Physically-based groundwater vulnerability assessment using sensitivity analysis methods

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Abstract Management of water resource systems requires adequate decision making to protect the water-related functions of fundamental importance to human life, ecosystem preservation and economic development. Groundwater vulnerability assessment studies are useful tools for land-use planning and groundwater protection. A generalized physically-based method using numerical models of groundwater flow is proposed for quantifying the impact on groundwater resources to external pressures, in terms of both quantity and quality. The proposed method is based on the definition of groundwater state sensitivity and groundwater vulnerability coefficients. The vulnerability coefficient is defined as a ratio that reflects the “distance” between the current state of degradation of the water resource system and the “damaged state”. Different numerical methods are proposed to compute the sensitivity coefficients. The uses of these concepts in risk assessment for groundwater resources are discussed and the computation algorithms are illustrated using a simple, yet insightful case study.

Key words vulnerability; sensitivity; physically-based; artificial recharge; risk assessment

INTRODUCTION

Vulnerability assessment studies are becoming increasingly popular as they are useful tools for land-use planning and related decisions about groundwater protection. The most popular are index-based overlay methods such as DRASTIC, EPIK and GOD used to produce a grided vulnerability map. The major advantage is their simplicity and few data requirements (Neukum et al., 2008). However, there are many similar techniques proposed in the literature creating confusion as their respective results, on a same case study, can be very dissimilar. As a need to verify the results, physically-based methods are now proposed (e.g. Frind et al., 2006). While most groundwater vulnerability assessment methods and studies have focused on contamination issues, it is now recognized that there are several other pressures that are likely to threaten the groundwater system such as predicted changes in precipitations and groundwater recharge.

We present a general and physically-based method for evaluating and quantifying the potential impact on groundwater resources of external pressures, both in terms of quantity and quality, using numerical models of groundwater flow. The proposed method is based on the definition of groundwater state sensitivity coefficients and vulnerability coefficients. The concept of sensitivity relies on the definition of physically-based indicators of changes for the groundwater state as affected by pressures. It is extended to vulnerability by introducing a new approach borrowed from socio-economical sciences that involve the concept of likelihood of falling below a threshold.

Sensitivity analysis methods are traditionally used for automatic calibration or inverse modelling, to assess the sensitivity of a model to its parameters and boundary conditions (Sykes et al., 1985), for uncertainty analysis (Jyrkama & Sykes, 2006) and for optimization. Here, the sensitivity analysis is used as a tool for decision making and aquifer management support; we assume that a numerical model is already calibrated and we use sensitivity analysis methods to evaluate the vulnerability of the state of the system to local variations in external pressures. The emphasis is put on the choice of a sensitivity analysis method with respect to the management objectives in order to generate insightful vulnerability coefficients while minimizing the computational burden. Different numerical methods are proposed to compute the sensitivity coefficients.
METHODS

Underlying concepts

Development of human activities poses a threat to functions of fundamental importance to human life, ecosystem and economic development, assured by groundwater systems. They represent first elements of causal links which lead to quantity and quality stresses on groundwater reservoirs. Prior to the development of a groundwater vulnerability assessment method (GWVA), it is necessary to identify which are the stress factors that will be considered and which are the affected characteristics of the groundwater resource. A general framework adopted by the European Community is the Drivers-Pressures-State-Impacts-Responses (DPSIR) approach (Kristensen, 2004) that describes the interactions between society and the environment. It is used to analyse the relationships between stress factors, groundwater characteristics and resulting possible impacts. It generalizes the concept of groundwater vulnerability in a greater diversity of settings by including any kind of stress factors that can affect groundwater. This is further refined by splitting up the groundwater system (i.e. state elements) to define physical components of the state related to the pressures and the impacts to be used in a mathematical equation. This allows us to develop a conceptual model that relates the DPSIR approach with a systematic and physical representation of the groundwater system. The State component of the DPSIR chain is identified explicitly by: (1) the physical factors that relates the state variables to the pressures, called “state upstream factors” (e.g. groundwater recharge, well abstraction), and (2) the physical factors that relate the state variable with the impacts, called “state downstream factors” (e.g. reduction in baseflow, change in groundwater levels) (Fig. 1).

![Fig. 1 Schematic presentation of the generalized concepts of GWVA in the DPSIR framework.](image-url)

Refinement within the State component of the DPSIR chain allows us to make a further distinction between groundwater resource vulnerability (GRV) and groundwater source vulnerability (GSV). GRV reflects the vulnerability of the whole aquifer with respect to a given pressure while GSV is the vulnerability of specific components of the groundwater system with respect to given pressures (generally with a specific location in space), such as a pumping well or discharge gallery (Fig. 1).

