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Development and application of simple water balance models to understand the relationship between climate and water resources

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ABSTRACT Simple water balance models, one linear and analytic the other nonlinear and numerical, show that runoff is highly sensitive to the range of climatic change expected to be caused by increasing greenhouse gases in the atmosphere. In the southeastern quadrant of the U.S. arid places will be much more affected than humid; low flows, more than high. Water resources are more sensitive to precipitation change than to potential evapotranspiration.

APPROACH

How much will a changed atmosphere actually affect our water supply? Our goal is to find a simple scientific basis, using water balance models, to calculate how the change in incoming water, outgoing evaporation and changed temperatures will change the water stored in soil and running in rivers where we can store it in reservoirs and channel, pump and pipe it to water crops, quench thirst and flush waste.

We borrow the important concept of <u>elasticity</u> from the field of economics to quantify our estimates of sensitivity. Accordingly, elasticity, Φ, of runoff to a climate variable, X, is defined as:

$$\frac{\delta Q}{Q} = \Phi \frac{\delta X}{X} \qquad (1)$$

so that a change in X of δX will produce a change in runoff of δQ. In percentage terms, the percentage change in runoff will be Φ times the percentage change in X. If Φ is greater than 1.0, the change in runoff would be greater than the change in precipitation. Then, runoff would be "elastic" with respect to X.

In the following sections it will be shown that runoff in parts of the U.S. is extremely sensitive to certain kinds of climatic change. Values of Φ range from less than 1.0 to more than 4.0.

<u>Water Balance Models</u>

Water balance models have been formulated in varying degrees of complexity. An example of a simple one is that of Thornthwaite and Mather (1955). They related climate and runoff in one equation. More complex, computer models were developed in the 1960's and 1970's. These were termed "soil moisture accounting models" and are operated by time steps of a few minutes to a few hours. Examples of soil moisture accounting models are the Stanford Watershed Model

(1966) and the Sacramento Model (1973). The simplest water balance models operate by monthly steps whereas the soil moisture accounting models operate by steps of hours or minutes.

Two different basic approaches are possible to water balance models. The usual one is to construct a computer simulation model and then calculate numerical solutions to specific cases. A second approach is to express the processes in a few equations and then solve them analytically. The advantage of the first approach is that highly complex hydrologic processes can be simulated. The advantage of the second approach is a better theoretical basis for conclusions about model behavior over a wide range of conditions. In this study we take both approaches.

First, a linear, analytical water balance model (LINEAR MODEL) was constructed for broad insight into how precipitation and potential evapotranspiration affect runoff. LINEAR MODEL was kept simple to permit analytical solution. Then a more realistic simulation model, referred to as the nonlinear water balance model (NONLINEAR MODEL) was constructed. The influences of physical processes can be easily seen in the linear model, giving clear insights into the effect of changing climate on runoff. On the other hand, the nonlinear model should test whether the processes glossed over in the simplifications needed for the linear model are sufficiently important to cause its simplicity to mislead us about changes in runoff following arrival of a new climate.

INSIGHTS FROM A LINEAR MODEL

The continuous, dynamic water balance of a river basin obeys the fundamental equation of hydrology:

$$I - O = \frac{dS}{dt} \qquad (3)$$

Neglecting transfers of groundwater across basin boundaries, inflow, I, is the average precipitation, P, over the basin. The outflow, O, is the sum of the evapotranspiration, E_T, and the runoff, Q. Therefore, the water balance is:

$$P - E_T - Q = \frac{dS}{dt} \qquad (4)$$

The term $\frac{dS}{dt}$ is the rate of change of all water stored within the basin. This equation involves no approximation. It is the mass conservation law of physics. The trick in hydrologic modeling is to approximate how these variables, averaged over the basin and time, depend on each other. The idea is to reduce complex space-time relationships to simple mathematical approximations.

One way to give more physical meaning to the storage variable, S, is to introduce the deficit variable, D, defined as:

$$D = S^* - S \qquad (5)$$

where S^* is the maximum or limit of S. When D is zero: the storage system is saturated to its limit; water evaporates and transpires at the rate of evaporation from a free water surface (called the rate of potential evapotranspiration), E_P; and, precipitation cannot infiltrate and becomes storm runoff. When D is not zero, evapotranspiration, ET, is slower than the potential rate, E_P; much of the precipitation infiltrates into the soil; and a portion, α, of the precipitation becomes storm runoff.

Runoff is the sum of a storm component, Q_s, and a groundwater component, Q_g. A simple expression for Q is:

$$Q = Q_s + Q_g = \alpha P + k S \tag{6}$$

where α and k appear as constant coefficients for storm runoff, α, and groundwater runoff, k. In reality, α and k are not constant **and** will be reconsidered below in more detail when we present the nonlinear model. For now, the variability of α and k will be ignored. Maximum Q_g occurs when $Q_g = k S^*$. Parameter k is a measure of how fast water flows from groundwater to streams. This can be estimated from the exponential decay of streamflow hydrographs during dry spells.

