Determination of "Diffusion" Erosion Coefficients for Some Tributaries of Oaklimiter Creek, North-Central Mississippi

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ABSTRACT The lowering of base-level is a cause of accelerated erosion and gullying. Quantitative estimates of different stages of development of gullies are necessary in order to provide an effective tool for land management.

A simple mathematical model that describes channel response to base-level lowering was proposed, the governing equation of which being a diffusion equation, was applied to 10 tributaries of Oaklimiter Creek in North-Central Mississippi, which were entrenched after channelization of Oaklimiter Creek took place in 1965.

The "diffusion" erosion coefficients were determined by comparing the 1965 longitudinal profiles to those surveyed in 1981. In order to be a predictive tool for other streams in the area, "diffusion" coefficients (K, in square feet per day) were regressed on drainage area (A, in square miles), with the resulting equation:

$$K = 203.5\, A^{1.66} \qquad (r = 0.935;\ n = 10;\ 0.05 < A < 7.3)$$

The "diffusion" erosion coefficient governs the rate of erosion in different locations along the channel at different times and therefore this equation permits a fair first-approximation estimate of channel response.

INTRODUCTION

The lowering of base level is a cause of accelerated erosion in existing channels, and may be the cause of initiation of gullies. Daniels (1960) showed it to be the triggering mechanism of gullying in a wide area, and other studies also show that degradation follows base-level lowering in alluvial channels within a time-scale of some days (Kirkby and Kirkby, 1969), weeks (Mosley, 1984), years or decades (Schumm and Hadley, 1957; Nordin, 1964; Livesly, 1975; Pickup, 1975; Simons and Li, 1980; Kellerhals, 1982).

The development of gullies is significant to agriculturalists and conservationalists because it has several undesirable effects, as follows: the headward growth of gullies destroys agricultural lands and undermines structure; the entrenchment of gullies causes lowering of water-tables in valleys with the resulting negative effects of farming and grazing; the increased sediment discharge causes loss of channel capacity, increased flood probability and loss of reservoir capacity (Heede, 1975; Curtis, 1976; Hadley and Shawn 1976).

Several causes may lead to the lowering of base level of streams: tectonic upwarping of an area close to a previous base level (Kirkby and Kirkby, 1969), downfaulting (Wallace, 1977), drawdown of a lake (Mosley, 1984), channelization of a main stream, leading to lowered water elevation for its tributaries (Daniels, 1966; Schumm et al., 1984), lowering water elevation of a main stream through dam regulation (Simons

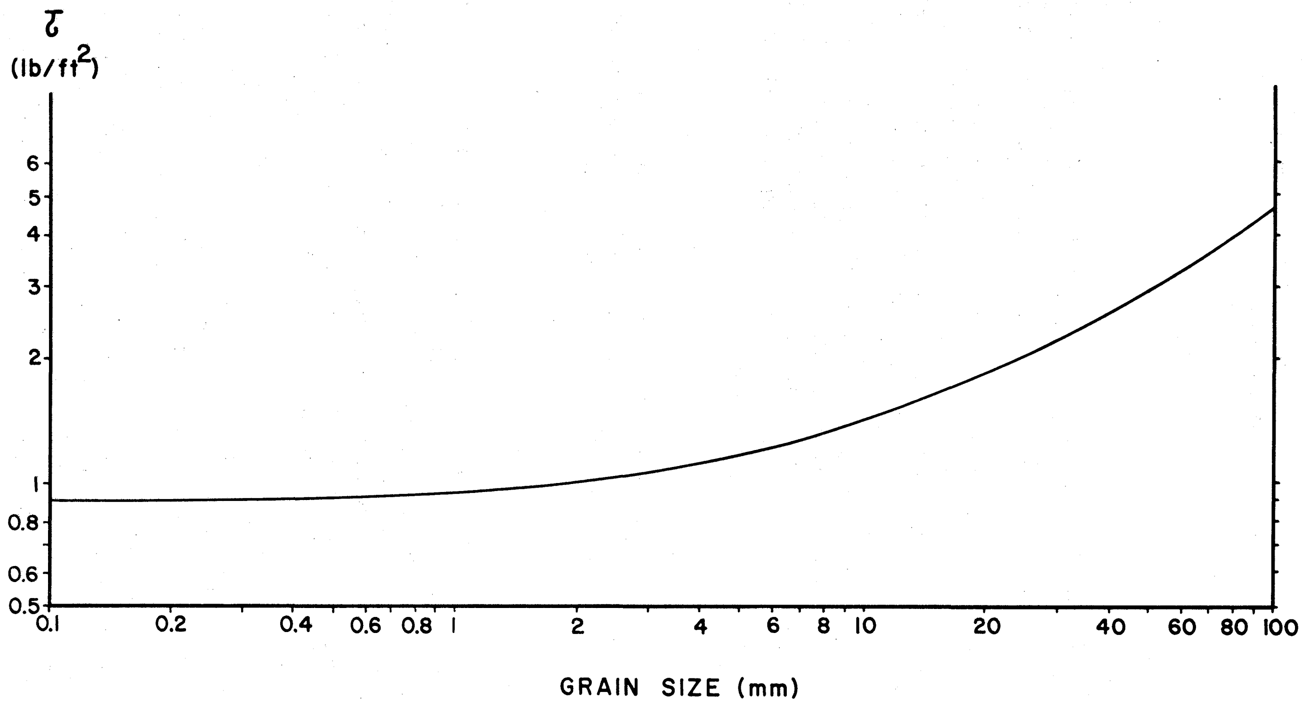

Figure 1. Diagram relating flow shear stress to channel grain size for an error of 5% in the approximation $(\tau-\tau_{cr})^{1.5} \cong \tau^{1.5}$.

and Li, 1980; Kellerhals, 1982) or gravel mining (Simons and Li, 1980). The delayed upstream effects of base level lowering may sometimes hinder full recognition of the significance of this phenomenon.

