Mathematical model for the computation of sediment yield from a basin

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ABSTRACT A mathematical simulation model for the computation of sediment yield due to upland erosion from a large basin on a daily basis is presented in this paper. The model was tested with data of suspended load at the inlet of the "Forggensee" Reservoir (Bavaria, FRG), i.e. at the outlet of the corresponding basin ($\simeq$ 1500 km^2). For more detailed calculations of the sediment yield the basin was divided into subareas. Components of this model are the Universal Soil Loss Equation (USLE; Wischmeier and Smith, 1978) for computation of the soil detachment in each subarea, the rainfall-runoff-model of Lutz (1982) for computation of the runoff parameters in USLE and the concept of overland flow sediment transport capacity (Foster and Meyer, 1972) for computation of the sediment yield at the outlet of each subarea and at the outlet of the whole basin.

INTRODUCTION

An important aspect for water resources planning and management is the deposition of sediment in a reservoir. This sediment is the product of the erosion by water in the corresponding basin.

The erosion in a basin may be divided into two general types: the upland erosion and the channel erosion.

Erosion by water embodies the processes of detachment, transportation and deposition of soil particles (sediment) by the erosive and transport agents of raindrop impact and runoff over the soil surface (Foster, 1982).

The quantification of the erosion processes can be made more exact, if it is applied to small land areas. The processes quantified in this work are:

(a) The upland erosion and especially the soil detachment by the raindrop impact and runoff in each subarea.

(b) The sediment delivery due to upland erosion at the outlet of each subarea (sub-basin) and at the outlet of the whole basin. This process includes implicitly the subprocesses of transportation, deposition and detachment of sediment over the soil surface.

(c) The surface runoff.

The channel erosion and channel runoff are neglected.

The model presented in this paper was applied to the 1500 km^2 basin of the "Forggensee" Reservoir (Bavaria, FRG) (Fig. 1). The largest part of the basin is in Austria and the main stem channel is the "Lech" River. The basin consists of forest-, prairie-, urban and rock-areas (over 2000 m in altitutde).

The following data were available:

(a) Suspended load at the outlet of the basin for 12 years (1966-1977), on a daily basis.

(b) Daily rainfall amounts from five rainfall stations in the basin for these same 12 years.