Typically, throughout the world, the long term average potential evapotranspiration varies during the year periodically with a dominant annual period:

$$E_P = E_{Pavg} \left[1 + \epsilon \sin(wt) \right] \tag{7}$$

where E_{Pavg} is the annual average, ϵ is the amplitude and wt varies from 0 to 2π during a period of 1 year. A map of E_{Pavg} for the U.S. is available (Farnsworth and Thompson, 1982). On average, ϵ is about 0.70, causing the calculated E_P to change from 30 percent of average during winter to 170 percent during summer.

The actual evapotranspiration, E_T, is less than or equal to E_P. A simple equation for this statement is :

$$E_T = \left[\frac{S}{S^*} \right] E_P \tag{8}$$

When equations 6 to 8 are substituted into equation 4, the following differential equation for S results:

$$\frac{dS}{dt} + \left[\frac{kS^* + E_{Pavg}}{S^*} \right] \left[1 + \frac{\epsilon E_{Pavg}}{kS^* + E_{Pavg}} \sin(wt) \right] S = (1 - \alpha) P \tag{9}$$

Equation 9 emphasizes the forcing of storage and evaporation by precipitation, P. The equation is written in this particular form to emphasize the relative magnitude of the time-varying part of the coefficient of S which also aids in solving the equation. From an equation for P, as a function P(t), an analytical solution may be found for S(t). Then, equation 6 can be used to find the hydrograph of runoff, Q(t). Together, equations 6 and 9 form the Linear Analytical Water Balance Model (LINEAR MODEL). In the special case, where P is taken as a constant, P_{avg}, the average annual precipitation; the steady state solution (to first order approximation) for Q is:

$$Q_{avg} = \left[\frac{1 - \alpha}{1 + \beta} + \alpha \right] P_{avg} = C\, P_{avg} \qquad (10)$$

where Q_{avg} is the mean annual runoff. Here C is a runoff coefficient reflecting the relative contributions from groundwater runoff and storm runoff, respectively. Coefficient C is a function of only two parameters, α and β. Recall that α is the proportion of precipitation that becomes storm runoff. Parameter β is the ratio of the average potential evapotranspiration rate to the maximum rate of groundwater runoff:

$$\beta = \frac{E_{Pavg}}{kS^*} \qquad (11)$$

Recall that kS^* represents the maximum rate of groundwater runoff. Accordingly, β measures the relative balance between evaporation and groundwater runoff, a ratio usually much greater than 1.0.

The LINEAR MODEL of equation 10 achieves our goal of showing how runoff will change with changing climate. Although necessarily approximate, LINEAR MODEL simply and elegantly shows the change C, or $d(Q_{avg})/d(P_{avg})$, of runoff with precipitation and its dependence on storm runoff, α, and the ratio β of evaporation to outflow from groundwater. According to the LINEAR MODEL, the elasticity of runoff to an increase in precipitation is

$$\Phi_P = 1.0 \qquad (12)$$

This means a 10 percent change in precipitation would be expected to produce a 10 percent change in runoff. This particular result occurs because this simple water balance model is a linear model. It is well known, from experience with more complex water balance models, that natural river basins behave nonlinearly with respect to precipitation. This means that Φ_P is actually greater than 1.0 and varies depending on the climate. Therefore, in the next sections, a nonlinear monthly water balance model is developed and used to simulate the behavior of specific river basins in the southeast.

The elasticity of runoff with respect to potential evapotranspiration, according to equation 10, is

$$\Phi_E = -\frac{\beta\,(1 - \alpha)}{C\,(1 + \beta)^2} \qquad (13)$$

Equation 13 shows that Φ_E depends on both α and β. The elasticity is always negative, meaning that an increase in potential evapotranspiration will cause a decrease in runoff. These LINEAR MODEL results suggest runoff elasticity to evapotranspiration:

a. is less than runoff elasticity to precipitation;

b. increases as evapotranspiration increases;

c. decreases as the storm component of runoff increases.

The magnitude of the elasticities from the LINEAR MODEL are different than the values that would be calculated by more detailed representation of nonlinear processes. This will be examined below using the nonlinear monthly water balance model presented in the next section.

NONLINEAR MONTHLY WATER BALANCE MODEL

The LINEAR MODEL worked reasonably well to explain the long-term seasonal variability of runoff from a humid basin in eastern China. But it failed to explain the runoff from a semi-arid basin in Oklahoma. The main reason seemed that α is not a constant but depends on both precipitation, P, the moisture deficit, D, and the way both P and D vary throughout the basin. The failure of the LINEAR MODEL to account for important processes in the hydrologic cycle was not surprising. The heterogeneity and complexity of river basins, makes impossible any hydrologic model's explaining all details. The processes seemed far too complex to proceed further with an analytical model and required numerical methods to solve the equations. A hydrologic simulation model was required, and the strategic issue was how to approach its development.

