Recent Developments in the Explanation and Prediction of Erosion and Sediment Yield (Proceedings of the Exeter Symposium, July 1982). IAHS Publ. no. 137.

Erosion initiation

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ABSTRACT Critical conditions for both cohesionless and cohesive materials are analysed in terms of applied power. Power applied by a stream along its bed in maintaining motion is compared to the power required to dislodge bed particles. In the case of cohesionless materials, unique mathematical relationships have been derived for both laminar and turbulent flow conditions. A theoretical relationship has also been established for cohesive materials. This relationship was calibrated with field data for cases of erosion on steep slopes.

INTRODUCTION

A stream will transport sediment only if the critical condition is exceeded. The critical stage is reached when the transporting capacity of a stream equals that which is required to dislodge material from the channel margin. Criteria which indicate whether sediment will be transported under given conditions are very important in erosion studies. The most important pioneering contributions regarding critical conditions are probably those of Hjulström (1935), Shields (1936) and Liu (1957). Considering existing relationships for critical conditions, the authors concluded that:

(a) Critical conditions for cohesionless particles are quantified best in the Liu diagram.

(b) Information on critical conditions for cohesive materials indicate a large degree of scatter e.g. Lane (1953).

(c) Complete logical mathematical description of critical conditions is lacking.

(d) It would be necessary to express both transporting capacity and sediment characteristics in equivalent functions to obtain a unique relationship for critical conditions. This led the authors to investigate erosion initiation in terms of applied unit stream power.

APPLIED STREAM POWER

Consider a uniform one-dimensional stream with depth of flow D and slope s (Fig.1). The power which is applied in maintaining fluid motion is provided through the steady release of potential energy by descending elements. A translating element releases potential energy at the rate of ρgsv per unit volume where ρ = mass density of fluid, g = acceleration of gravity, s = (energy) slope and v = velocity of

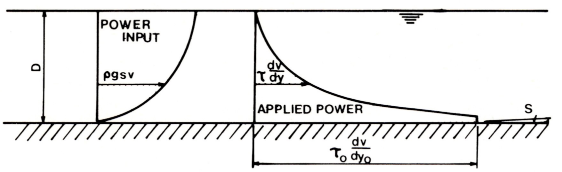

FIG.1 Variation in power input and applied power (one-dimensional flow).

flowing element.

As may be expected, the total amount of applied power equals the total power input (Rooseboom, 1974) i.e. the enclosed areas in Fig.1 are equal. The value of $\tau dv/dy$ (applied unit stream power) reaches a very high maximum near the bed and it is a characteristic of fluid flow that where two alternative modes of flow exist, the mode which requires the least power per unit volume will occur, e.g. a laminar sublayer next to a smooth boundary below turbulent flow conditions (Rooseboom, 1974).

In the case of rough turbulent flow, the velocity profile can be described by the log-deficiency equation:

$$v = \sqrt{gDs}/\kappa \ln y/y_o$$

with v = velocity at a distance y from the bed, D = depth of flow, κ = von Kármán constant, y_o = ordinate where the velocity = 0. The value of y_o is proportional to the absolute roughness (k). The maximum value of $\tau dv/dy$ in this case equals $\rho gsD\sqrt{gDs}/\kappa y_o$ and as the value of $\kappa = 0.4$ and $y_o \simeq k/29.6$ (Rooseboom, 1974) (or d/29.6 in this case), this maximum value for rough turbulent flows becomes:

$$\tau\left(\frac{dv}{dy}\right)_{max} = 74\rho gsD\sqrt{gDs}/k \qquad (1)$$

with s = energy slope, D = depth of flow, d = particle diameter, k = absolute roughness value = d in the case of cohesionless particles forming a smooth bed.