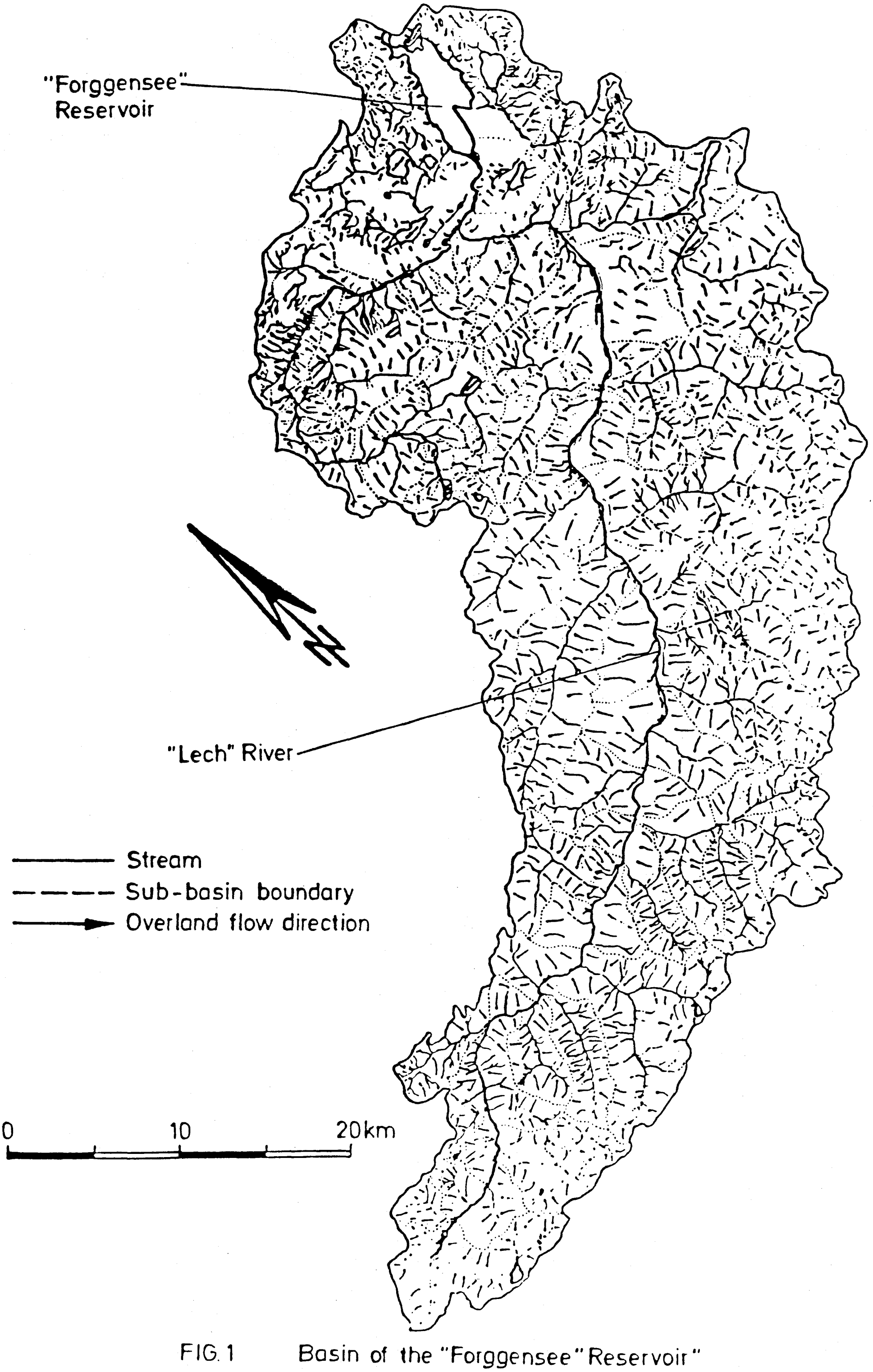

FIG. 1 Basin of the "Forggensee" Reservoir

The sediment yield at the outlet of the basin is computed by this model on a daily basis. These daily values of the sediment yield are added to produce the annual value of the sediment yield at the outlet of the basin.

Upland erosion mainly produces sediment transported in the streams as suspended load. Therefore, it is possible to compare the computed annual values of sediment yield due to upland erosion with the measured annual values of suspended load at the outlet of the basin. The two categories of values do not have to coincide necessarily. The measured values of suspended load are rather indications for the verification of the mathematical model.

AREA AND TIME UNIT IN THE CALCULATIONS

The example basin was divided into 85 sub-basins by means of a quadrangular grid. Each square had an area of 25 km^2 (5 km x 5 km). The geometrical symmetry of the quadrangular grid facilitates the performance of the calculation on the one hand, but on the other hand destroys the "natural image" of the basin because the sides of the squares are not natural boundaries of the sub-basins.

The selection of the "day" as the time unit in the calculations is a very good approximation of the sediment delivery problem of a large basin.

It was assumed that uniform conditions exist in each subarea and steady conditions exist throughout each day for the runoff and erosion processes.

Daily rainfall occurrences were treated as individual storm events.

UNIVERSAL SOIL LOSS EQUATION (USLE)

The classical form of the USLE (Wischmeier and Smith, 1978) is:

$$A = R\ K\ LS\ C\ P \qquad (1)$$

where:

A soil loss due to upland erosion (weight per unit area),
R rainfall erosivity factor (energy per unit area x rainfall intensity unit),
K soil erodibility factor (A-unit/R-unit),
LS topographical factor (-),
C cropping management factor (-) and
P erosion control practice factor (-).

