

Statistical self-similarity of spatial variations of snow cover and its application for modelling snowmelt runoff generation in basins with a sparse snow measurement network

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Abstract An analysis of snow cover measurement data in a number of physiographic regions and landscapes has shown that fields of snow cover characteristics exhibit statistical self-similarity property. This property is useful when, because of a sparse measurement network, the spatial variability of snow cover can be determined only for large enough basins. Small-scale variability of snow cover can be estimated by scaling of the spatial variance determined for a large basin. A physically-based distributed model of snowmelt runoff generation developed for the Kolyma River and the Sosna River basins has been used to estimate the sensitivity of snowmelt dynamics and flood hydrographs to scaling of maximum snow water equivalent variance. It was shown that this scaling allows improvement of the description of snowmelt dynamics both within small areas and over the entire river basin. The flood hydrographs appeared to be sensitive to scaling of snow water equivalent mainly for small river basins and at certain hydrometeorological conditions.

Key words modelling; self-similarity; snow cover; snowmelt runoff; ungauged

INTRODUCTION

It was established long ago that the spatial variations of snow depths and snow water equivalents for a given area can be considered as random fields of the variables, the areal statistical distributions of which follow lognormal or gamma probability laws. However, these random fields may be strongly heterogeneous, and statistical parameters needed for the construction of probability distributions may vary in space and as a function of the area's size. The number of measurement points needed for reasonable estimation of spatial variance or higher statistical moments is commonly sufficient only for areas which are significantly larger than the grid cells of numerical runoff models or, *vice versa*, only for a small part of these grid cells. Thus, it is necessary to assign the statistical parameters for domains without measurements or to transfer these parameters from the larger to smaller domains.

An opportunity for a solution of this problem is associated with investigating regularities in the stochastic spatial structure of snow characteristics and searching for relationships between variations of these characteristics for different spatial scales.

Significant experience in this field has been accumulated on the basis of the theory of random fields with homogeneous increments in meteorology (Gandin, 1963; Monin & Jaglom, 1967), and in geology resulting in development of geostatistics (Matheron,

1967). On the other hand, the theory of fractals can be successfully used to receive the relationships between statistical parameters of different spatial scales, as has been shown for random fields of elevations (Mark & Aronson, 1984), rainfall characteristics (Lovejoy & Mandelbrot, 1985), and soil constants (Burrough, 1983). Both these approaches, under the specific properties of random fields commonly called statistical self-similarity, enable us to transfer statistical parameters for one scale to statistical parameters for other scales using simple scaling transformation of the random variable. The objectives of this study are the following: (a) to verify the hypothesis of statistical self-similarity for spatial distribution of the snow water equivalent, (b) to use relationships, following from this property, for estimation of the snow water equivalent statistical moments for ungauged or poorly gauged areas, and (c) to estimate the effectiveness of using these relationships in improving the modelling of snowmelt runoff generation.

According to Gupta & Waymire (1990), we shall call statistical self-similarity such a property of a given random variable $S(x)$, when the probability distribution of $S(x)$ within any cell F_k of an area F is the same as the distribution over the whole area F if a scaling transformation of this variable within F_k is made. Such a scaling transformation occurs when the variable $S(x)$ is multiplied by a factor r^H , where r is a constant depending on the ratio of F_k to F and H is a constant depending on a measure of spatial correlation of $S(x)$. In this case, the conditions of the equality of probability distributions of $S(x)$ within the areas F_k and F can be presented as the following relation between the corresponding statistical moments $E[S_k^n]$ and $E[S_F^n]$ of order n :

$$E[S_k^n] = r^{nH} E[S_F^n] \quad (1)$$

When the random field is heterogeneous but the increments $I(h) = S(x+h) - S(x)$ are assumed to be homogeneous and isotropic, it is possible to construct the variograms:

$$\gamma(h) = E[S(x+h) - S(x)]^2 \quad (2)$$

If the variogram of the value of $S(x)$ has the power structure:

$$\gamma(h) = \alpha h^{2H} \quad (3)$$

where α and H are constants, then for the increments with steps of h and rh the following equality can be written:

$$I(rh) = r^H I(h) \quad (4)$$

Determining the statistical moments for both sides of the equation (4), we derive equation (1) and, consequently, the random variables whose variograms have the power structures are statistically self-similar. If $H > 0.5$, the increments of this process are positively correlated and large-scale variations prevail; if $H < 0.5$, the increments are negatively correlated and small-scale variations prevail. As a measure of irregularity of a random surface and correlation of the large-scale and small-scale variations, one can also use the fractal dimension:

$$D = E + 1 + H \quad (5)$$

where E is the topological dimension.