As the perturbation in longitudinal profile caused by base level lowering is propagated quite long distances upstream within some years, gully entrenchment is a transient phenomenon on a time scale of years. Since this particular time scale is of direct interest from an engineering point of view, both the final state and the transient states of gullies should be determined. Quantitative determination of different stages of development of gullies are necessary in order to provide an effective tool for land management, if base level lowering is an expected outcome of some planned actions within a watershed. Such a quantitative assessment may be provided by state-of-the-art dynamic modelling which is "the development of water and sediment routing techniques that can be used to predict the channel response to natural and man-induced variations and/or changes in sediment supply and discharge" (Li and Simons, 1982, p. 471).

However, the physical processes governing stream responses may be very complex and therefore their mathematical expression must involve some uncertainty. There always exists a trade-off relationship between the difficulty (and cost) of obtaining an accurate solution, and the risk of an inadequate representation of the system (Overton and Meadows, 1976). A common feature of dynamic models of channel response is that the movement of sediment is traced by a numerical procedure step-by-step, while in each step the temporal and spatial conditions of water and sediment are considered. However, since the very basic problem of accurate prediction of sediment transport has not been satisfactorily solved, even a sophisticated dynamic model may contain considerable inaccuracy if not properly calibrated versus the real world situation. It should also be mentioned that as knickpoint migration involves rapid changes of relatively steep channel slopes, the response of a channel to the particular case of base level lowering may prove to be quite a formidable task.

On the other hand, a simple mathematical model that describes channel response to base level lowering was proposed, the governing equation of which being a diffusion, or heat, equation, and it was successfully applied in flume experiments (Begin et al., 1980a, 1980b, 1981; Begin, 1982). As will be shown below this model involves the simplifying assumption that sediment discharge, under appropriate conditions, may be considered to be directly, linearly proportional to the local bed slope. This assumption is then used in order to obtain a "diffusion" coefficient for the degradation process, and this coefficient lumps together the water discharge and its interaction with the channel bed sediments. As a result, the channel response is characterized by a single coefficient which is determined through the overall performance of the channel over a period of time. Calibration of the model, therefore, involves the characterization of the average performance of the channel using a single coefficient. The advantage of this approach is that the basic equation has a simple numerical solution, resulting in a computer program which is easy to handle and cheap to operate. With proper calibration, this model - though simplistic in nature - may provide practical, first-approximation predictions of the response of channels to base level lowering.

THEORETICAL BACKGROUND

In an alluvial channel with lateral inflow of sediment, conservation of matter is formulated by the two-dimensional equation of sediment continuity:

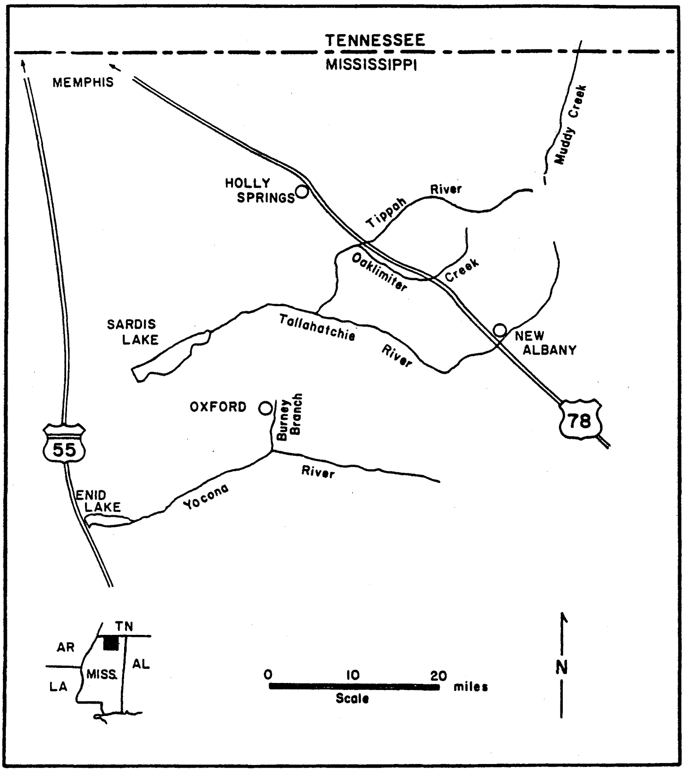

Figure 2. Location map of the studied streams.

$$\frac{\partial y}{\partial t} = \frac{1}{\gamma_s} \frac{\partial q_s}{\partial x} + B \qquad (1)$$

where q_s is sediment discharge by weight per unit width, γ_s is the bulk weight per unit volume of sediment, y is elevation of the channel bed, x denotes distance along the channel (positive upstream with x = 0 at the outlet), t denotes time, and B is the volume of lateral inflow of sediments, per unit flow width, and per unit length of channel.

As noted by Gessler (1971), many sediment transport equations can be brought into the form:

$$q_s = C_1(\tau - \tau_c)^p \qquad (2)$$

where C_1 and p are empirical constants; τ and τ_c are the bottom shear stress and the critical bottom shear stress, respectively. In particular, in some sediment transport equations the power p is 3/2. In these cases, if $\tau >> \tau_c$, equation (2) reduces to:

$$q_s = C\ \tau^{3/2} \qquad (3)$$

which leads to (Begin et al., 1981):

$$q_s = k\ \frac{dy}{dx} \qquad (4)$$

from which

$$\frac{\partial q_s}{\partial x} = k \frac{\partial^2 y}{\partial x^2} \tag{5}$$

with

$$k = C\, q_w \sqrt{(\gamma_w^3 f/8g)} \tag{6}$$

where f is the Darcy-Weisbach friction factor; g is acceleration due to gravity; q_w is the water discharge per unit width, and γ_w is water density.

