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Mathematical simulation of erosion on graded terraces

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ABSTRACT A physically-based mathematical model is developed to simulate flow on a graded terrace idealized as an erodible and pervious overland flow plane and a terrace channel. A nonlinear kinematic wave scheme is adopted for routing the water and the sediment discharges through the overland and the channel flows. The erosion and the sediment transport capacities, the erosion rate and the sediment concentration in the overland flow are interrelated through a closed form equation. The transport capacity of the channel is determined by using the unit stream power concept. A finite difference method is employed to solve the partial differential equations of the mathematical model. The model is verified by a set of observed data.

INTRODUCTION

Analytical treatment of terrace flow is difficult, if not impossible, due to the simultaneous interactions of the many processes involved relating to water, soil and the plants. However, with the aid of digital computers and numerical analysis techniques, physically-based mathematical models can be devized to simulate a variety of terrace flow conditions.

From the hydraulic engineering viewpoint, most graded terraces can be idealized as conjunctive flow systems, consisting of an erodible and pervious overland flow plane and a terrace channel constructed perpendicular to the overland flow direction. The major processes and the flow components involved are the rainfall, infiltration, overland flow, channel flow and the detachment, transport and the deposition of soil particles throughout the system as shown schematically in Fig.1. These flow components have been treated individually or in pairs in many sophisticated mathematical models reported in the past. For instance, overland and channel flows on impervious surfaces were studied by Kareliotis & Chow (1971), infiltration by Whisler & Klute (1966), overland flow on pervious surfaces by Akan & Yen (1981), flow in alluvial channels by Chen & Simons (1973), and erosion by surface runoff by Meyer & Wischmeier (1969). The mathematical model introduced here considers all the major components of the terrace flow and accounts for the dynamic equilibrium among them.

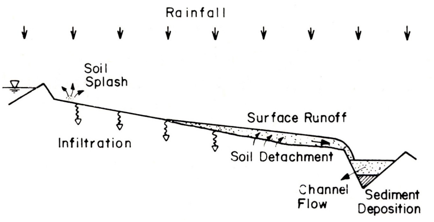

FIG.1 Schematic representation of terrace flow.

DESCRIPTION OF THE MATHEMATICAL MODEL

The proposed model attempts to represent as many components as possible by physically-based equations. However, several parametric equations are also included since no better information is available in the literature describing some of the processes involved in terrace flow.

Rainfall

Rainfall is the main source of water for terrace flow. It also causes soil erosion due to the impact action of the raindrops, especially prior to the commencement of the surface runoff. As proposed by Bubenzer & Jones (1971), the quantity of soil detached and splashed by rainfall is estimated from

$$S_p = m\ (2.78 \times 10^{-7}\ I)^{\alpha}\ k_e^{\beta}\ P_c^{-\lambda} \tag{1}$$

in which S_p is the soil splash in kg m^{-2}, I is the intensity of rainfall in mm h^{-1}, k_e is the total kinetic energy of the raindrops in J m^{-2}, p_c is the percentage of clay in the soil, and the parameters take the values of m = 1.5-3.0, α = 0.25-0.55, β = 0.83-1.49, and λ = 0.40-0.60. As suggested by Wischmeier & Smith (1958) the kinetic energy, k_e, is estimated by using

$$k_e = 24.16\ P + 8.73\ P \log\ (I/25.4) \tag{2}$$

in which the depth of rainfall P is in mm.

Infiltration

The rate of infiltration is computed by employing a set of expressions proposed by Mein & Larson (1971) and verified through the numerical solutions of the Richards equation. The infiltration equations adopted are

$$t_s = (\Psi\ SMD)/\left[I(I/K - 1)\right] \tag{3}$$

$$f_p = K\left[1 + (\Psi\ SMD/F_v)\right] \tag{4}$$

$$F_v = (\Psi\ SMD)/\left[(I/K) - 1\right] + \int_{t_s}^{t} f\ dt \tag{5}$$

where t_s is the time from the start of the rainfall to the saturation of the surface soil, Ψ is the average suction head at the wetting front, SMD is the initial soil moisture deficiency, K is the hydraulic conductivity, f_p is the infiltration capacity which is equal to the rate of infiltration under the conditions examined and F_v is the accumulated infiltration up to time t. The integration in equation (5) is evaluated by using the trapezoidal rule. Equations (3)-(5) are dimensionally homogeneous.