By averaging $S(x)$ and the variances for two embedded circles or squares with the centre at a point X_0 and with areas F_k and F , and taking into account (4), we obtain:

$$m_k - S_0 = r^H (m_k - S_0) \quad (6)$$

$$\sigma_k^2 = r^{2H} \sigma_F^2 \quad (7)$$

where $S_0 = S(x_0)$; m_k and σ_k^2 are the means and the variance of $S(x)$ over the area F_k , respectively; m_F and σ_F^2 are the means and the variance of $S(x)$ over the area F respectively; $r = \sqrt{\frac{F_k}{F}}$. These relationships can be applied for scaling of statistical parameters with changing areas.

VERIFICATION OF THE HYPOTHESIS OF STATISTICAL SELF-SIMILARITY OF SNOW COVER

The available information about snow cover distribution includes two forms of snow cover data: the point measurements along straight-line snow courses and snow cover data received on the basis of the point measurements and averaging the courses measurements. The snow courses are commonly chosen to represent the micro and mesoscale variability of snow cover for different types of landscape and relief. The required length of snow courses varies from 100 m to several km. To analyse the statistical structure of large-scale snow cover fields, it is necessary to have a sufficient number of snow courses inside the area under consideration. The spatial snow density variation is small in comparison with the snow depth, so the fields of the snow depth and the snow water equivalent have similar structure. Taking into account these peculiarities of snow cover measurements we have investigated the statistical structure of snow depth for the snow courses and snow water equivalents for large-scale areas.

Figure 1 shows the variograms and fractal dimensions of snow depth spatial variations obtained on the basis of the snow point measurements in the following

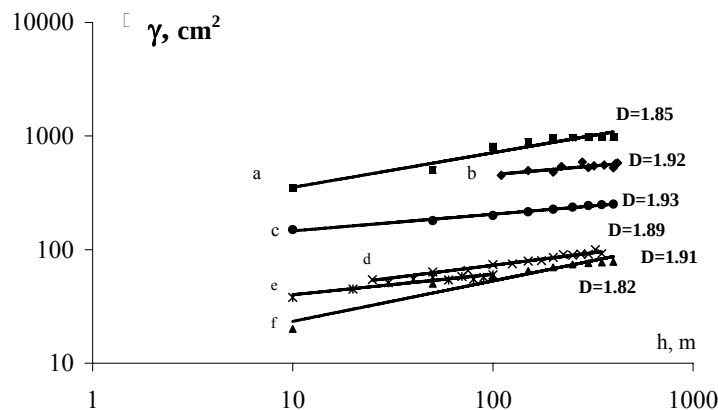


Fig. 1 Variograms of snow depth for scales from tens to hundreds metres: (a) Tien-Shan region; (b) Alaska; (c) Valday region; (d) Oka River basin; (e) Don River basin; (f) Lower Volga region.

