Monte Carlo experiments for uncertainty investigation of glacier melt discharge predictions through surface energy balance analysis

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Abstract The spatial representativeness of point records is a concern in glacier discharge predictions. A Monte Carlo-based global sensitivity approach is used to investigate the predictive uncertainty in the net radiation (Rn) as the major component driving glacier melt in the Bolivian Andes. The Rn is inferred through the Surface and Energy Balance Algorithm, calibrated with point dry-season records monitored on a glacier’s ablation area. High uncertainties are expected in the vicinity of the monitoring station (surface albedo (α) between 0.81 and 0.79, specific melt discharge (SMD) between 72 and 88 L s⁻¹ km⁻²); smaller uncertainties are expected on the glacier boundaries (α between 0.10 and 0.08, SMD between 128 and 143 L s⁻¹ km⁻²). Thus, with the incoming long wave radiation (Rl↓) as the most sensitive model parameter, the spatial variability in α determines the spatial variability in the SMD predictive uncertainties.

Key words tropical Andes; sensitivity analysis; remote sensing

INTRODUCTION
The spatial representativeness of point records that arises from the complexity observed in natural systems, is a concern to mathematical modellers. In watershed numerical modelling (as a valid example), the classical “calibration” as a means to assure the adequacy of a given model has long been brought into question by several hydrological publications. These publications established our limitations in representing complex natural systems (e.g. Beven, 1993; Wagener et al., 2004), emphasizing the need for the uncertainty reduction in predictions to be a primary aim. Thus, similar to watershed hydrology, the investigation of predictive uncertainty is a demanding topic in the prediction of glacier-melt discharge in remote areas, due to limited knowledge of the spatial distribution of surface processes resulting from the low spatial density of ground observations.

Our aim in this study is to investigate the uncertainty in the spatial representativeness of point data recorded on a glacier in the tropical Andes, through a sensitivity analysis approach inspired by the equipollutability concept. The equipollutability idea suggests that, given current levels of knowledge and measurement technologies, rather than a unique representation of a given system, the existence of a universe of behavioural models is likely. Beven (1993) formally introduces the equipollutability idea in the hydrological literature. It became an inspiration for Wagener et al. (2004), who among others provide the basis for Tang et al. (2007) and Soria & Kazama (2011), whose ideas are applied in this study. The approach includes the application of remote sensing techniques as a tool to infer the spatial distribution of surface energy balance processes on glacial formations.

STUDY AREA AND DATA
The investigation is on the Zongo glacier in the Cordillera Real, situated in the tropical Andes in Bolivia. From a water resources engineering perspective, the melt from the ice caps of the Cordillera Real are worth studying because they provide freshwater for nearby ecosystems and for neighbouring urban settlements, La Paz and El Alto (approximate population of 1 000 000 people). The Zongo glacier (approx. 2 km² of ice cover, horizontal view) is a unique source of information for the study of tropical glaciers in the Andes of Bolivia.

The surface energy balance is investigated using point meteorological observations acquired on 26 July 2005 (at 5050 m a.m.s.l.) on the ablation zone of the Zongo glacier. The spatial distribution of the energy balance is inferred from the processing of a Landsat ETM+ 30-m
horizontal resolution scene. The meteorological data are provided by the GLACIOCLIM (Les GLACIers, un Observatoire du CLIMat), and the Landsat scene was obtained from the US Geological Service (USGS) Earth Resources Observation and Science (EROS). The Landsat scene was acquired during the dry season (in the austral winter, 26 July 2005) at 10:30 h (local time). This scene was selected considering the small discrepancy with ground observations as observed in Soria & Kazama (2010). The calibration of the Landsat scene was carried out with the procedure suggested in Chander et al. (2009). The glacier-covered area was calculated from false colour composites (as in Soria & Kazama, 2009). The topographic information at 90-m horizontal resolution is from the Shuttle Radar Topography Mission Digital Elevation Model (SRTM DEM).

METHODS

The net radiation is assumed to be the major source of energy for glacier melt in the Cordillera Real. The instantaneous net radiation at the Landsat sensor acquisition time, $R_n$, is estimated through the Surface Energy Balance algorithm SEBAL (Bastiaanssen, 2000). The $R_n$ is calibrated with the GLACIOCLIM point data. The uncertainty analysis is carried through a Monte Carlo variance-based global sensitivity analysis (Chan et al., 2000).