Overland flow

Overland flow is approximated by a nonlinear kinematic wave. The equation of continuity for a unit width is written as

$$(\partial y/\partial t) + (\partial q/\partial x) + (\partial y_s/\partial t) - I + f = 0 \quad (6)$$

where y is the depth of flow, $q = q_w + q_s$ is the combined discharge, q_w and q_s are respectively the water and the sediment discharges, y_s is the depth of surface soil detached, I and f are respectively the rates of rainfall and infiltration, x is the displacement in the flow direction and t is time.

The momentum equation is simplified by assuming the friction slope can be approximated by the bottom slope of the overland flow plane. Hence

$$f^* \, q^2/(8gy^3) = S_O \quad (7)$$

where f* is the Darcy-Weisbach friction coefficient, g is the gravitational acceleration and S_O is the bottom slope. In the proposed model, the friction coefficient is calculated by using an approximate form of the Moody diagram as suggested by Kareliotis & Chow (1971).

The equation of continuity for sediment transport in the overland flow is

$$\partial(c\,y)/\partial t + \partial(c\,q)/\partial x = E_r \quad (8)$$

where c is the volumetric sediment concentration averaged over a flow section and E_r is the rate of soil detachment. By definition, $E_r = (1 - p)\partial y_s/\partial t$ where p is the porosity of the surface soil. The rate of detachment and the sediment concentration in a flow section are interrelated by the closed form equation

$$(E_r/D) + (c\,q/T) = 1 \quad (9)$$

in which D and T are respectively the detachment and the transport capacities of the overland flow. As suggested by Foster & Meyer (1971), D and T are evaluated as $D = C_d\tau^{1.5}$ and $T = C_t\tau^{1.5}$ where $\tau = \gamma y S_O$ is the unit tractive force over the flow bed and γ is the specific weight of the fluid. The coefficients C_d and C_t take the values $C_d = 0.001$-0.5, and $C_t = 0.004$-0.8 as D, T, and τ are respectively evaluated in kg $m^{-2}s^{-1}$, kg $m^{-1}s^{-1}$ and kg m^{-3}.

Overland flow solutions are obtained by using a finite difference technique. At each time step, equations (6) and (7) are solved through the use of an explicit characteristic scheme, and the values of y and q at various flow sections are determined. Then, knowing y and q, the values of c and E_r are calculated from equations (8) and (9) by use of a direct explicit scheme. Total soil loss up to time t is computed as

$$E_{RT} = \int_0^t \int_0^L E_r \, dx \, dt \tag{10}$$

where L is the flow length, and the integral is evaluated numerically. The details of the solution procedure have been given by Ezen (1979) and will not be repeated here.

Channel flow

Flow in the terrace channel is also represented by a nonlinear kinematic wave. Sediment deposition over the channel bed as well as the lifting of the deposited material is allowed. The equation of continuity for the channel flow is

$$(\partial A/\partial t) + (\partial Q/\partial x) + (\partial A_d/\partial t) - q_L = 0 \tag{11}$$

in which A is the flow area, $Q = Q_w + Q_s$ is the combined discharge, Q_w and Q_s are respectively the water and the sediment discharges, A_d is the volume of the sediment deposition per unit length of the channel, q_L is the combined lateral inflow of water and sediment from the overland flow, x is the displacement in channel flow direction and t is time. The direct rainfall input into the channel and the seepage from the channel are neglected.

Similar to the formulation of the overland flow, the friction slope of the channel flow is evaluated by the Darcy-Weisbach formula and equated to the channel bottom slope S_b to yield

$$f^* \, Q^2/(2 \, g \, A^2 \, R) = S_b \tag{12}$$

in which R is the hydraulic radius.

The equation of continuity for the sediment phase in the terrace channel is

$$\partial(c_s \, Q)/\partial x + (1 - p_s)\partial A_d/\partial t + \partial(c_s \, A)/\partial t - q_{sL} = 0$$

where $c_s = Q_s/Q$ is the sediment concentration averaged over a flow section, p_s is the porosity of the sediment deposition over the channel bed and q_{sL} is the lateral sediment inflow from the overland flow plane.