CRITICAL CONDITIONS FOR COHESIONLESS SEDIMENT

In terms of the principle of least resistance, the stream will begin to entrain particles when the power required to suspend the particles becomes less than the power required to maintain the status quo. The power which is required to suspend a particle with settling velocity V_{ss} and mass density ρ_s equals $(\rho_s - \rho)gV_{ss}$ and therefore movement will begin when:

$$(\rho_s - \rho)gV_{ss} \propto 74\rho gsD\sqrt{gDs}/d$$

According to the general equation for settling velocity (Graf, 1971):

$$V_{ss} \propto \left[\frac{(\rho_s - \rho)gd}{\rho C_d}\right]^{\frac{1}{2}} \qquad (2)$$

Assuming that C_d, the drag coefficient, is a constant, which is true for larger diameters, then the condition of incipient sediment motion under rough turbulent flow conditions is depicted by

$$\sqrt{gDs}/V_{ss} = \text{constant} \tag{3}$$

Rough turbulent conditions will prevail while values of $\sqrt{gDs}.d/\nu$ are high and this function is used to define boundary layer conditions as in the Shields and Liu diagrams.

As can be seen in Fig.2, this relationship fits measured data as compiled by Yang (1972) very well. The value of the constant equals 0.12 for values of

$$\sqrt{gDs}.d/\nu > 13$$

Similarly, in smooth turbulent flow as well as in completely laminar flow the unit applied stream power along the bed equals (Rooseboom, 1974)

$$(\rho gsD)^2/\rho\nu$$

The corresponding equation for settling velocity (2) under viscous conditions (Stokes' law) states that

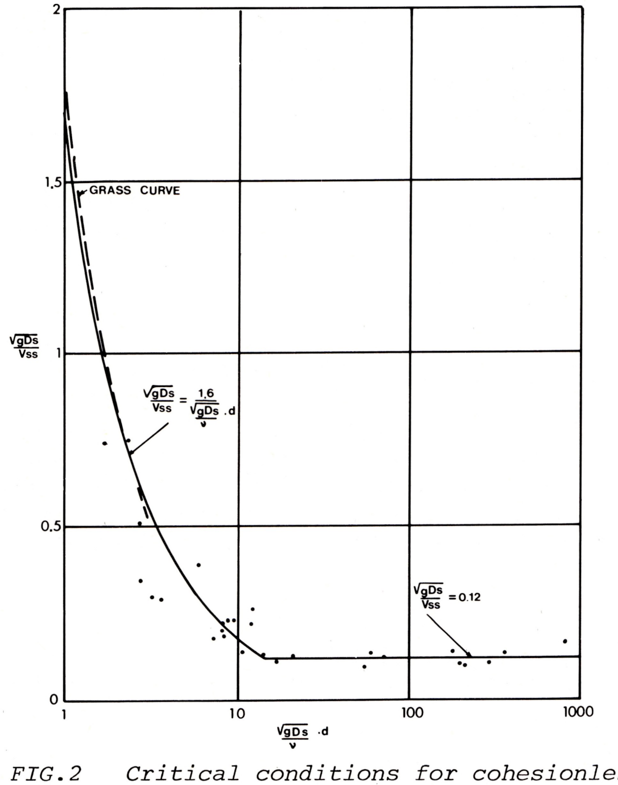

FIG.2 Critical conditions for cohesionless sediment (Measured data from Grass (1970) and Yang (1972)).

$$V_{ss} \propto d^2 g(\rho_s - \rho)/\rho\nu \quad (4)$$

Accordingly, the relationship for values of $\sqrt{gDs}\, d/\nu < 13$, calibrated with data by Grass (1970) and Yang (1972) is found to be

$$\frac{\sqrt{gDs}}{V_{ss}} = \frac{1.6}{(\sqrt{gDs}/\nu)\, d} \quad (5)$$

It is noteworthy that the above analyses logically lead to a Liu-type diagram and provide complete and logical mathematical relationships which describe the shape of this diagram. Particles of the same sieve diameter size but differing shapes require different fluid transporting capacities for entrainment and transportation. As the shape of a particle is reflected in its settling velocity, this variable is superior to particle diameter in sediment transport analyses.

When sediment particles are bound together by cohesive forces, determination of critical conditions becomes more complicated.

CRITICAL CONDITIONS FOR COHESIVE SEDIMENT

The basic problem in defining critical conditions for cohesive sediments remains the same. How does one express both the transporting capacity of a stream and the characteristics of its channel sediments in equivalent terms? Direct comparison becomes possible if both fluid and sediment movement are considered in similar terms. Incipient movement of sediment is therefore treated as flow of a medium with high viscosity. Viscous flow is generally defined in terms of the Newtonian equation:

$$\tau = \mu\, dv/dy \quad (6)$$

with τ = applied shear stress, μ = dynamic viscosity and dv/dy = vertical velocity gradient, equal to the velocity of rotation of a flowing element (Rooseboom, 1974).