Energy balance equation and simplifications

The energy available for glacier melt $Q_M$ in W m$^{-2}$ is investigated through equation (1), and the melt depth is calculated with equation (2) (Paterson, 1999):

$$ Q_M = R_s + R_{L,n} + Q_H + Q_{LE} + Q_G + Q_P $$

$$ M = \frac{Q_M \cdot \Delta t}{D_W \cdot \lambda_f} $$

In equation (1), $R_s$ is the net shortwave radiation, $R_{L,n}$ is the net longwave radiation, $Q_H$ is the turbulent sensible heat, $Q_{LE}$ is the turbulent latent heat flux, $Q_G$ is the conductive-energy flux in the snow/ice or subsurface flux, and $Q_P$ is the heat flux supplied by precipitation (Paterson, 1999). In equation (2), $M$ (in m) is the melt depth for a time interval $\Delta t$ due to $Q_M$ in W m$^{-2}$, the density of water $D_W$ is 1000 kg m$^{-3}$, and the latent heat of fusion of ice $\lambda_f$ is 0.334 $10^3$ kJ kg$^{-1}$.

We assume that the calculations of $Q_M$ are well represented by the net radiation $R_n$ (i.e. the sum of $R_s$ and $R_{L,n}$). The points below, (a) to (c), discuss such an assumption.

(a) The $R_s$ and the incoming longwave radiation $R_L$ are the most relevant sources of energy and can not be neglected. The $R_s$ controls the variability of the energy balance during the melt season (which coincides with the glacier accumulation and the glacier ablation seasons), whereas the $R_L$ is the main energy source for melting (Wagnon et al., 1999).

(b) The turbulent fluxes $Q_H$ and $Q_{LE}$ tend to cancel each other during the melt season (the wet season) (Sicart et al., 2008). This observation suggests that $Q_H$ and $Q_{LE}$ can be neglected for analysis conducted during the melt season. For the winter season (the dry season), the relevance of $Q_H$ and the $Q_{LE}$ is high due to the dry air at high elevations. However, for our analysis we assume that such error may not be large, given that average daily melt discharge rates during winter are notoriously low in comparison to melt season discharge rates (e.g. in the year 1999–2000, daily melt discharge in winter was about 10% of the peak melt discharge in the melt season (Sicart et al., 2007).

(c) The $Q_P$ is negligible because precipitation on the glacier always falls as snow (Wagnon et al., 1999). The $Q_G$ is negligible because it is excessively small in melt season; compared to $R_n$ and turbulent fluxes, the $Q_G$ is also small in winter (Wagnon et al., 2009).

Other assumptions for glacier melt estimations

The equilibrium line is at approx. 5250 m a.m.s.l. (Sicart et al., 2007). We assume that the runoff limit is at some distance above the equilibrium line (Rick, 2008). In the ablation zone, it is
Monte Carlo experiments for the uncertainty investigation of glacier melt discharge predictions

assumed that all melt contributes to runoff (Rick, 2008). In the accumulation zone, it is assumed that some melt is retained by refreezing of the percolated melt (Rick, 2008). We assume that $Q_M$ is effective on the glacier portion below 5300 m a.m.s.l.. Above 5300 m a.m.s.l., we assume that melt refreezes before it reaches the glacier catchment outlet.

Estimation of the net radiation through remote sensing

The SEBAL inferences are carried out at the SRTM DEM resolution. The $R_n$ in W m$^{-2}$ is estimated using equation (3), where $\alpha$ is the dimensionless surface albedo, $R_{L\downarrow}$ is in W m$^{-2}$, and $R_{L\uparrow}$ in W m$^{-2}$ is the outgoing longwave radiation. The $\epsilon_0$ is the dimensionless surface emissivity. The $\epsilon_0$ is 0.999 on snow surfaces (Morse et al., 2000). For our analysis, $\epsilon_0$ is considered to be an uncertain parameter.

$$ R_n = (1 - \alpha) R_s + R_{L\downarrow} - (1 - \epsilon_0) R_{L\uparrow} $$  \hspace{1cm} (3)

