since by its intensity and duration.

The study of the persistent drought, using Markov chains, showed that two successive dry years is more persistent in the basin of the River Bia.

REFERENCES

Adjanohoun, E. & Guillaumet, J. L. (1971) *La végétation. in : Le milieu naturel de Côte d'Ivoire.* Mémoire ORSTOM 50, 161–162.

Akyuz Dilek Eren, Bayazit Mehmetcik, & Onoz Bihrat (1998) Markov chain models for hydrological drought characteristics. *Journal of Hydrometeorology* 13, 298–309.

Bodian, A., Dacosta, H. & Dezetter, A. (2011) Caractérisation spatio-temporelle du régime pluviométrique du haut bassin du fleuve Sénégal dans un contexte de variabilité climatique. *Physio-Géo - Géographie Physique et Environnement*, 2011, volume V p. 107–124

Boyer, Lubes–Niel, J. F., Fritsch, H., Masson, J. M. & Marieu, B. (1998) Identification, caractérisation et conséquences d'une variabilité hydrologique en Afrique de l'Ouest et centrale, 323–337. IAHS Publ 252. IAHS Press, Wallingford, UK.

Doudja, S-G, Nour, Ed & Abd, E. M. B. (2007) Simulation de la pluviométrie journalière en zone semi-aride par l'analyse en composantes principales. *Sécheresse* 18(2), 97–105.

Durand, J. & Guiral, D. (1994) *Hydroclimat et hydrochimie. Environnement et ressource aquatique de Côte d'Ivoire. (Tome 2).* Les milieux lagunaires. *Mémoire ORSTOM, Numéro 50.* Paris, 129–136.

Girard, G., Sircoulon, J. & Toucheboeuf, P. (1971) *Aperçu sur les régimes hydrologiques*. In: *Le milieu naturel de côte d'ivoire Mémoire ORSTOM* 50,109–155.

Goula, B. T. A, Savané, I., Konan, B., Fadika, V. & Gnamien, K. B. (2006) Impact de la variabilité climatique sur les ressources hydriques des bassins de N'Zo et N'Zi en Côte d'Ivoire (Afrique tropicale humide). *Vertigo* 7(1), 1–12.

James, E. & Caskey, J. R. (1963) A Markov chain model for the probability of precipitation occurrence in intervals of various length. *Monthly Weather Review* 91(6), 293–297.

Jan, L., Anastassia, B. & Deliang, C. (2008) Modelling precipitation in Sweden using multiple step markov chains and a composite model. *Journal of Hydrology* 363, 42–59.

Javier, M-V. & Lidia, G. (1999) Regionalization of peninsular spain based on the length of dry spells, *International Journal of Climatology* 19, 537–555.

Justin, G. M. (2008) Markov Chain modeling of sequences of lagged NWP ensemble probability forecasts: an exploration of model properties and decision support applications. *Monthly Weather Review* 136, 3655–3669.

Kouassi, K. L. (2007) *Hydrologie, transport solide et modélisation de la sédimentation dans les lacs des barrages hydro-électriques de Côte d'Ivoire : cas du lac de Taabo (Côte d'Ivoire).* Thèse de doctorat, Université d'Abobo-Adjamé, Abidjan, Cote d'Ivoire.

Kouassi K. L., Kouamé K. I., Konan K. S., Angoulo M. S., Demé M. & Meledje N. H. (2013) Two-Dimensional Numerical Simulation of the Hydro-Sedimentary Phenomena in Lake Taabo, Côte d'Ivoire. *Water Resources Management*, 27(12): 4379-4394. doi 10.1007/s11269-013-0417-x.

Laborde, J. P. & Mouhous, N (1998) *Notice d'installation du logiciel HYDROLAB.* Equipe de Gestion et Valorisation de l'Environnement. Université de NICE, France. 43p.

Lazri, M,, Ameur, S. & Haddad, B. (2007) Analyse de données de précipitations par approche markovienne *Larhyss Journal* 6, 7–20.

Mahe, G. & Olivry, J. C. (1995) Variations des précipitations et des écoulements en Afrique de l'Ouest et centrale de 1951 à 1989. *Sécheresse* 6(1), 109–117.

Noufé, D., Lidon, B., Mahé, G., Servat, E., Brou, Y. T., Koli, Bi Z. & Chaléard, J. L. (2011) Variabilité climatique et production de maïs en culture pluviale dans l'Est ivoirien. *Hydrological Sciences Journal* 56(1), 152–167.

Pablo, F. H. A. & Javier, M. V. (2011) Regionalization of the probability of wet spells and rainfall persistence in the Basque Country (Northern Spain). *Int. J. Climatol.* 32, 1909–1920.

Pita, M. F. (2000). Un nouvel indice de sécheresse pour les domaines méditerranéens. Application au bassin du Guadalquivir (sud-ouest de l'Espagne). *Publications de l'Association Internationale de Climatologie* 13, 225–234.

Zhi, L., Francois, B. & Jie, C. (2013) Assessing the applicability of six precipitation probability distribution models on the Loess Plateau of China. *Int. J. Climatol.* 33, doi: 10.1002/JOC3699.