Frequency domain analysis of sediment delivery processes

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ABSTRACT. Frequency domain analysis is outlined as a method for determining the behaviour of sediment delivery processes during storm events. The discussion centers on the use of the spectrum as a descriptor of frequency behaviour. Due to the relatively short nature of storm event series, the Maximum Entropy Method is suggested for calculating spectral estimates.

INTRODUCTION

Research on aspects of sediment delivery problems has benefited greatly from improved monitoring and data logging equipment. One of the major benefits has been the collection of data sets of a temporal nature and these data sets can be used to assess the variation in sediment delivery during storm events. An example of such an approach is the study of responses of suspended sediment concentration to discharge using sedigraphs and hydrographs, or alternately, combining concentration and discharge into a single graph in the form of a hysteresis plot. The important aspect of this type of analysis is that it considers the sequential nature of the data series or indicates temporal changes in the relationship between variables.

Though storm event data sets have proven useful for insights on sediment delivery processes, most of the analysis on them has been in the time domain where simple temporal plots are examined. With a transfer to the frequency domain, there is a possibility that further information on the dynamic nature of delivery processes can be obtained. This information is in the form of the spectrum which indicates the important frequencies or scales operating in a process.

The following discussion gives details on the application of frequency domain analysis to data related to sediment delivery. The general theory of spectral analysis is briefly outlined in order to indicate the nature of the information which is contained in the spectrum. Methods of estimating the spectrum are then detailed. These methods overcome the theoretical implications resulting from frequency domain analysis of short time series which are usually the case for most storm event data sets. Finally, an example is given in which spectra are used to differentiate between suspended sediment response types.

SPECTRAL ANALYSIS

The spectrum

Spectral analysis is simply a statistical technique which reorganises a time series, or set of sequentially ordered data, into a framework in which the series can be better understood. The series is considered to be the sum of a set of symmetrical wave functions with each wave having a specific frequency, amplitude and phase (Jenkins & Watts 1969) and the reorganistion is based on the properties of these wave functions.

The spectrum of a time series is best understood by means of an analogy taken from optics. When white light, which is made up of a number of frequencies, is passed through a prism, the spectrum is the result. The prism has, in effect, decomposed the mixture of frequencies in the original light into distinct frequency bands or colours. The spectrum is simply a frequency domain representation of the properties of the light and gives information concerning the nature of the light's frequency components.

In a similar fashion, a time series can be decomposed into orthogonal frequency components by means of generalised Fourier analysis (Jenkins & Watts 1968) and the spectrum generated. Fig. 1 gives an example of such a spectrum and is simply the plot of the spectral estimate against frequency. An important property of spectra, calculated in this manner, is that the area under the curve is equivalent to the variance of the original time series. Thus, a peak in the spectrum indicates an important frequency component in terms of explaining a large proportion of variance in the original series.

The information given by the spectrum can be used to determine the dynamic behaviour of any time series. The frequencies corresponding to peaks in the spectrum are important in terms of variance explained and can be viewed as dominant scales or levels which are operating in the process represented by the time series. It should be stressed though, that the spectrum is merely a starting point to understanding processes and while it isolates major scales at which a process is operating, these scales still have to be given physical meaning.

The basic concepts underlying the spectrum form the cornerstone for the majority of other techniques used in time series analysis. Filtering theory is based on the fact that a set of weights moved over a series either amplifies or attenuates various frequencies in the series. Cross spectral analysis is an extension of spectral analysis and determines the relationships between the frequency components in two or more series. It can be interpreted as dynamic regression in that the relationship between the two series is done at the frequency or scale level (Pickup & Rieger 1979).

Estimating the spectrum

The characteristics of storm event time series play an important role in selecting one of the various methodologies which have been developed for spectral estimation. The series tend to be relatively short in terms of number of observations. Where automatic water samplers are used, the series of suspended sediment concentration may contain 20-100 observations, though the number of observations may be larger in the case of turbidity meters. The series may also contain truncated low frequency components which is especially true for a single rise discharge series.

Because of these characteristics, the more traditional methods of spectral estimation, as outlined by Jenkins & Watts (1969), cannot be used for storm event series. Lacoss (1971) details the problems associated with the application of the traditional methodology to such series. Briefly, the problems relate to the stability of the estimates and to poor resolution. In the case of poor resolution,important peaks may be missing or peaks may be shifted relative to their true frequencies.

The Maximum Entropy Method (MEM) of calculating spectral estimates overcomes the above problems related to the nature of storm event series. MEM was first developed by geophysicists who were confronted with data series similar to those generated during storm events in drainage basins. MEM has the properties of giving high resolution for short series and being able to resolve truncated frequency components within a series (Ulrych 1972). Ulrych & Bishop (1974) detail the theory underlying MEM, give examples of its resolution properties and provide the source code of a Fortran subroutine which can be used for MEM calculations.

FREQUENCY DOMAIN ANALYSIS OF SEDIMENT RESPONSES

The analysis of suspended sediment response to discharge during storm events can be performed in either the time or frequency domains. In effect, both types of analysis complement one another. Time domain analysis can first be used to give an indication of the complexity and variation of response patterns, then frequency domain analysis can be used to isolate the dominant scales which might be part of the process operation between discharge and suspended sediment concentration. Such an approach was used by Rieger & Olive (in press) and is briefly outlined here to demonstrate the merits of the frequency domain approach.