Substituting equation (5) into equation (1) yields:

$$\frac{\partial y}{\partial t} = \frac{k}{\gamma_s} \frac{\partial^2 y}{\partial x^2} + B \tag{7}$$

defining:

$$K = \frac{k}{\gamma_s}$$

equation (7) finally becomes:

$$\frac{\partial y}{\partial t} = K \frac{\partial^2 y}{\partial x^2} + B \tag{8}$$

This is a version of the well-known diffusion (or heat) equation, and K of equation (8) thus becomes a 'diffusion coefficient', with dimensions of $\{L^2\}/\{T\}$. The elevation y at any time t, and distance x, may be found once equation (8) is solved for y(x,t), and the solution is dependent upon the boundary and initial conditions which are specified for the problem at hand.

For a channel having an initial linear profile described by: $y = Y + ax$, for which base level is lowered by amount Y, the solution of equation (8) (assuming B = 0) is:

$$y_{(x,t)} = Y \text{ efr } (x/2\sqrt{KT}) + ax \tag{9}$$

where erf is the error function.

It is important at this point to determine the conditions under which equation (8) is a reasonable approximation. In other words we should explore the range of hydraulic conditions and sediment sizes under which equation (4) is reasonably close to well established sediment transport equations, that is: we explore under what conditions equation (3) is practically close enough to equation (2) (with p = 3/2), or $\tau^{3/2}/(\tau-\tau_{cr})^{3/2}$ $1 + m$, where m is the error. Defining $h = \tau_{cr}/\tau$ we have $\tau^{3/2}/\tau^{3/2}$ $(1-h)^{3/2} = 1 + m$, or $(1-h)^{3/2}(1+m) = 1$. Solving for different h values with a specified error, m, and taking values of τ_{cr} for different grain sizes from the Shield's relationship (using Vanoni, 1977, Fig. 2-44), it is possible to determine flow shear stress values for different levels of accuracy of equation (3) (Fig. 1). It should be noted that two factors combine to largely increase the flow shear stress in a channel influenced by base level lowering. First, the bankfull discharge increases as the channel is being incised, the amount of incision given by equation (9). Second, the slope of the channel increases, depending on the amount of base level change, the distance from the stream mouth

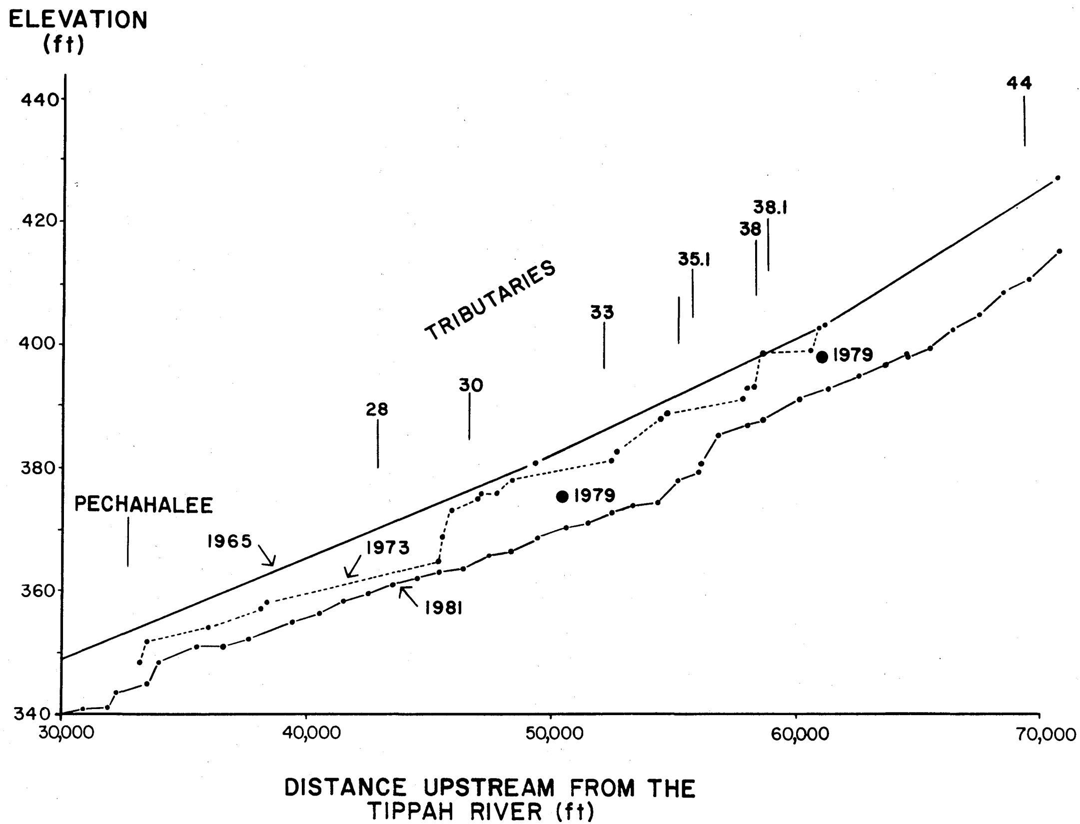

Figure 3. Longitudinal profiles of Oaklimiter Creek in 1965 (constructed profile), 1973 and 1981, with two points surveyed in 1979. Vertical bars represent confluence of tributaries identified by their numbers. Source of data: SCS construction plans and SCS surveys.