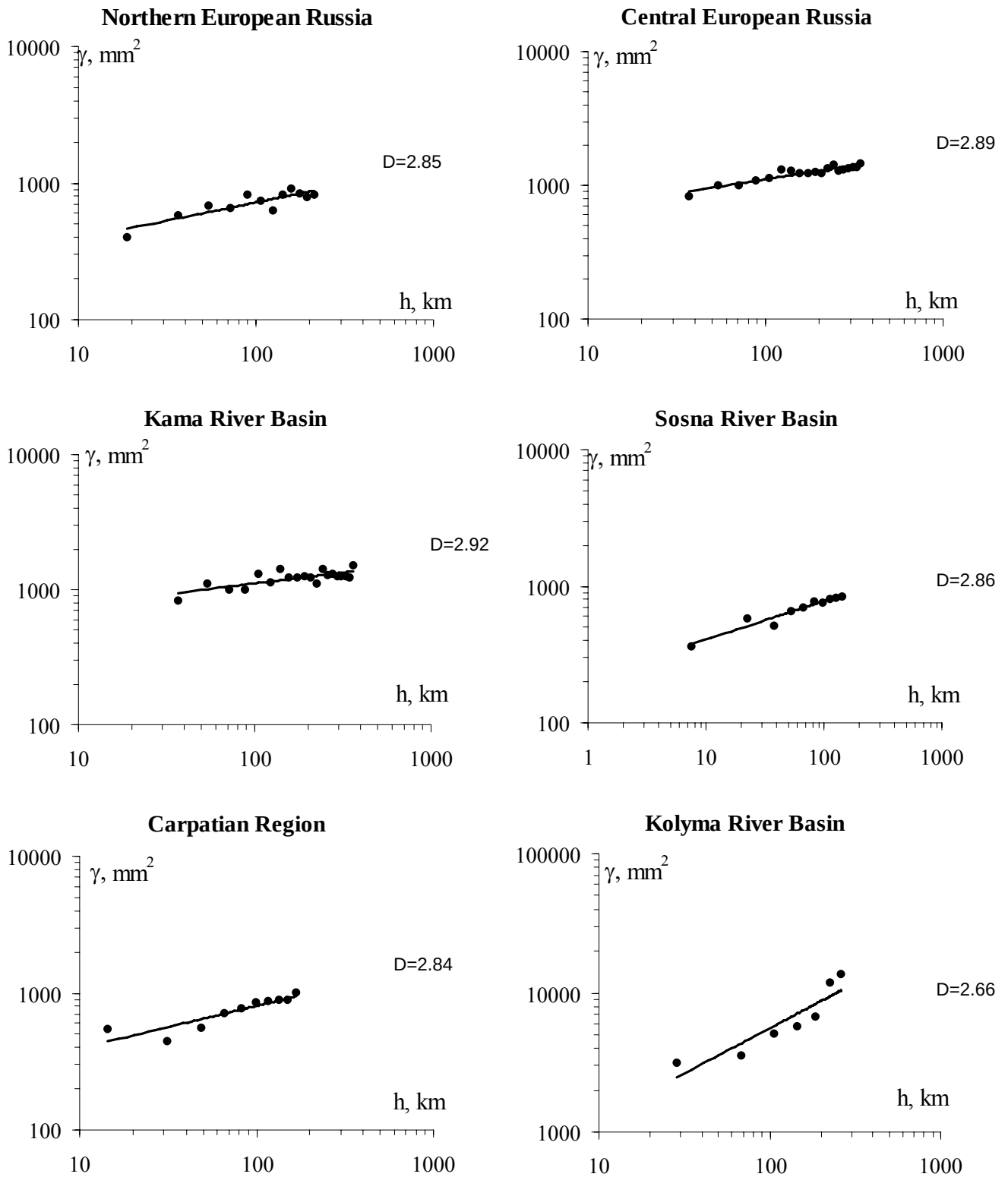


Fig. 2 Variograms of snow water equivalent for scales from ten to hundreds of km.

regions: (1) Valday region (forest zone), (2) Don River basin (forest steppe zone), (3) Lower Volga region (steppe zone), (4) northern Alaska (tundra), (5) Tien-Shan

region (mountain terrain), and (6) Oka River basin. As can be seen from Fig. 1, we can consider the spatial variations of snow depth at chosen snow courses as fractional Brownian processes.

The spatial variograms and the fractal dimensions of the maximum before melting snow water equivalents for six physiographic regions are shown in Fig. 2. These physiographic regions are as follows: (1) the region of 48 000 km² partially includes the drainage area of the North Dvina, Mesen, Pechora and Upper Volga (a plain relief with dominantly forest vegetation); (2) the upper and middle part of the Volga River basin (mainly a plain rugged relief with forest vegetation; the area of the region is 31000 km²); (3) the region is situated in the Kama River basin and occupies 42 000 km² (the upper part of the region is a mountainous area; the middle and lower part of the region is a hilly plain); (4) the region is located in the Don River basin and covers an area of 21 000 km² (rugged plain in the forest-steppe zone); (5) the region covers an area of 34 000 km² in the western part of the Ukraine (the significant portion of the region is the foothills of the Carpathian Mountains with the forest vegetation; the rest of the region is a forest-steppe zone); and (6) the upper part of the Kolyma River basin (the drainage area is 99 400 km²) is a mountainous area covered mostly by tundra and taiga vegetation.