The narrow band albedo ($\alpha_{TOA}$) is transformed into $\alpha$ using equation (4) (Bastiaanssen, 2000), where $\omega$ is a dimensionless weighting coefficient, $\rho$ is the dimensionless planetary reflectance at the top of the atmosphere, each Landsat band is denoted as $\Lambda$, $\alpha_p$ is the sun radiation reflected from the atmosphere, and $\tau_{SW}$ is the dimensionless atmospheric transmissivity of clear skies. The standardized mean solar exo-atmospheric spectral irradiance $ESUN$ in W m$^{-2}$ $\mu$m$^{-1}$ is used to calculate $\omega$ (equation (5)) (Chander et al., 2009). The $\alpha_p$ is assumed to be around 0.03 (Morse et al., 2000). For our analysis, $\alpha_p$ is considered an uncertain parameter. The $\tau_{SW}$ is estimated with equation (6) (Bastiaanssen, 2000), where $z$ is the SRTM DEM surface elevation in m a.m.s.l. Further uncertain topographic corrections are neglected (Riaño et al., 2003).

$$ \alpha = (\alpha_{TOA} - \alpha_p) \tau_{SW}^{-2} - \left[\Sigma(\omega A \cdot \rho A) - \alpha_p\right] \tau_{SW}^{-2} $$  \hspace{1cm} (4)

$$ \omega_A = \frac{ESUN_A}{\Sigma(ESUN_A)} $$  \hspace{1cm} (5)

$$ \tau_{SW} = 0.75 + 2 \cdot 10^{-5} \cdot z $$  \hspace{1cm} (6)

$Rs$ in W m$^{-2}$ is estimated with equation (7) (Morse et al., 2000), where the solar constant $Gsc$ is 1367 W m$^{-2}$, $90 - \beta$ is the sun elevation angle, $d^2$ is the Earth–sun distance in astronomical units:

$$ Rs = Gsc(90 - \beta) \tau_{SW} \cdot d^2 $$  \hspace{1cm} (7)

The $R_{L\uparrow}$ and $R_{L\downarrow}$ in W m$^2$ are estimated with the Stefan-Boltzmann Law using equations (8) and (9), respectively (Morse et al., 2000), where $Ts$ (in K) is the surface temperature, $Ta$ (in K) is the absolute air temperature at the reference height, and the Stefan-Boltzmann constant $\sigma_{SB}$ is $5.67 \cdot 10^{-8}$ W m$^{-2}$ K$^{-4}$. The $Ts$ is estimated from the brightness temperature detected by the sensors $Tb$ and $\epsilon_o$ using equation (10) (Chander et al., 2009), where $K1$ and $K2$ in W m$^{-2}$ sr $\mu$m$^{-1}$ are Landsat calibration constants, and $L_A$ in W m$^2$ sr $\mu$m$^{-1}$ is the spectral radiance at the sensor’s aperture. The $\epsilon_{eff}$ is the non-dimensional effective atmospheric emissivity (about 0.7 on snow and ice covered surfaces, Morse et al., 2000). For our analysis, $\epsilon_{eff}$ is considered an uncertain parameter.

$$ R_{L\uparrow} = \epsilon_o \sigma_{SB} T_s^4 $$  \hspace{1cm} (8)

$$ R_{L\downarrow} = \epsilon_{eff} \sigma_{SB} T_a^4 $$  \hspace{1cm} (9)

$$ T_s = \frac{1}{\epsilon_o} T_b = \frac{1}{\epsilon_o} \frac{K2}{\ln(K1/L_A + 1)} $$  \hspace{1cm} (10)

$Ta$ may be approximately equal to $Ts$ on areas where most of the energy is spent on sublimation (e.g. Morse et al., 2000). For our analysis in winter season, the latter mentioned assumption may be erroneous, because of which the spatial distribution of the $Ta$ is inferred under the assumption that the temperature gradient between $Ts$ and $Ta$ at the observation site is constant along the entire surface of the glacier.
Uncertainty analysis, numerical experiments, and uncertain model parameters