The unit stream power approach of Yang (1972) extended to unsteady flow is employed to evaluate c_s from

$$c_s = 2.02 \, (3.28 \, Q \, S_b/A - 2 \times 10^{-5})^{1.35} \tag{14}$$

where Q is in $m^3 s^{-1}$ and A is in m^2.

Equations (11)-(14) are solved simultaneously by use of a finite difference scheme to determine the channel flow conditions. The solution procedure is similar to that of the overland flow and has been reported in detail by Ezen (1979).

APPLICATION OF THE MATHEMATICAL MODEL

The proposed mathematical model is applied to a hypothetical graded terrace and the results are verified qualitatively. The overland flow plane of the hypothetical terrace has a flow length of 25 m, a width of 100 m and a slope of 0.10. The terrace channel is triangular with side slopes of 1/1 and 1/5. The channel is 100 m long with a bottom slope of 0.04. The parameters employed in the simulation are $m = 3.0$, $\alpha = 0.41$, $\beta = 1.14$, $\lambda = 0.52$, $p_c = 20$, $\Psi = 0.15$ m, $K = 4 \times 10^{-7}$ m s^{-1}, $p = 0.40$, SMD $= 0.30$, $C_d = 0.10$ and $C_t = 0.60$. A rainfall with $I = 64.8$ mm h^{-1} and $t_d = 400$ s is assumed to occur over the hypothetical terrace.

The simulated rainfall hyetograph as well as the computed hydrographs of infiltration, overland flow and channel flow are shown in Fig.2. The slight difference between the areas enclosed under the total flow graphs of the overland and the channel flows implies that sediment deposition occurs in the channel.

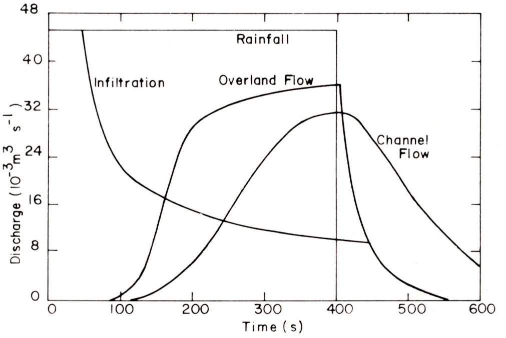

FIG.2 The infiltration and the surface flow hydrographs.

The computed rates of soil detachment by the surface runoff are shown in Fig.3. The first peak around $t = 110$ s represents the lifting of the soil particles already detached by the raindrops. The second peak is due to the increased flow depths as the overland flow develops. This causes a high sediment content requiring a larger portion of the flow energy to be consumed for the sediment transport. So, it is followed by a reduction in the detachment rates. Later, as the heavy sediment load is carried downstream, the detachment rate

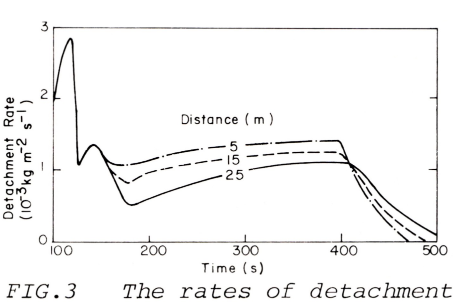

FIG.3 The rates of detachment by the overland flow.

increases again starting from the upstream flow sections until the recession of the overland flow.

Shown in Fig.4 are the water and sediment discharge hydrographs computed at various flow sections of the overland plane. In accordance with the spatially varied flow properties, the discharges increase in the main flow direction.

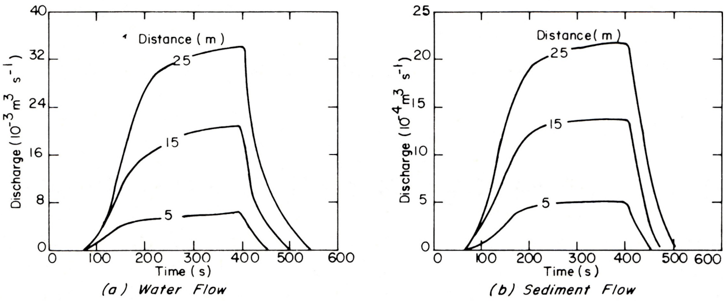

FIG.4 The discharge hydrographs of the overland flow.

The accumulated amount of sediment deposition calculated at various flow sections along the terrace channel is shown in Fig.5. Expectedly, smaller velocities at the upstream flow sections cause larger amounts of deposition.