As can be seen from Figs 1 and 2, all variograms in logarithmic coordinates are approximated quite well by linear functions and, consequently, the condition of the self-similarity (3) is satisfied. The calculated values of the fractal dimensions both for the snow courses in different landscapes, and for the physiographic regions, have relatively small differences. The values of H for all variograms are less than 0.5, and, thus, the spatial increments of the snow depth or the snow water equivalent appear to be relatively noisy and the short-range effects in their variations dominate (the sign of derivations from the mean values of these functions often alternates). These anti-persistent properties of snow cover are also typical for the spatial distribution of rainfall (Gupta & Waymire, 1990), and topography (Mark & Aronson, 1984). However, regardless of the dominance of short-range effects, the correlation of increments can extend over arbitrarily large spatial scales.

APPLICATION OF THE HYPOTHESIS OF STATISTICAL SELF-SIMILARITY OF SNOW COVER

In order to estimate the sensitivity of modelled snowmelt runoff to scaling of snow cover spatial variance, distributed runoff generation models developed for the Upper Kolyma River basin (Kuchment *et al.*, 2000) and the Sosna River basin (Kuchment & Gelfan, 1996) were applied.

The first model is based on a finite-element schematization of the river basin and describes the following main processes of runoff generation in the permafrost regions: snow cover formation and snowmelt, thawing of the ground, evaporation, basin storage dynamics, overland, subsurface and channel flow. In the model of the Sosna River runoff generation, the basin was represented by rectangular strips located along the river channels and on which a plane-parallel overland flow occurs. The model describes snow cover formation and snowmelt, thawing and freezing of the ground, infiltration, evaporation, overland and channel flow.