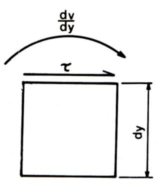

FIG.3 Definition sketch of a rotational element.

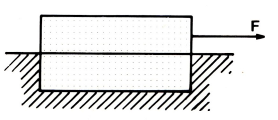

FIG.4 Schematic view of the shear box.

If the velocity of rotation of cohesive particles, which are subjected to shear stresses, can be determined, then the "viscosity" μ_s of the soil would be defined:

$$\mu_s = \tau/(dv/dy) \quad (7)$$

With this value known, it is possible to determine the applied unit power

$$\tau \, dv/dy = \tau^2/\mu_s \qquad (8)$$

For the purpose of determining values of μ_s use was made of a standard shear box apparatus. (Farnell SM8 with shear box size 61.5 x 61.5 mm and loading 52.4 kg.)

The container of the apparatus is filled with soil and the upper section of soil pulled across the lower section at constant speed v_a while the required force (F) is measured.

With the shear stress τ across the soil interface determined, and the translation velocity (v_a) known, the value of μ_s can be determined. As the detached particles rotate individually, the velocity gradient or angular velocity of the particles can be equated to v_a/d, with d = particle diameter. The power which is required to dislodge a layer of particles with thickness d and area A according to equation (8) = (τ^2/μ) Ad, where A = plan area of sample and d = particle size. The power required in the shear box is also = Fv_a and, as v_a is kept constant, F may be taken to represent the applied power. Typical results of shear box tests are shown in Fig.5:

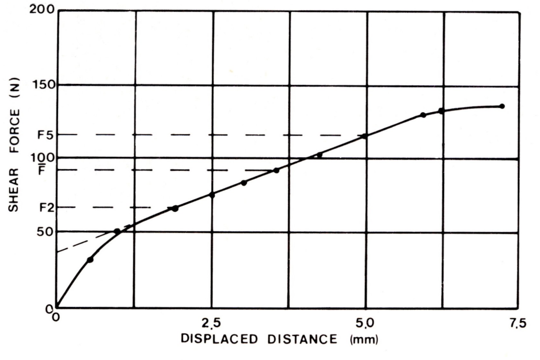

FIG.5 Typical shear box results.

It was generally found that the relationship between shear stress and displaced distance was linear for displacements between 2 and 5 mm. In order to have a common standard it was decided to accept the average of the shear stresses for these displacements as the representative soil shear stress (τ) in equation (8); v_a = a standard value of 1.2 mm min^{-1}.

As in the case of cohesionless material, the power which is applied in maintaining fluid motion along a bed (in rough turbulent flow) is equal to $74\rho gsD\sqrt{gDs}/k$ (equation (1)). The absolute roughness value k represents the size of irregularities on the bed. In the case of cohesive materials this value is generally larger than say the d_{90} particle size. With both the power required to dislodge bed particles (equation (8)) and the power applied by the stream (equation (1)) defined, these values can now be correlated as will be shown in the next section.

CRITICAL FIELD CONDITIONS

The latter part of this study was undertaken specifically to establish criteria for predicting and preventing rainfall induced erosion on steep slopes. Modern railway lines for instance are often built to high standards, resulting in high cuts and fills being formed. Serious erosion problems often develop on these slopes.

Serious erosion typically starts at a specific distance (L) from the top of a slope (Fig.6), where the transporting capacity of the accumulated discharge reaches the critical value.

FIG.6 Initiation of serious erosion.

To be able to quantify the applied stream power it is necessary to determine the depth of flow at this point. Crawford & Linsley (1966) developed a formula for the flow depth (D) under conditions of accumulating discharge (Fig.7). For small values of D, this formula can be reduced to:

$$D = \left[\frac{n\ I\ L^{\frac{8}{3}}}{C\ \ S_o^{\frac{1}{2}}}\right]^{\frac{3}{5}} \qquad (9)$$