Variance-based techniques have the advantage for interpreting the uncertainty contributions of mutual parameter interactions to the total output variance. In summary, the technique estimates the contributed variance of \( u \) model parameters (each denoted by sub indices \( i,j,...k \)) to model output \( Y = f(u_i, u_j, ..., u_k) \). After theoretically decomposing the total variance of the model output \( V(Y) \) into summands of decreasing dimensions, sensitivity indices measure the relevance of the parameter contribution to total variance (Chan et al., 2000). Each term on the decomposition is computable by Monte Carlo integrations (see Chan et al., 2000). In this analysis, the interpretation of the numerical experiments is carried through the total order index \( S_{Ti} \) at every grid cell. The \( S_{Ti} \) denotes the main effect of parameter \( u_i \) as well as its interactions, and it is interpreted as the expected percentage of variance that remains if all parameters were known but \( u_i \) (Chan et al., 2000). As an importance measure, \( S_{Ti} \) evaluates the importance of a parameter as the percentage of the output variance associated with it (Chan et al., 2000). Equation (11) calculates the \( S_{Ti} \), where \( V_i \) denotes the influence on the variance of all the factors except \( u_i \). For details on applications, the reader is referred to Tang et al. (2007) and Soria & Kazama (2011):

\[
S_{Ti} = (1 - V_i) \cdot \left[ V(Y) \right]^{-1}
\] (11)

For our analysis we carried out 1024 experiments with a sample size of 128. The uncertain parameters are those that could be calibrated. Three model parameters were tested in the sensitivity analysis of the net radiation equation: \( \alpha \) on behalf of the \( \alpha \) and \( Rs \) component, \( \varepsilon_0 \) on behalf of the \( RL \uparrow \) component, and \( \varepsilon_{eff} \) on behalf of the \( RL \downarrow \) component. The uncertainty bounds assumed for each uncertain parameter are the same for each grid cell. Table 1 summarizes the uncertain parameters and the corresponding uncertainty bounds. The Sobol quasi-random sequence for non-correlated parameters is used to generate the sample (Chan et al., 2000), 1024 values of \( M \) are calculated, and the set of \( M \) values is interpreted through the \( S_{Ti} \) calculated for each grid cell.

<table>
<thead>
<tr>
<th>Model component</th>
<th>Model parameter</th>
<th>Uncertainty range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>( \alpha_{TOA} )</td>
<td>None (non-calibratable parameter)</td>
</tr>
<tr>
<td></td>
<td>( \omega_{\alpha} )</td>
<td>None (non-calibratable parameter)</td>
</tr>
<tr>
<td></td>
<td>( \alpha_p )</td>
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<td>( z )</td>
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</tr>
<tr>
<td></td>
<td>( \tau_{SW} )</td>
<td>None (non-calibratable parameter)</td>
</tr>
<tr>
<td>( Rs )</td>
<td>( G_{sc} )</td>
<td>None (non-calibratable parameter)</td>
</tr>
<tr>
<td></td>
<td>( 90 - \beta )</td>
<td>None (non-calibratable parameter)</td>
</tr>
<tr>
<td></td>
<td>( d^2 )</td>
<td>None (non-calibratable parameter)</td>
</tr>
<tr>
<td>( RL \uparrow )</td>
<td>( \sigma_{SB} )</td>
<td>None (non-calibratable parameter)</td>
</tr>
<tr>
<td></td>
<td>( \varepsilon_0 )</td>
<td>0.900 to 0.999 (calibratable parameter)</td>
</tr>
<tr>
<td>( TS )</td>
<td>( Tb )</td>
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</tr>
<tr>
<td></td>
<td>( K1 ) and ( K2 )</td>
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</tr>
<tr>
<td></td>
<td>( L_A )</td>
<td>None (non-calibratable parameter)</td>
</tr>
<tr>
<td></td>
<td>( Ts )</td>
<td>Function of ( \varepsilon_{eff} )</td>
</tr>
<tr>
<td>( RL \downarrow )</td>
<td>( \varepsilon_{eff} )</td>
<td>0.6 to 1.0 (calibratable parameter)</td>
</tr>
<tr>
<td></td>
<td>( Ta )</td>
<td>None (non-calibratable parameter)</td>
</tr>
</tbody>
</table>