Starting from the observation that there are others threats to the groundwater resource than local or diffuse contamination sources, an extension of the physically-based groundwater vulnerability assessment approach presented in Brouyère et al. (2001) and Popescu et al. (2004) is proposed to any kind of stress factors: The concept of groundwater vulnerability reflects the natural mechanisms and processes that make the aquifer more or less sensitive to any kind of pressure. Using the P-S-I causal chain, the concept of groundwater vulnerability is generalized to any kind of stress factor by reflecting, the easiness that the groundwater system (the “state”) leads to pressures into impacts, or, using the terminology defined earlier: the easiness that changes in the “upstream factors” transmit changes in the “downstream factors”, whatever the kind of pressure and resulting impact, thus based on the groundwater system properties only.
A general methodology for groundwater vulnerability assessment

The most general methodology consists in evaluating the sensitivity $S_{ij}$ of impact $(I)$ "$i$" (e.g. decrease of water availability) to pressures $(P)$ "$j$" (e.g. climate change) within equation (1):

$$S = S_{ij} = \frac{\partial I_i}{\partial P_j}$$  \hspace{1cm} (1)

This very general and specific relationship can be made clear to the case of GSV and GRV. They allow evaluating how a change in a given upstream factor $(UF)$ "$j$" (e.g. changes in groundwater recharge over the basin) has knock-on effects on downstream factors $(DF)$ "$i$" (e.g. baseflow to rivers) or groundwater state component $(ST)$ "$i$" (e.g. safe yield), respectively:

$$S = S_{ij}^{GS} = \frac{\partial DF_i}{\partial UF_j} \quad \text{or} \quad S = S_{ij}^{GR} = \frac{\partial ST_i}{\partial UF_j}$$  \hspace{1cm} (2)

where $S_{ij}^{GS}$ is the groundwater source sensitivity and $S_{ij}^{GR}$ the groundwater resource sensitivity. The larger $S_{ij}$, the more sensitive is the groundwater state, thus potentially vulnerable in the sense that it will transmit more easily a pressure (upstream factor $UF_j$) to an impact ($-DF_i$).

Fig. 2 Flowchart presenting the main steps of the general methodology for GWVA.

The generalized definition of groundwater vulnerability emphasizes the sensitivity of the groundwater resource/source to pressures. However, it does not take into account the margin available between the present state of the groundwater resource/source and the critical state from where it should be damaged. Luers et al. (2003) extended the concept of sensitivity ($S$) as defined above to a definition of system vulnerability ($V$) by integrating a ratio ($D_w$) that reflects the “distance” between the current degradation of the water system and its “damaged state”:

$$V = f(S, D_w) = \left[ \frac{\partial W}{\partial X} \right] \left[ \frac{W_0}{W} \right]$$  \hspace{1cm} (3)

where $V$ varies according to sensitivity $(S)$ and state relative to a threshold $(D_w)$. The sensitivity is the absolute value of the derivative of well-being, $W$, with respect to the stress factor, $X$, i.e. $|\partial W/\partial X|$. $W$ is the current state degradation, i.e. equivalent to the downstream factor $DF_i$ or state variable $ST_i$ depending on whether GSV or GRV is targeted, and $W_0$ is the threshold of well-being over which the system is said to be damaged. $|W/W_0|$ is precisely the ratio $D_w$ that reflects the “distance” between the current degradation of the water resource and the “damaged state”. $W_0$ may represent a “minimal” acceptable value (e.g. lowest acceptable groundwater level). In that case:

$$D_w = \left| \frac{W}{W_0} \right|$$  \hspace{1cm} (4)
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In summary, the GWVA is as follows: in a DPSIR analysis, stress factor, affected characteristics of the groundwater resource/source and the relationships are identified. Next, the physically-based indicators have to be defined, i.e. variables that are able to quantify the sensitivity of the changes in the: (1) upstream factors, (2) groundwater state variables (e.g. GW quantity), and (3) downstream factors. Their reduction enables calculation of sensitivity coefficients. Finally, the definition of a threshold enables calculation of vulnerability coefficients that depends on the site-specific conditions and consists of a maximal/minimal acceptable value of well-being (Fig. 2).

**Sensitivity analysis method**

The key elements of the GWVA methodology are the derivatives that express the sensitivity of groundwater state variables or downstream factors to upstream factors. Three methods to compute the sensitivity coefficients have been implemented into HydroGeoSphere – HGS (Therrien et al., 2007). The first two methods consist of a differential approach, which relies on the derivative of the performance measure with respect to the system parameter/boundary conditions. The second approach is a variational approach, also known as the adjoint operator method.

In the perturbation method (PM), the parameter of interest is manually perturbed and the sensitivity coefficient of, for instance, hydraulic heads $h_i$ with respect to parameter $\alpha_k$ is obtained:

$$\frac{\partial h_i}{\partial \alpha_k} = \frac{h_i(\alpha_k + \Delta \alpha_k) - h_i(\alpha_k)}{\Delta \alpha_k} \quad i = 1, N_{\text{nodes}} \quad k = 1, N_{\text{parameters}}; \Delta \alpha_k = \xi \alpha_k, \quad 10^{-3} \leq \xi \leq 10^{-2}$$  \hspace{1cm} (5)

In the sensitivity equation method (SEM), a set of sensitivity equations are obtained by taking the partial derivative equation of the governing equation and boundary conditions with respect to each of the parameters $\alpha_k$. The sensitivity equation is solved in the same way that the primary problem, i.e. fluid continuity equation. For hydraulic head, equation (6) has to be solved:

$$\frac{\partial}{\partial \alpha_k} \left[ \frac{\partial}{\partial x_j} \left( K_{ij} \frac{\partial h}{\partial x_j} \right) + Q_i \right] = 0 \quad (i, j = 1,2,3)$$  \hspace{1cm} (6)

The PM and SEM are most efficient for assessing the GRV. They enable the calculation of a spatially-distributed sensitivity coefficient to a local change in boundary conditions/parameters. For example, the sensitivity of the water table to a change in abstraction at pumping well (Fig. 3).