USLE is intended to estimate average soil loss over an extended period, e.g., average annual soil loss (Foster, 1982). The raindrop impact is only taken into account in this equation to estimate soil loss.

An improved erosivity factor was introduced by Foster et al. (1977) to take also into account the runoff shear stresses effect on the soil detachment for single storms:

$$R = 0.5\ R_{st} + R_R = 0.5\ R_{st} + 0.5a\ Qq_p^{0.33} \qquad (2)$$

where:

R modified erosivity factor,
R_{st} rainfall erosivity factor,
R_R runoff erosivity factor,
Q runoff volume per unit area,

q_p peak runoff rate per unit area and
a a constant depending on the units.

Schwertmann (1981) converted the USLE from its original English units to metric units and presented tables for the evaluation of the factors K, C and P for Bavaria. The units and tables of Schwertmann were used in this work.

The rainfall erosivity factor is defined as the product of two rainstorm characteristics: Kinetic energy and the maximum 30 minute intensity. The computation of this factor for a rainfall event requires a continuous record of rainfall intensity. But only rainfall amounts were available for the example basin. Therefore, a regression analysis was used to estimate the factor R_{st} as a function of the daily rainfall amount. Data for the rainfall erosivity factor and rainfall amount were available from a small basin ($\simeq$ 14 km^2) in the neighbouring state of Baden-Württemberg.

The topographical factor LS was evaluated with the following equation (Wischmeier and Smith, 1978; Kent Mitchell and Bubenzer, 1980):

$$LS = (\lambda/22.13)^m \, (0.065+0.045s+0.0065s^2) \qquad (3)$$

where:
λ slope length (m),
s slope gradient (%) and
m an exponent depending on the slope gradient.

For extreme slope gradients greater than 30%, the topographical factor was estimated with the following relationship proposed by Van Vuuren (1982):

$$LS = (\lambda/22.13)^m \, 16 \, [\sin 2(\arctan(s/100))]^{1.6} \qquad (4)$$

In addition to the tables of Schwertmann a soil, a topographical and a vegetation map were required for estimating the factors K, LS and C.

RAINFALL-RUNOFF-MODEL

The model of Lutz (1982) was used to compute the runoff volume per unit area Q for USLE (factor R_R). This model is based on the model of Anderl (1975) which forecasts the rainfall excess for a given storm by using region- and event-dependent parameters. The model of Anderl is valid for daily rainfall amounts which may include only one event or multiple events. This model was modified by Lutz (1982) to be extrapolated in extreme storm events. The model of Lutz is expressed mathematically by the following equation:

$$Q = (N-Av)c+(c/ao) \, [\exp(-ao(N-Av))-1] \qquad (5)$$

where:
N daily rainfall amount (mm),
Q rainfall excess (mm),
Av initial abstraction consisting mainly of interception, infiltration and surface storage and depending on the land use (mm),
c maximum end runoff coefficient after a very long storm event depending on the land use and the hydrological soil group and
ao proportionality factor (mm^{-1}) which is given by the following equation

$$ao = P_1 \exp(-2.0/WZ)\exp(-2.0/QB) \quad (6)$$

where:

P_1 region-dependent parameter,

WZ week number which designates the season and

QB base flow which designates the antecedent moisture condition ($l\ s^{-1}km^{-2}$).

The following formula of the Soil Conservation Service (SCS) of the USA was used for determining the peak runoff rate of a sub-basin (Huggins and Burney, 1982):

$$q_p = 0.278\ F\ Q/T_A \quad (7)$$

where:

q_p peak runoff rate (m^3/s)

F sub-basin area (km^2),

Q rainfall excess (mm) and

T_A time of rise of the hydrograph (h) (time from the beginning of runoff to the time of peak runoff).

This formula is based upon the SCS triangular hydrograph analysis procedure for approximating the manner in which an incremental volume of rainfall excess is translated into a time distribution of runoff at the basin's outlet.