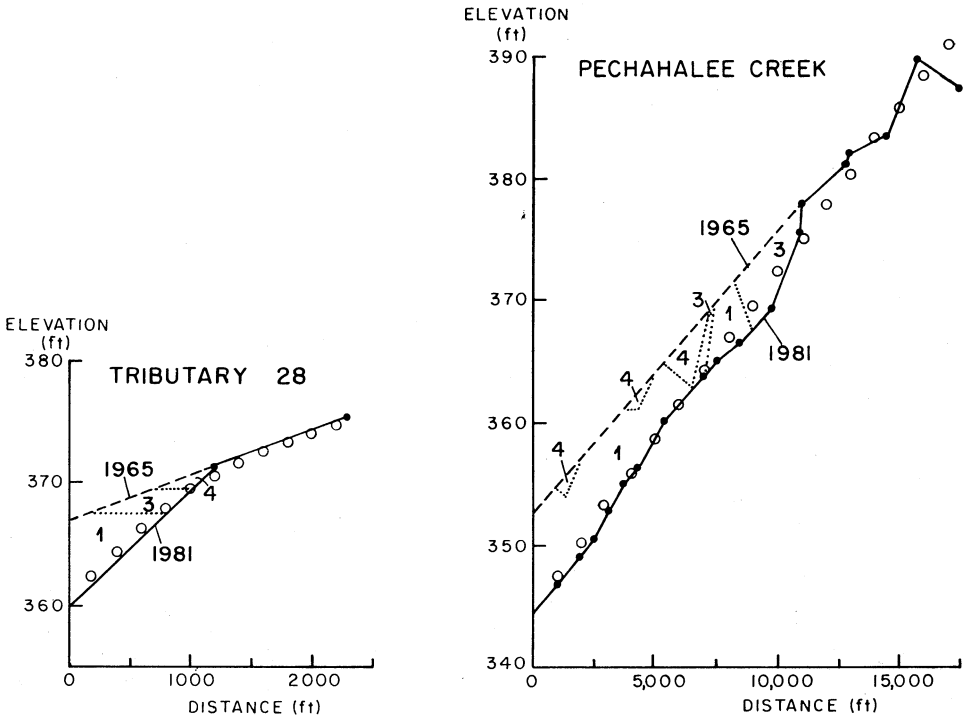

Figure 4. Longitudinal profiles of tributaries to Oaklimiter Creek in 1965 and in 1981. Open circles denote best-fit profile computed through Programme ERFUS5. Numbers denote lithological units: 1,3 - Meander Belt Units; 2-Lacustrine Unit; 4 - Post Settlement Alluvium.

as well as on the time since base level lowering. The change in slope is determined by differentiating equation (9) with respect to distance:

$$\frac{\partial y}{\partial x} = \frac{Y}{\sqrt{(\pi K t)}} \exp\left(-x^2/4Kt\right) + a \tag{10}$$

APPLICATION TO SMALL NORTHERN MISSISSIPPI STREAMS

Entrenchment of Oaklimiter Creek

Some streams in the upper Yazoo Basin of the north-central portion of the State of Mississippi were channelized in order to alleviate the problem of frequent flooding. One of these streams is Oaklimiter Creek, a tributary of the Tippah River which in turn is a tributary of the Little Talahatchie River, which flows into the Sardis Reservoir (Fig. 2). Oaklimiter Creek was channelized in 1965, channelization entailing realignment of the natural sinuous channel and steepening of its slope. As a result of these works Oaklimiter Creek was entrenched into the alluvium by as much as 15 feet between 1965 and 1981, the deepening being accompanied by a great increase of channel width. The dramatic response of Oaklimiter Creek to channelization was analyzed in detail by Schumm et al., 1984.