Fig. 3 Illustration of the two main approaches to calculate the sensitivity coefficients. (a) Theoretical aquifer with water levels given by $h(x)$ in which there are $n$ pumping wells and an observation point $h(x_p)$. (b) Differential approach where the sensitivity of the aquifer water levels are given with respect to a change in pumping rate at well $Q_i$ (resource sensitivity). (c) Adjoint operator approach where the sensitivity of the water level at point $h(x_p)$ is calculated with respect to a change in every pumping well $Q_i$ (source sensitivity).

The adjoint operator method (AOM) uses a measure of the state of the system, called a performance measure and its variation due to a local change in the model parameters is evaluated. This method requires: (1) solving the forward problem, (2) solving the adjoint problem, and (3) calculation of sensitivity coefficients using the solution of the forward and the adjoint problems. The AOM is most appropriate for GSV because the solution is the sensitivity of a selected (and often
local) performance measure, to a spatially-distributed set of changes in the boundary conditions/parameters. For instance, it could be used to obtain the sensitivity of a water level in an observation well to a change in pumping rate at every pumping well in the aquifer (Fig. 3).

While all the methods will provide the same results, the choice is very important when dealing with large models. SEM is more accurate than the PM because no choice about the selection of a perturbation increment has to be made. However, the AOM is most efficient to evaluate GSV. For example, the number of simulations to solve the problem in Fig. 3(c) is 2 (forward problem + adjoint model). With the differential approach it would be, at best, equals to \( n \), the number of wells. In other problems, \( n \) could be orders of magnitude (10^4) larger (i.e. if we consider recharge instead of pumping wells) (Fig. 4(c)). Then, using the differential approach would be inefficient.

**NUMERICAL MODEL**

A synthetic alluvial aquifer with concerns related to water supply has been implemented within HGS. It is derived from Hill & Tiedeman (2007) and is used for calibration, sensitivity, predictions and uncertainty analysis on MODFLOW-2000 and UCODE_2005. The flow system consists of two confined aquifers separated by a confining unit. The lower aquifer presents a mild degree of heterogeneity, while the upper one is homogeneous. This 3-D model emphasizes groundwater management problem, quantity issues and environmental impacts arising in distributed systems (Fig. 4(a)). The GWVA method is demonstrated through the ability of the model to be analysed in terms of relationships between stress factors, groundwater characteristics and possible impacts (Fig. 5).

![Fig. 4](image-url)

Fig. 4 Results with three sensitivity methods: (a) conceptual model: the volume of groundwater in the reservoir (GWV) is essentially recharged by infiltration. Pumping well is being completed in both the upper and lower aquifers to supply hypothetical and various needs. (b) Spatially-distributed sensitivity coefficient of a selected variable (i.e. \( h \)) with respect to a uniform variation of groundwater intake, obtained with the PM and SEM. The variation in the hydraulic head in the pumping well is up to seven times greater than the difference in the infinitesimal change in well discharge rate. (c) Single point sensitivity (AOM): the sensitivity of the water level at well \( p_1 \) with respect to a non-uniform spatially-distributed variation of recharge. (d) Single point sensitivity (AOM): the sensitivity of the water level at well \( p_1 \) and \( p_0 \) is calculated at \( p_1 \) with respect to a change in the pumping well \( p_1 \). Results from (b) and (d) are equivalent, as stated before (same colour bar).
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RESULTS

Three sensitivity analysis methods have been used for the most frequent possible types of P-S-I causal chains. From a DPSIR perspective (Fig. 5), pressure affecting the groundwater resource is mainly related to changes in pumping operations (groundwater intake: $Q_{GWI}$) and also to the natural replenishment of the groundwater resource (groundwater recharge: $Q_{GWR}$). $Q_{GWI}$ and $Q_{GWR}$ are the physical criteria defined as being indicators of changes in the upstream and downstream factors. $\Delta H_{pz}$ is the hydraulic head.

DISCUSSION AND PERSPECTIVES

Physically-based indicators can be proposed for the groundwater state, as affected by pressures. Decisions are required to set a threshold for vulnerability indicators. Each ensemble or spatially distributed indicator has its own significance in terms of decision making. For multidimensional environmental problems in terms of acting pressures and impacts, it may be useful to “merge” all the information provided by the indicators into a single reference indicator, through aggregation and multi-criteria analysis. This could be used to test the efficiency and more specifically the ranking of alternatives (e.g. artificial recharge) in terms of reduction of the vulnerability.

REFERENCES