OVERLAND FLOW SEDIMENT TRANSPORT CAPACITY

The following relationships of Beasley et al. (1980) was used to compute the sediment transport capacity of the overland flow in each subarea:

$$TF = 146\ sq^{1/2} \quad \text{for } q \leq 0.046\ m^3\ min^{-1}m^{-1} \quad (8)$$

$$TF = 14600\ sq^2 \quad \text{for } q > 0.046\ m^3\ min^{-1}m^{-1} \quad (9)$$

where:

TF transport capacity of flow ($kg\ min^{-1}m^{-1}$),

s slope gradient (-) and

q flow rate per unit width ($m^3\ min^{-1}m^{-1}$).

The first equation is valid for laminar and the second for turbulent flow.

The relationships of Beasley et al. are based upon an equation of Yalin (1963), who assumes that the mechanism of sediment transport by a shallow flow (e.g. by the overland flow) is similar to the mechanism of bed load transport in channels and that a critical shear stress exists acting on the soil at the beginning of sediment transport.

COMPUTATION OF SEDIMENT YIELD

The sediment yield due to upland erosion at the outlet of a subarea results by means of the following consideration:

If the available sediment in a subarea exceeds the sediment transport capacity of the overland flow, deposition occurs, and the sediment transported to the next subarea equals the sediment transport capacity. If the available sediment in a subarea is less than the sediment transport capacity of the overland flow, and if the flow's erosive forces exceed the resistance of the soil to detachment by flow, detachment occurs; in this case the sediment transported to the next subarea equals the available sediment. It is symbolized by the following relationships:

$$FLO = TF, \quad \text{if } VS > TF \tag{10}$$
$$FLO = VS, \quad \text{if } VS \leqslant TF \tag{11}$$

where:
FLO sediment yield at the outlet of a subarea,
TF sediment transport capacity of the overland flow and
VS available sediment in the subarea.
The available sediment VS is given by the following equation:

$$VS = FLI + A \tag{12}$$

where:
FLI sediment inflow to the subarea and
A soil detachment rate in the subarea computed by USLE.
The amounts FLO, TF, VS, FLI and A in the Equations (10), (11) and (12) are expressed in the same units, i.e. in tonnes or kilograms.

The above considerations are based on the following equation of Foster and Meyer (1972) expressing an interrelationship between deposition and sediment load:

$$(E_f/E_c) + (G/TF) = 1 \tag{13}$$

where:
E_f overland flow detachment rate (weight per unit area per unit time),
E_c overland flow detachment capacity (weight per unit area per unit time),
G sediment load (weight per unit width per unit time) and
TF overland flow transport capacity (weight per unit width per unit time).

SYNTHESIS

The three component submodels, i.e. USLE, the rainfall-runoff-model and the concept of the overland flow transport capacity, were combined to form the final model represented by the flow chart in Fig. 2

The application of this model requires the use of a sediment transport plan, which specifies the motion of the sediment from subarea to subarea.

RESULTS

The ratio of the computed annual values of sediment yield due to upland erosion to the measured annual values of suspended load at the outlet of the example basin is presented in the Table 1.