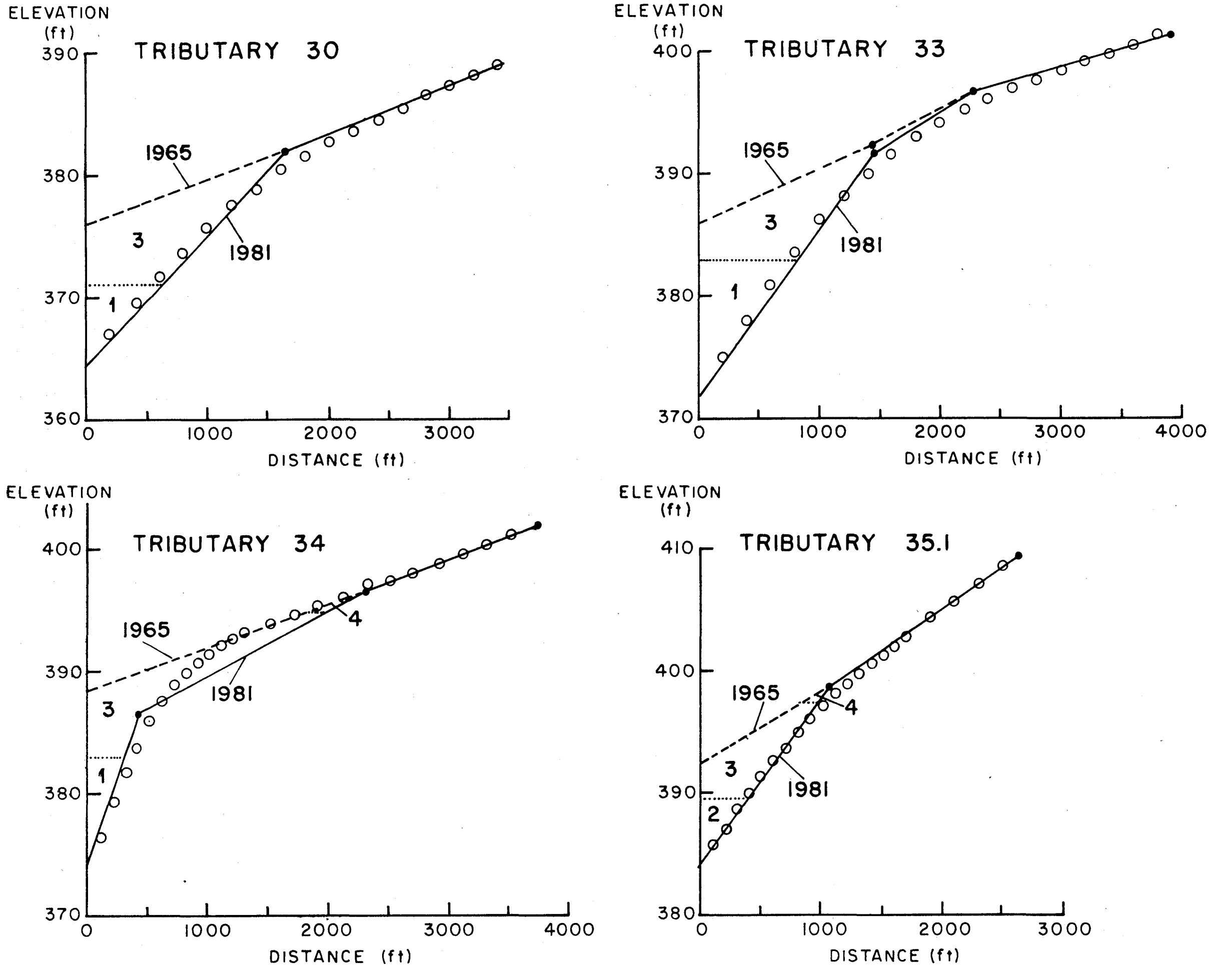

Figure 4. (Continued)

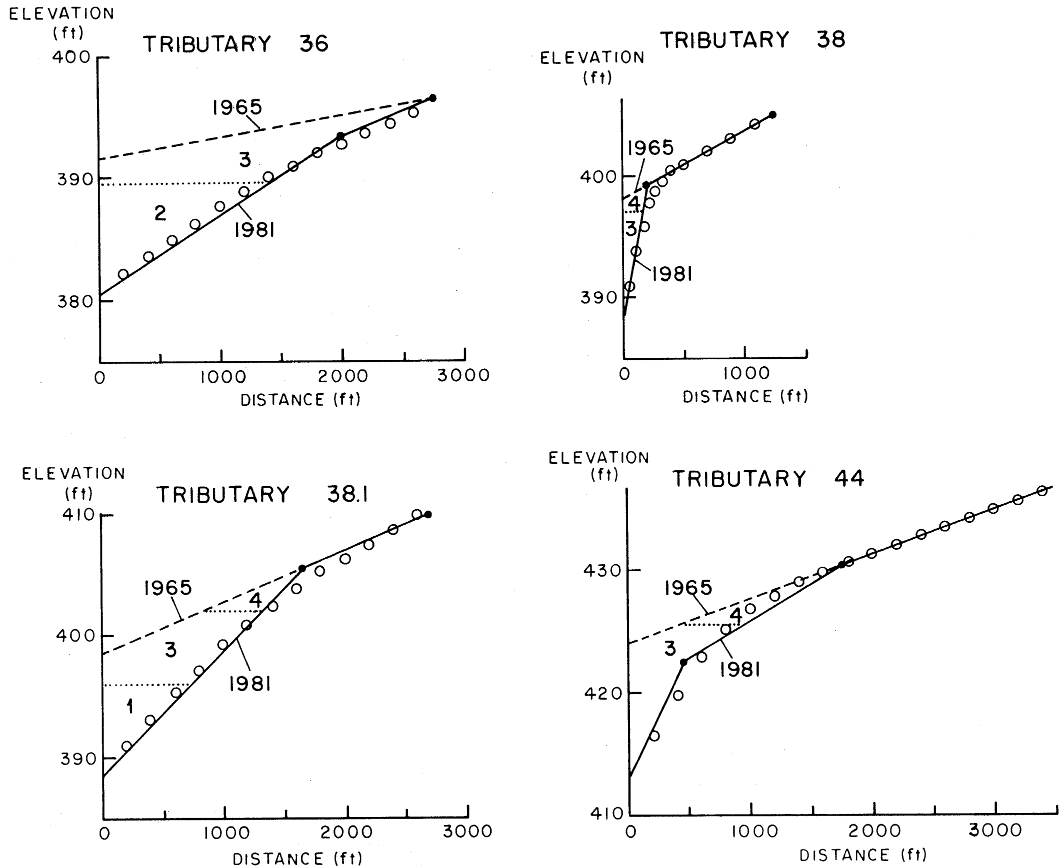

Figure 4. (Continued)

The entrenchment of Oaklimiter Creek resulted in the lowering of base level of its tributaries, thereby causing their entrenchment. The response of ten tributaries, ranging in drainage area from 0.05 to 7.3 square miles is analyzed here, in order to determine their "diffusion" erosion coefficients, in an attempt to present an estimate of these coefficients for other streams in that region.

Lithology

Four litho-stratigraphic units of Quaternary age, which were exposed in tributaries in the studied area, were described by Schumm et al., 1984. The lower unit, Meander Belt 1, is a fine grained meander belt deposit, consisting mainly of readily erodible, cohesionless, medium and coarse grained sand ($0.03 \leq D_{50}(mm) \leq 0.5$). Lacustrine deposits are found in places above this unit. These consist of cohesive loess, which has a well developed polygonal columnar structure. The inter-column seam materials are cohesionless, very fine grained sands which provide the failure planes for bed and bank erosion.

The Meander Belt 2 unit overlies either of these units. It is a fine-grained meander belt deposit, similar to MB1 but less cohesive and more erodible. The unit covering all previous ones is the Post Settlement Alluvium, comprising cohesionless, fine-grained sediments.

These units are shown on the longitudinal profiles of the studied tributaries (Fig. 4).