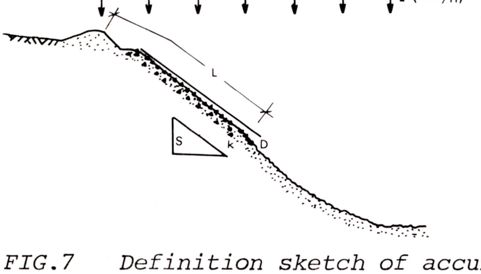

FIG.7 Definition sketch of accumulated runoff on a slope.

with D = depth of flow in m, L = slope length in m, n = applicable Manning roughness coefficient, I = rainfall intensity in mm h^{-1}, S_O = slope and C = empirical coefficient with value = 4.97×10^5 (SI units). The Manning n-value can be expressed in terms of the absolute roughness value (k)

$$n = \frac{k^{\frac{1}{6}}}{26} \quad \text{(Webber, 1971)}$$

The applied stream power along the bed according to equation (1)

$$E = \frac{74 \rho g s_O D \sqrt{g D s_O}}{k} \tag{10}$$

and, with the value of D known from equation (9), it is possible to calculate the value of E.

It would be practically impossible to model erosion conditions on steep slopes in laboratories. As the end results are to be used in field applications anyway it was decided to calibrate the developed formulae in the field. For this purpose it was fortunate that high intensity rainstorms occurred in the Pretoria region during 1978 and that the rainfall intensities were accurately recorded.

Values of L, S, k and I were determined where serious erosion had been initiated by these storms. Samples of the eroding material present were taken and subjected to shear box tests. Thus information was obtained on critical conditions for different soil types which occurred on different constructed slopes. Fig.8 was developed from these results for practical application. The upper quadrants contain the variables required to define applied stream

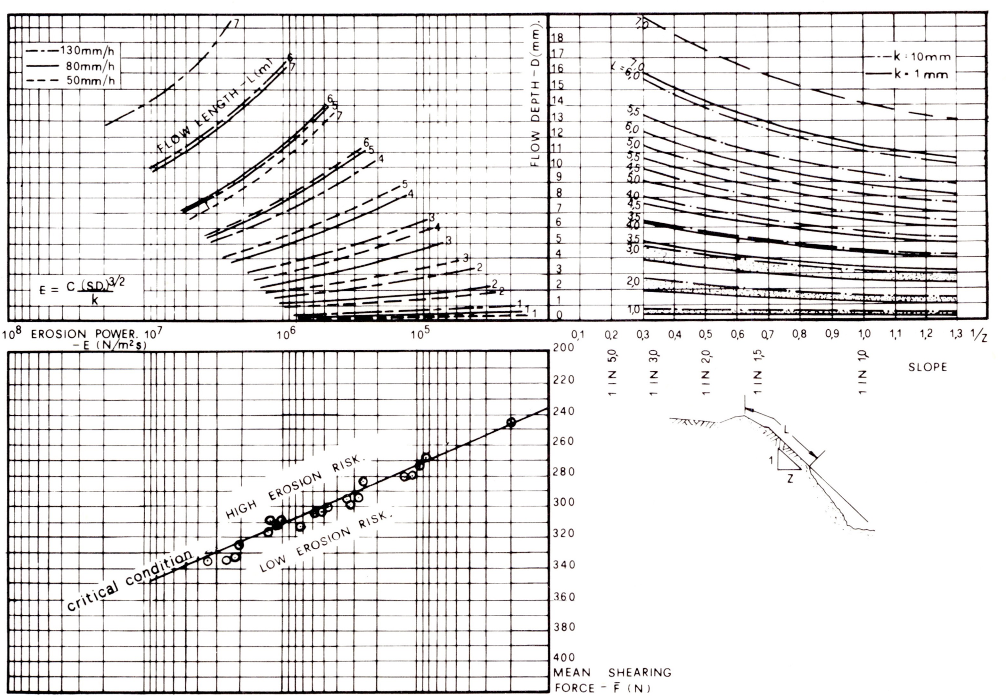

FIG.8 Critical conditions for cohesive soils.

power (E) which is represented along the left hand horizontal axis. As shown before, F represents the power required to dislodge the bed particles.

A definite relationship is obtained between applied stream power and shear force F which confirms the validity of the applied power approach for erosion on steep slopes.

CONCLUSIONS

Analysis of critical erosion conditions in terms of applied stream power leads to well defined relationships. This approach holds much promise for providing universal relationships to describe critical conditions.

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