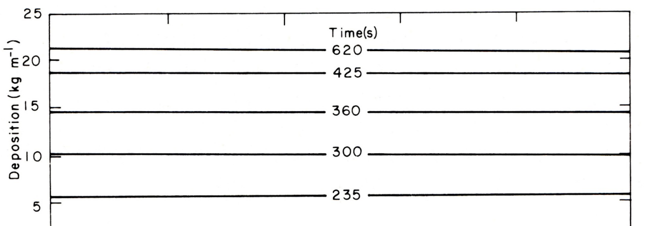

FIG.5 Sediment deposition in the terrace channel.

Fig.6 illustrates that the conservation of mass principle is perfectly satisfied in the mathematical model for both water and solid phases. The ordinate of a computed point in Fig.6(a) is determined as the numerical integration up to time t of the instantaneous rates of net water input to the system, that is the rate of rainfall minus the rate of infiltration minus the rate of channel water outflow. The abscissa of the same point is calculated as the summation of the instantaneous storages of water over the overland plane and in the terrace channel at time t. In a similar manner, Fig.6(b) is constructed for the sediment phase.

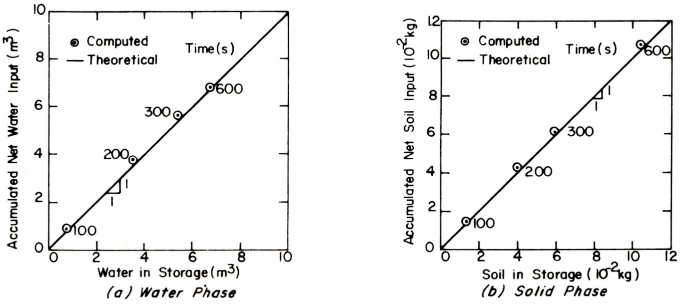

FIG.6 The conservation of mass in the model.

VERIFICATION OF THE MATHEMATICAL MODEL BY OBSERVED DATA

The mathematical model is verified by a set of data observed in the experimental terraces of the Soil-Water Central Research Institute in Ankara, Turkey. The details of the experimental terraces and the measurement techniques have been reported by Dogan *et al.* (1977). The measurements were taken at the exits of the overland flow planes and are employed to verify only the overland flow component of the mathematical model.

The composition of the soil examined is 22% clay, 24% silt and 54% sand. It has a porosity of 0.40, a hydraulic conductivity of 9×10^{-8} m s^{-1} and an initial moisture deficit of 0.20. The parameters adopted for this soil in the numerical simulation are: $m = 3.0$, $\alpha = 0.41$, $\beta = 1.14$, $p_c = 20$, $\lambda = 0.52$, $p = 0.40$, SMD = 0.20, $\Psi = 0.40$ m, $C_d = 0.008$ and $C_t = 0.045$.

Eight different flow conditions examined are summarized in Table 1. In the first four cases, the overland flow length is 23 m with a bottom slope of 0.10. In the remaining cases, the overland flow plane is 21.9 m long and has a bottom slope of 0.17. Also shown in Table 1 are the observed and the computed results. A good agreement exists between the mathematical model and the experiments for all the flow cases investigated.

TABLE 1 Comparison of computed and observed results

Case no.	*Rainfall*		*Soil loss (kg)*		*Runoff volume (m^3)*	
	I(mm h^{-1})	*t_d(s)*	*Observed*	*Computed*	*Observed*	*Computed*
1	18.0	600	0.014	0.017	0.001	0.001
2	23.8	900	0.070	0.067	0.040	0.044
3	37.8	720	0.115	0.092	0.100	0.106
4	10.8	2100	0.055	0.050	0.026	0.024
5	18.0	600	0.022	0.030	0.002	0.002
6	23.8	900	0.120	0.115	0.048	0.044
7	37.8	720	0.170	0.160	0.110	0.106
8	10.8	2100	0.090	0.092	0.027	0.024

CONCLUDING REMARKS

The physically-based mathematical model introduced and verified in this study is not the most sophisticated one possible. It is rather a usable model which satisfies the basic principles of hydraulics, yet requires input data not unreasonably detailed and difficult to collect. Such a model can be used not only for analysing the terrace flow as a distributed system, but also for testing the validity of the empirical formulae commonly employed in terrace design practice.

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