Hydrology

Discharge-drainage area relationship for the Oaklimiter watershed were worked out by the Soil Conservation Service (Table 1).

Flow computations for Pechahalee Creek were carried out by Michael Baker Inc. for different recurrence interval. According to these computations the one year flow produces a flow depth of about 3.5 feet, and the 2.33 year flow produces a flow depth of about five feet. As the median grain size of the units eroded by the stream is smaller than 0.5 mm, a shear stress of 0.9 lb/ft^2 is needed in order that the assumption of equation (3) will be a reasonable approximation (Fig. 1). Such a shear stress is not obtained with the original channel slope (0.0025) for $Q_{2.33}$ but as base level is lowered by 10 feet, the appropriate slope of 0.0029 is obtained as far as 12,000 feet from the mouth. This calculation is done through equation (10) with $K = 25629\ ft^2/day$ (Table 1).

Changes in Base-Level

The initial base level for the studied channels is considered to be the 1965 elevation constructed in Oaklimiter Creek, records of which are filed with the Soil Conservation Service (Fig. 3). The 1981 profile is the result of joining minimum bed elevation points on cross sections surveyed by Michael Baker Inc. Another survey of cross sections of Oaklimiter Creek was carried out by the Soil Conservation Service in 1973, and some additional, sporadic cross sections were measured in 1979.

The longitudinal profiles of Oaklimiter Creek (Fig. 3) indicate that the studied tributaries underwent a slightly different history. By 1973 the base level of Pechahalee Creek and Tributary 28 was alredy several feet below the 1965 base level, while the base level for tributaries 30, 33, 36 and 35.1 was only slightly lower than their 1965 elevations. For

Table 1: Data on the studied tributaries of Oaklimiter Creek

Tributary	Location* (ft)	Length (ft)	Drainage Area (sq. miles)	Discharge (cfs) Q_1	$Q_{2.33}$	Q_5	Q_{50}	Modelling: Time,** T (yrs)	ΔX (ft)	Average squared deviation (ft^2)	Diffusion Coefficient K (ft^2/day)
Pechahalee	32640	15500	7.28	417.4	763.3	1190.6	2584.0	16	420	1.19	25629.3
28	42900	4000	0.54	3.1	4.9	5.8	7.4	16	50	0.14	46.0
30	46850	5000	1.05	16.9	27.8	36.5	59.2	10	50	0.20	157.1
33	52225	3910	1.17	22.3	36.8	49.4	83.0	10	40	0.46	130.4
34	53525	3720	0.33	0.9	1.4	1.5	1.6	10	40	1.12	32.7
36	55250	3500	2.06	94.7	161.2	237.9	486.9	10	40	0.24	327.7
35.1	55820	2620	0.38	1.3	2.0	2.2	2.5	10	30	0.11	71.6
38	58500	1250	0.05	-	-	-	-	8	15	0.11	2.8
38.1	58850	2690	0.71	6.2	10.0	12.3	17.4	8	30	0.28	197.4
44	69500	4320	0.86	10.1	16.5	21.0	31.7	8	45	0.22	51.7

* Distance of mouth from Tippah River, measured along Oaklimiter Creek.

** Assuming that base level was lowered with a constant rate for this period.

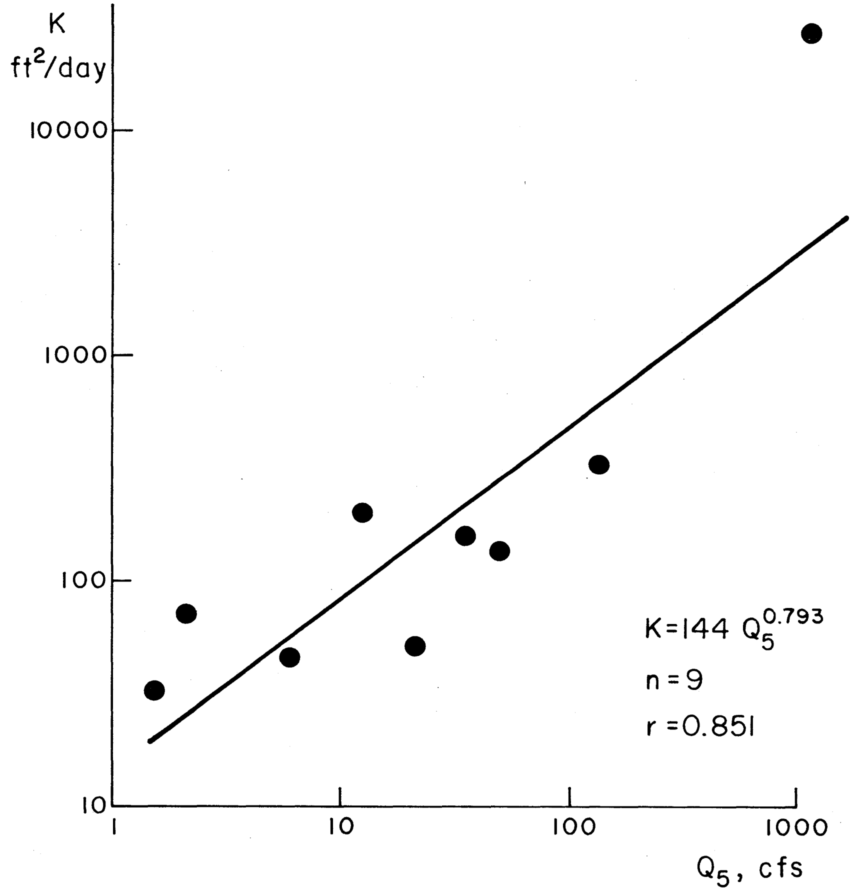

Figure 5. The dependence of K on Q_5 in the studied streams.

tributaries 38, 38.1 and 44 base level did not change up to 1973, but elevations measured in Oaklimiter Creek in 1979 indicate that degradation was well under way upstream of station 50,000. It may be concluded that the base level lowering - and degradation - of all 10 tributaries occurred in 8-16 years. However, there was a phase-difference between the two lower tributaries, which underwent most of their level change between 1965 and 1973, and the other tributaries which changed their base level during the period 1973-1981. The assumed time of erosion for the different streams is presented in Table 1.