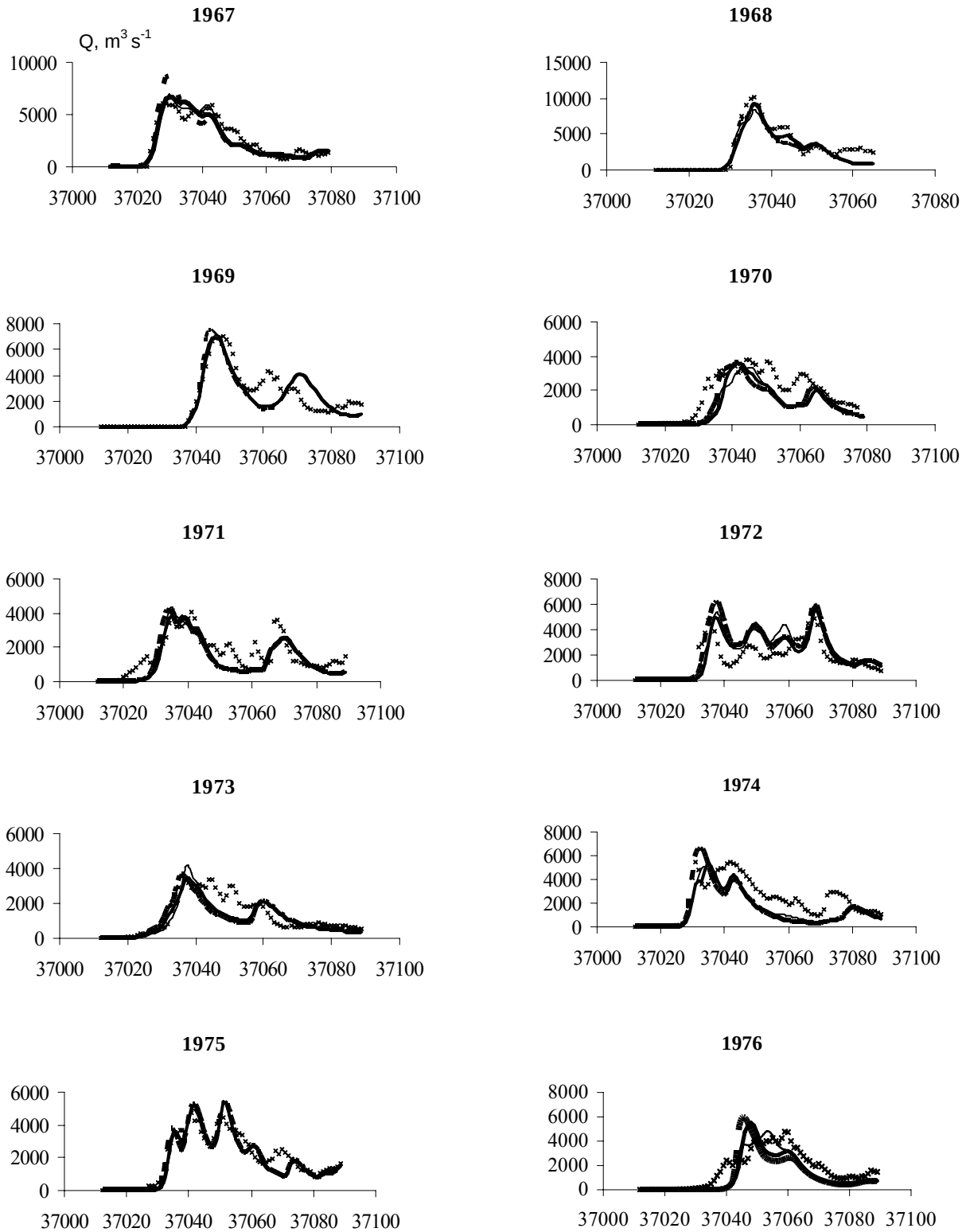


Fig. 3 Observed (points) and calculated hydrographs of the Kolyma River (dashed line: the variance of the initial $S_{\max}$ depends on the size of subgrid areas; bold line: the variance of the initial $S_{\max}$ is assigned as the constant for the whole basin; thin line: the coefficient of variation is assigned as the constant for the whole basin).