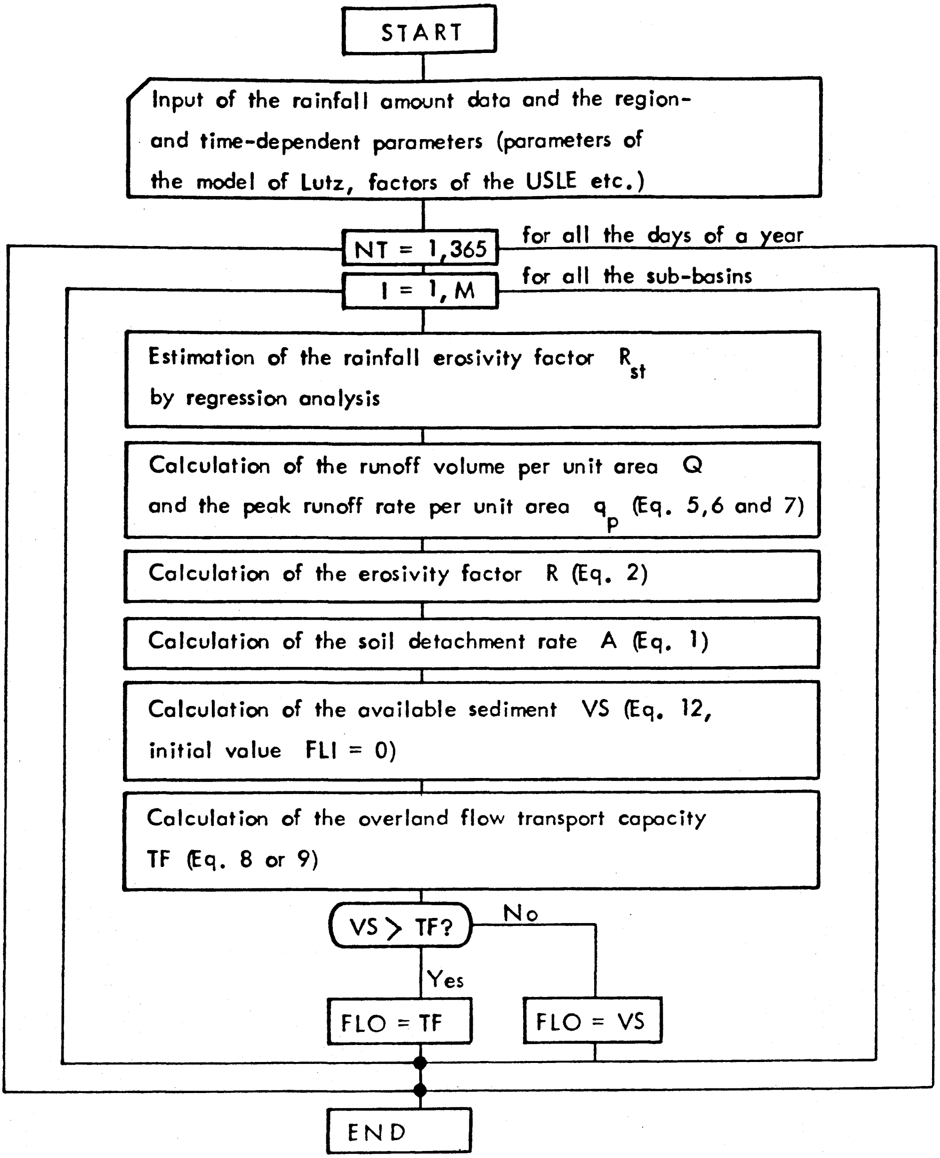

FIG. 2 Flow chart of the final model.

TABLE 1 Results of the calculations

Year	Measured values (t)	Computed values / Measured values	Year	Measured values (t)	Computed values / Measured values
1966	488000	2.73	1972	65872	14.51
1967	293000	4.54	1973	340293	2.85
1968	312000	3.47	1974	270031	5.49
1969	205000	3.26	1975	621322	2.12
1970	971000	1.79	1976	263143	3.48
1971	271710	3.34	1977	260021	4.54

A sensitivity analysis has shown that the rainfall amount, the sub-basin area and the factors of USLE (K,LS,C and P) strongly affect the daily sediment yield at the outlet of the whole basin.

CONCLUSIONS

The work presented in this paper was an attempt to combine USLE with the concept of overland flow sediment transport capacity for estimating the soil detachment and the sediment yield due to upland erosion in a large basin.

The results were relatively satisfactory considering the large basin area and the fact that the computation was performed on a daily basis.

An important factor contributing to the deviation between computed and measured values of sediment yield is the neglection of the channel erosion, which alters the continuity of the physical processes. However, taking into account channel erosion requires large amounts of mostly unavailable data for the geometry and hydraulics of the entire stream system.

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