Selection of Tributaries

Thirty tributaries of Oaklimiter Creek were surveyed in 1981, with only two or three cross sections surveyed along each one. In only ten of these tributaries can the initial 1965 profile be reconstructed with reasonable accuracy. Eight of the selected tributaries are those for which the 1965 base level - as taken from the data on Oaklimiter Creek (Fig. 3) - lie on the continuation of the upstream segment of the longitudinal profile. For Pechahalee Creek it was assumed that the thalweg upstream of 12,000 feet is the intact 1965 thalweg, and a similar assumption was applied to Tributary 36, upstream of 2750 feet.

RESULTS

Based on equation (8) Program ERFUS5 was written in order to enable determination of the "diffusion" erosion coefficient K for channels which in response to base level lowering degraded from a given initial longitudinal profile to a given profile after a certain known time period. In each iteration a new profile is calculated according to a numerical solution of equation 8 (Begin, 1984). After each iteration the resulting profile is compared to the final profile and the mean square error is calculated.

When a minimum value of the mean square error is reached, the n-th iteration profile is considered to best approximate the real final channel profile and computation is stopped. The diffusion coefficient is calcualted through:

$$K = \frac{0.5n(\Delta X)^2}{T}$$

where ΔX is the space interval along the channel (15 to 420 ft), and, T is specified in Table 1. The calculations are based on the assumption that base level was lowered in a constant rate for the period T.

The results of the simulations for the 10 tributaries are presented in Fig. 4. For msot of them there is a good fit between the true and the modelled 1981 profile.

The K values for 9 of the 10 studied tributaries were regressed on their corresponding discharge values Q_5 (there are no data on discharge for tributary 38). The results of this regression (Fig. 5) as well as the regression of K on $Q_{2.33}$ and Q_{50} show a high correlation between discharge and K. This indicates that the "diffusion" erosion coefficients as determined are indeed meaningful.

Lithological differences do not seem to affect the K values. The Meander Belt units are similar to each other and most tributaries have been entrenched in these. Tributaries 35.1 and 36 are partly entrenched in the Lacustrine Unit, but they conform to the general behavior of the studied streams. It seems that the erodibility of the Lacustrine Unit is similar to that of the other units because it erodes along its columnar structures.

In order to be a predictive tool for other streams in the area where discharge measurements are not available, it is possible to use drainage area as a surrogate for discharge, and so K values were regressed on drainage area (Fig. 6) and the regression equation was calculated to be:

$$K = 203.5\ A^{1.664}$$

($r = 0.935$ for 10 data points) with K in square feet per day and A in square miles.

CONCLUSIONS

The results of this study indicate that the process of stream degradation in response to base level lowering can be realistically treated with the approximation of linearity for the relationship between rate of sediment transport and bed slope. This leads to the possibility of describing degradation by resorting to only one parameter - the

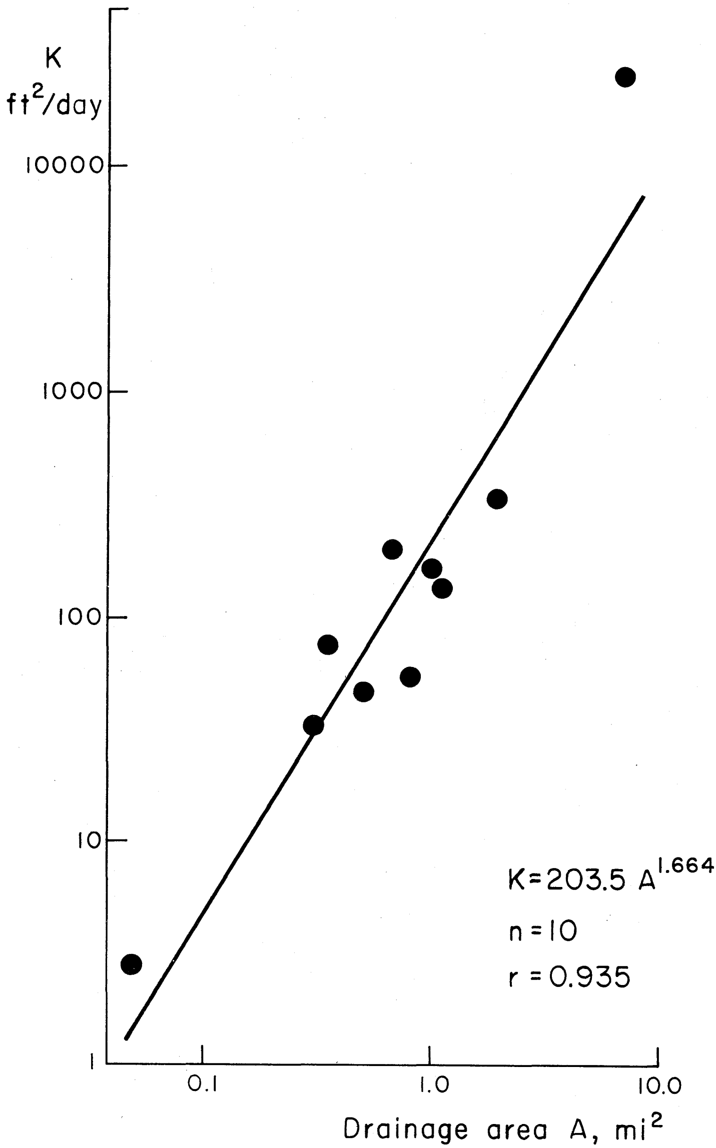

Figure 6. The dependence of K on drainage area in the studied streams.

diffusion erosion coefficient. In a field situation this coefficient is determined on the basis of the long-term performance of streams. The simplicity of the basic assumption leads to an algorithm which serves as the core of a computer program which is simple and cheap to operate.