\( \alpha \): surface albedo; \( \alpha_{TOA} \): albedo at the top of the atmosphere; \( \omega_{\alpha} \): weighting coefficient for \( \alpha_{TOA} \); \( \alpha_p \): sun radiation reflected from the atmosphere; \( z \): surface elevation; \( \tau_{SW} \): atmospheric transmissivity; \( Rs \): incoming shortwave radiation; \( G_{sc} \): solar constant; \( 90 - \beta \): sun elevation angle; \( d^2 \): Earth–sun distance; \( RL \uparrow \): outgoing longwave radiation; \( \sigma_{SB} \): Stefan-Boltzmann constant; \( \varepsilon_0 \): surface emissivity; \( Ts \): surface temperature; \( Tb \): at-sensor brightness temperature; \( K1, K2 \): calibration constants; \( L_A \): spectral radiance at the sensor’s aperture; \( RL \downarrow \): incoming longwave radiation; \( \varepsilon_{eff} \): effective atmospheric emissivity; \( Ta \): absolute air temperature.
RESULTS

The uncertainty analysis summarized in Fig. 1 compares the $S_{Ti}$ calculated for $\epsilon_0$ and $\epsilon_{eff}$ and the $\alpha$ values. Changes in $\alpha_p$ show a very low sensitivity on the $\alpha$ component, because of which the corresponding results are not presented in Fig. 1. The sensitivity of $\epsilon_{eff}$ is higher compared to the sensitivity of the other two model parameters tested, which not only suggests the dominance of the $R_{L\downarrow}$ on the sensitivity of the net radiation calculations over the Zongo glacier during the winter season, but interestingly reveals the high predictive uncertainty expected on melt estimates over the ablation area around the region where the monitoring station is installed (marked as a white circle on the central region of the glacier in Fig. 1). However, the results show that in the region where the highest uncertainties in net radiation predictions are likely, low rates of specific melt discharge (SMD) should be expected because of the high surface albedo (the calculated SMD uncertainty range in the central zone of the ablation area is 72–88 L s$^{-1}$ km$^{-2}$).

As the surface albedo decreases in the vicinity of the lateral moraines and the glacier terminus, it also decreases the predictive uncertainty of the net radiation, as well as the sensitivity of the net radiation component terms. Simultaneously, as the surface albedo values decrease, the rates of predicted SMD increase considerably; in consequence, it increases the relevance of a predictive uncertainty that is apparently smaller than the predictive uncertainty observed in the glacier zone with high surface albedo (the calculated SMD uncertainty range on the boundaries of the glacier is in the range 128–143 L s$^{-1}$ km$^{-2}$).

In the melt season, the coincidence with the accumulation season should cause the glacier surface area having high albedo to grow; in consequence, the areal extent of the glacier where high melt predictive uncertainties are expected should also increase. However, considering that the melt rates on the region with high albedo are likely to be lower than the melt rates on the glacier boundaries, the overall uncertainty in melt predictions during the melt season should be expected to be smaller than the overall uncertainty expected in melt predictions during the dry-winter season over the glacier caps of the Cordillera Real.

![Fig. 1 Spatial distribution of the dimensionless total order index $S_{Ti}$ on the Zongo glacier calculated for 26 July 2006 (10:30 h, local time). The figures correspond to the: (a) surface emissivity $\epsilon_0$; (b) effective atmospheric emissivity $\epsilon_{eff}$; and (c) spatial distribution of the surface albedo $\alpha$ for the Monte Carlo run with the highest calculated values. The white circles indicate the approximate location of the monitoring station; white triangles indicate the pixels where the smallest (small $S_{Ti}$) and the largest uncertainties (large $S_{Ti}$) are expected. Also shown are the ranges of specific melt discharge values (SMD) in L s$^{-1}$ km$^{-2}$ calculated on the latter mentioned locations.](image-url)
CONCLUSIONS

The dominance of the albedo on the estimation of the spatial distribution of the net radiation is a known fact that has been demonstrated throughout our results. From the perspective of the net radiation investigation, the highest net radiation predictive uncertainties should be expected in glacier areal portions with higher surface albedo. Conversely, the numerical experiments described reveal that, when the aim is the calculation of glacier melt, a high predictive uncertainty should be expected over the glacier portions with low surface albedo, because those areal portions determine the areas with higher melt potential. For glacier melt estimations on the central part of the glacier with high surface albedo, the high predictive uncertainty may be attenuated by the low potential discharge associated with a high surface albedo. The conclusions also apply for assessing the installation of a monitoring network, because a more dense measuring network would be desired on the glacier areal portions that drive higher uncertainties in the predictions.

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