The initial time domain analysis (Olive & Rieger 1985) used hysteresis diagrams to determine patterns of suspended sediment response. Data for the analysis consisted of a total of 39 storm event series of concentration and discharge from five small forested drainage basins in south east New South Wales. The analysis showed seven distinct types of sediment response which can be grouped into two major categories. The first category consisted of recognisable patterns such as: sediment lag for both single rise and multiple rise storms, sediment lead for both single and multiple rise storms,sediment lead and lag for multiple rise storms, and sediment-discharge correlation for single rise storms. The second category consisted of events which showed no recognisable pattern and occurred in 40% of the events analysed. Overall, the patterns of response showed spatial variation in that different responses occurred in the five basins for a any given storm event, and temporal variation with a number of different responses occurring in a basin during a two year period.

Frequency domain analysis was then used in an attempt to differentiate between these two categories of response type. Because the analysis was based on the possible process operating between suspended sediment concentration and discharge, the data for each of these variables were combined to give a new variable which manifested that process. This new variable was generated by calculating the slope between concentration and discharge for adjacent points in time during a storm event. The resulting series represented the changing sequential relationship between the variables as measured by the slope angle along the hysteresis plots.

The spectra were calculated using MEM for the 39 storm event series and the generalised results are given in Fig 1. All the storm events were dominated by a low frequency component which may be interpreted as the broad loop in the hysteresis plots. In a similar fashion, all the storm events showed a high frequency component in the range 0.37 to 0.50 cycles hr^{-1}, which likely corresponds to fluctuations about the broad hysteresis loops. The storm events which showed no

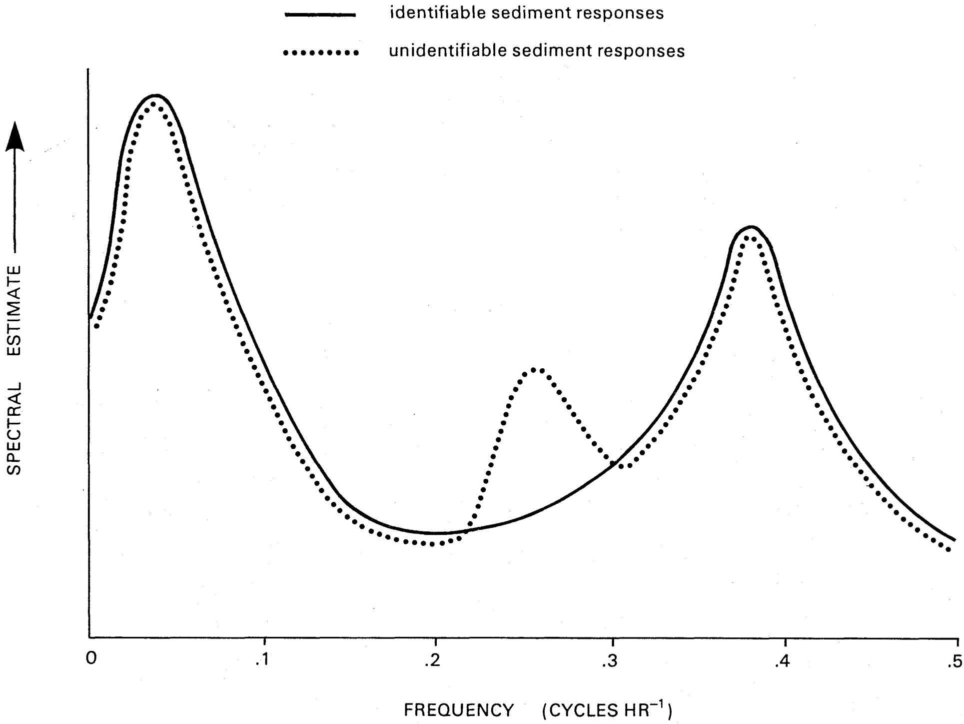

FIG 1 Generalised spectra for suspended sediment responses of storm event data.

identifiable response pattern were differentiated from events with recognisable patterns by a mid-frequency component in the range 0.20 to 0.33 cycles hr^{-1}. Present research by the authors is attempting to interpret this mid frequency component in terms of basin and/or storm characteristics.

CONCLUSIONS

Frequency domain analysis, in the form of spectral analysis, shows some merit as a technique for determining the behaviour of sediment delivery processes. The spectrum is similar to analysis of variance by frequency and can be used to identify important frequency components or scales which might be operating in a process. Recent developments in computational methods have overcome the problems associated with spectral estimation of the type of data series commonly collected by researchers studying storm event based phenomena. One of these methods is MEM and its appeal is based on its excellent resolution properties and its ease of implementation.

While one particular use of MEM spectra was outlined here, there are a number of other possible areas in sediment delivery research where it can be applied. One such area might be soil moisture measurements during storm events. Regardless of the application, researchers should never lose sight of the fact that frequency domain analysis simply reorganises data in a different framework. It is not a new, magical, analytical technique which will answer all questions, but rather it is the starting point for the possible better understanding of processes.

ACKNOWLEDGEMENTS

Funding for the research carried out in this paper was provided by the University of New South Wales and the Australian Research Grants Scheme. The Forestry Commission of New South Wales provided some monitoring equipment and the personel for equipment maintenance. We thank Susie Salisbury for her dedicated work as a research assistant and Professor B.G. Thom for his moral support.

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