The good correlation between the diffusion erosion coefficient K and the drainage area of the degrading streams can serve to calibrate K in order to apply the results to other streams in the same region. Also, the good correlation between K and discharge may permit extrapolation of similar results to streams in other regions. With an estimated K value, the response of streams to possible changes in their base level can be predicted, at least as a first approximation, as shown in the example application.

An advantage of the proposed method is that it permits a quantitative systematic view of the erosion of streams of different dize in different watersheds, through the designation of a K value to the degradation process. In order to turn this method into a useful engineering tool what is obviously needed is an assembly of additional well-documented cases under varied setups and hydrological conditions. The case of the tributaries of Oaklimiter Creek indicates that this method can be regarded as a promising one.

REFERENCES

Begin, Z.B. (1982) Application of "diffusion" degradation to some aspects of drainage net development. In: Bryan, R.B. and Yair, A. (eds.): Badland Geomorphology and Piping, Geo Abstracts Inc., Norwick, p. 169-179.

Begin, Z.B. (1984) ERFUSE - a computer program for the calculation of "diffusion" degradation of alluvial streams: Geol. Survey of Israel, Current Research 1983.

Begin, Z.B., Meyer, D.F. & Schumm, S.A. (1980a) Knickpoint migration in alluvial channels due to base level lowering: Amer. Soc. Civil Eng. Jour. Waterways Port. Coastal and Ocean Div., v. 106, p. 369-388.

Begin, Z.B., Meyer, D.F. & Schumm, S.A. (1980b) Sediment production of alluvial channels in response to base level lowering: Trans. Amer. Soc. Agric. Eng., v. 23, p. 1183-1188.

Begin, Z.B., Meyer, D.F. & Schumm, S.A. (1981) Development of longitudinal profiles of alluvial channels in response to base level lowering: Earth Surface Processes and Landforms, v. 6, p. 49-68.

Carslaw, M.S. and Jaeger, J.C. (1959) Conduction of Heat in Solids. Oxford, Clarendon Press, 510 pp.

Curtis, N.M. (1975) Erosion and sediment yield in New Mexico: Proc. 3rd Federal Inter-Agency Symp. on Sedimentation. Sedimentation Committee, Water Resources Council, p. 1-81-1-89.

Daniels, R.B. (1960) Entrenchment of the Willow Drainage Ditch, Harrison County, Iowa: Amer. Jour. Sci., v. 258, p. 161-176.

Gessler, J. (1971) Aggradation and degradation. In: Shen, H.W. (ed.) River Mechanics, Fort Collins, Colorado, v. 1, p. 8-1-8-24.

Hadley, R.F., and Shawn, L.M. (1976) Relation of erosion to sediment yield: Proc. 3rd Federal Inter-Agency Symp. on Sedimentation. Sedimentation Committee, Water Resources Council, 1-132-1-139.

Heede, B.H. (1975) Stages of development of gullies in the West. In: Present and Prospective Technology for Predicting Sediment Yields and Sources: U.S. Dept. Agriculture, Agr. Res. Service Report, ARS-5-40.

Kellerhals, R. (1982) Effects of river regulation on channel stability. In: Hey, R.D., Bathurst, J.C. & Thorne, C.R. (eds.): Gravel Bed Rivers, John Wiley & Sons, New York, p. 685-705.

Kirkby, M.J. & Kirkby, A.V. (1969) Erosion and deposition on a beach raised by the 1964 earthquake, Montague Island, Alaska. U.S. Geol. Prof. Paper, 543-H, 41 pp.

Li, R-M. & Simons, D.B. (1982) Dynamic modelling of channel response. In: Hey, R.D., Bathurst, J.C. & Thorne, C.R. (eds.): Gravel Bed Rivers, John Wiley & Sons, New York, p. 469-485.

Livesly, R. (1976) The sedimentary influence of a tributary stream growth of the Niobrara Delta. Proc. 3rd Federal Inter-Agency Symp.

on Sedimentation, Sedimentation Committee, Water Resources Council, 4-127-4-137.

Mosley, M.P. (1984) Response of the Ohau River and delta to lake level lowering: Earth Surface Processes and Landforms, v. 9, p. 181-187.

Nordin, K.F. (1964) Study of channel erosion and sediment transport. Amer. Soc. Civil Eng., Jour. Hydraulics Div., v. 90, p. 173-192.

Overton, D.E. & Meadows, M.E. (1976) Stormwater Modelling. Academic Press, New York, 358 pp.

Pickup, G. (1975) Downstream variations in morphology, flow conditions and sediment transport in an eroding channel. Z. Geomorphology, N.F., v. 19, p. 443-459.

Schumm, S.A. & Hadley, R.F. (1957) Arroyos and the semi arid cycle of erosion. Amer. Jour. Sci., v. 255, p. 161-174.

Schumm, S.A., Harvey, M.D. & Watson, C.C. (1984) Incised Channels - Morphology, Dynamics and Control. Water Resources Publications, Littleton, Colorado, 200 pp.

Simons, D.B. & Li, R.M. (1980) Erosion and sedimentation analyses of Dry Creek, Sonomo County, California. Simons, Li and Associates, Inc., Fort Collins, CO, Report, February, 1980, 168 pp.

Vanoni, V.A. (1977) Sedimentation Engineering. Amer. Soc. Civil Eng., New York, 745 pp.