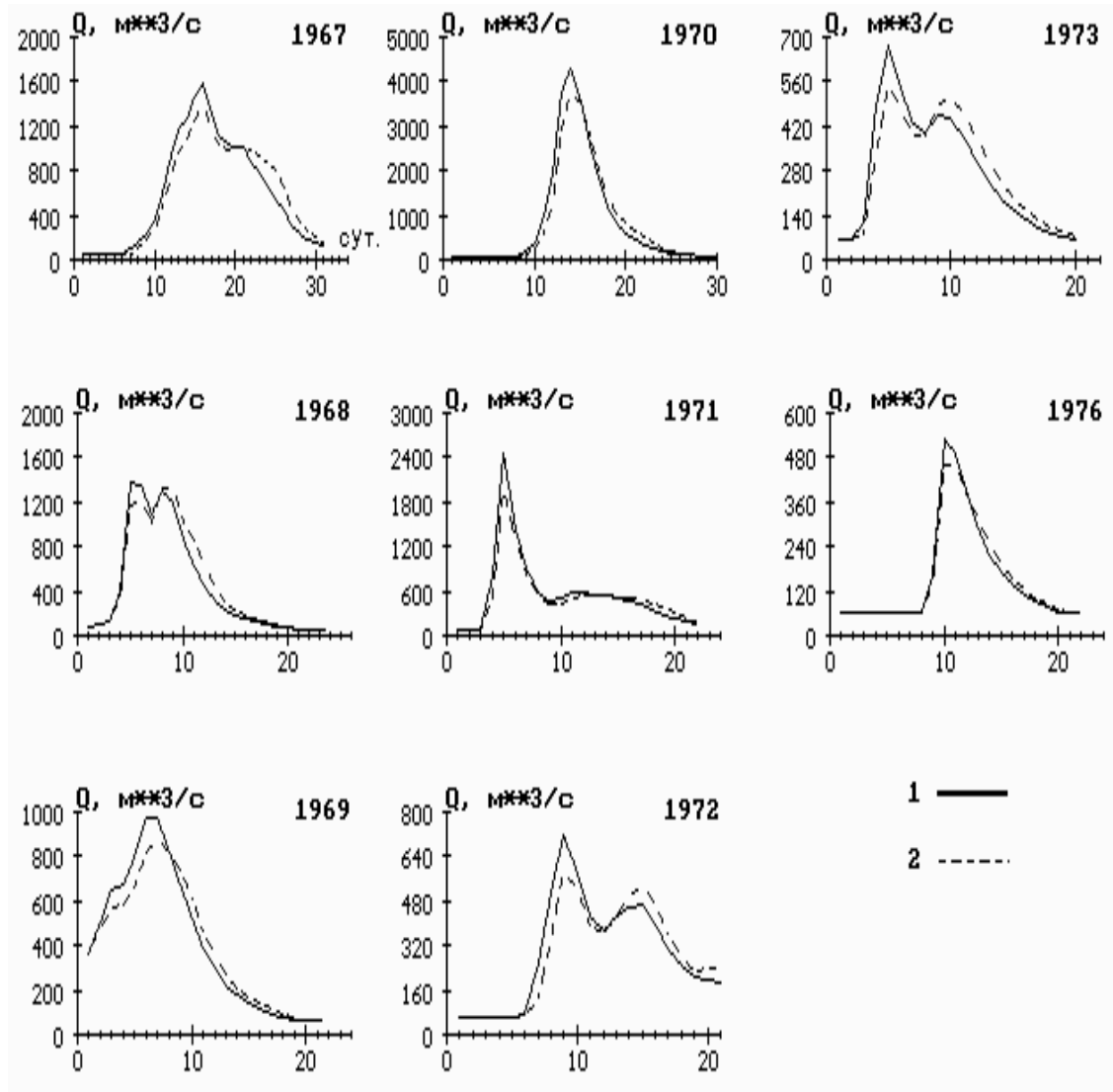


Fig. 4 Calculated hydrographs for the Sosna River (dashed line: the variance of the initial S_{max} depends on the size of subgrid areas; solid line: the variance of the initial S_{max} is assigned as the constant for the whole basin; thin line).

The maximum snow water equivalent S_{max} before the start of snowmelt was assigned as the initial conditions for calculation of runoff hydrographs in both models. The values S_{max} were determined for each grid area with aid of the Thiessen method using the records at 20 snow measurements stations for the Kolyma River and at 30 stations for the Sosna River.

It was assumed that the areal distribution of the initial value of S_{max} within each subgrid area satisfied a lognormal probability distribution. The mean value of S_{max} over the k th subgrid area (m_k) was assigned to be equal to value of S_{max} measured at the closest snow station. The variance of S_{max} over the k subgrid area (σ_k^2) was calculated according to (7).

Three sets of calculations were carried out: (a) the variance σ_k^2 was assumed equal to the measured one and constant for all subgrid areas; (b) the coefficient of variation

was equal to the value calculated over the whole basin and constant for all subgrid areas; and (c) the mean value and variance of S_{max} within the subgrid areas were determined from equations (6) and (7). The scaling of the variance of S_{max} has led to significant changes of the snowmelt dynamics. The decrease of subgrid variance reduces the portion of the subgrid area with large values of the snow water equivalent and the snowmelt completes earlier. For the Sosna River basin, the decrease in the snow cover variance caused by the transfer from the whole basin to the grid area led to a 3-day reduction of the snowmelt period.

Accounting for the spatial non-uniformity of snow cover has led to a 5-day prolongation of the snowmelt period and to some decrease in the maximum rate of outflow from the snow-pack. The decrease in the snow cover variance caused by the transfer from the whole basin to the grid area has led to a 3-day reduction of the snowmelt period.

Differences in flood hydrographs are less obvious because of smoothing due to the large lag times of channel flow (Figs 3 and 4). In most cases there is a perceptible increase of snowmelt peak discharges, when the size of area was taken into account in assigning σ_k . Such increases for the Kolyma basin reach 20% for 1974 and 30% for 1967.

Under some hydrometeorological conditions and for small river basins, this effect may lead to significant changes of the modelled hydrographs. At the same time, as can be seen from Fig. 3, in some cases in order to transfer from statistical parameters of the snow distributions for large areas to statistical parameters for small areas, it is a reasonable assumption that the coefficients of variation of these characteristics do not depend on the size of the area.

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