An essential requirement was readily available climate and streamflow data. Another was extensive calibration of parameters must not be required so that the model could easily be applied to a large number of basins. To minimize assumptions, the model was kept as simple as possible. Accordingly, the starting point was the water balance of the LINEAR MODEL. The model would be applied to river basins that range in size from about 10 square miles up to about 10,000 square miles. This is the range of area sizes with most data. Also, basins in this size range are more likely to be representative of the region where they are located than much smaller or larger basins.

The first modification was to keep account of a moisture deficit variable, D_t, rather than the storage, S. The maximum deficit, D_{max}, in an extreme drought when evapotranspiration ceases is introduced. It functions in a manner similar to S^*. The actual evapotranspiration, E_{Tt}, in any month, t, is a fraction of potential, E_{Pt}, decreasing from 1.0 when D_t is zero to zero when $D_t = D_{max}$:

$$E_{Tt} = E_{Pt} \left[\frac{D_{max} - D_t}{D_{max}} \right] \tag{14}$$

A monthly increment was selected because many years of monthly data were available. A major data processing effort would have been required for data having a daily or shorter time step. A shorter time step than a month would have been preferable because storm runoff occurs at shorter time steps. Storm runoff occurs when water at or near the surface runs off before it can percolate, meaning the temporal pattern of rainfall is important in estimating storm runoff. Different rainfall patterns could produce the same monthly rainfall but different storm runoff. Another reason to prefer less than monthly time steps is that some precipitation in a month will evaporate within that month by processes operating near the surface. Many rainy days in a month and much evapotranspiration leave little storm runoff from much rain. On the other hand, if the rain occurred in one storm, there could be some runoff even if the ground were dry.

To account for processes operating at time steps shorter than one month, a parameter θ was introduced. It is the proportion of E_{Tt} that must be satisfied from the precipitation in the current month before storm runoff or infiltration can occur. Precipitation in excess of $\theta \cdot E_{Tt}$ is split between storm runoff and infiltration.

The rule for computing the amount of storm runoff is based on a simple formula that satisfies some common sense boundary conditions. First a proportion, z, of the moisture deficit must be satisfied by infiltration before any storm runoff. This means precipitation contributing to storm runoff is:

$$PXX = P_t - \theta E_{Tt} - z D_t \tag{15}$$

If PXX is positive, storm runoff can occur. Another condition is that, in the limit as precipitation becomes very large, storm runoff approaches PXX less the moisture deficit at the beginning of the month. This limit should be approached asymptotically. The proportion of PXX that becomes storm runoff is assumed to increase accordingly. A mathematical statement of these conditions is:

$$Q_{st} = \left[\frac{PXX}{PXX + D_t} \right] PXX \tag{16}$$

A similar equation occurs as an intermediate step in the derivation of a runoff procedure known as the Soil Conservation Service Curve Number method. That method has been widely used in engineering practice throughout the world. In our model D_t varies with time and depends at least as much on climate as on the geologic and soil conditions in a basin.

It can be shown that if D_t and PXX are both exponentially distributed over a basin, and if the local storm runoff is the excess of PXX locally over D_t locally, then, equation 16 gives exactly the total amount of storm runoff from the basin! This is true even though D_t and PXX may each be spatially correlated, but not cross-correlated with each other.

Groundwater runoff is assumed to vary with deficit D_t. If D_t exceeds S_{max}, the ground water table is assumed to fall below the streams, and groundwater runoff ceases. The equation used is:

$$Q_{gt} = k\,(S_{max} - D_t) \tag{17}$$

If D_t should exceed S_{max}, Q_{gt} is zero and streams would percolate to groundwater. No attempt is made in our model to account for such losses of storm runoff from streams to groundwater.
Finally, the change in moisture deficit for the month is:

$$D_{t+1} = D_t - P_t + E_{Tt} + Q_{st} + Q_{gt} \tag{18}$$

The monthly water balance model was calibrated to the Oklahoma and China river basins first used to test the LINEAR MODEL. Then, values of the five parameters (D_{max}, θ, z, S_{max}, and k) were selected to simulate runoff from 52 river basins located throughout the southeastern U.S. The parameters were held constant from basin to basin. The idea was to see if the average annual runoff estimated by the model agreed well enough with observed averages to avoid calibrating the model separately to each basin. If this were possible, the parameters would be independent of climate and would not likely change if the climate were to change.

The estimated mean annual runoff according to the NONLINEAR MODEL for 52 basins is presented on a contour map in figure 2. This map should be compared to the map of observed mean annual runoff in figure 1. Observed and computed annual runoff for each of the 52 basins are compared in figure 3. Error bounds of ±20 percent are shown on either side of the line of equal observed and computed values; 49 basins lie inside these limits.

Another important test is whether the model preserves the year-to-year variability of the runoff measured by the standard deviation of the annual runoff. Observed and computed standard deviations of annual runoff for each of the 52 basins are compared in figure 4; 51 of all the basins lie within ±20 percent error limits.

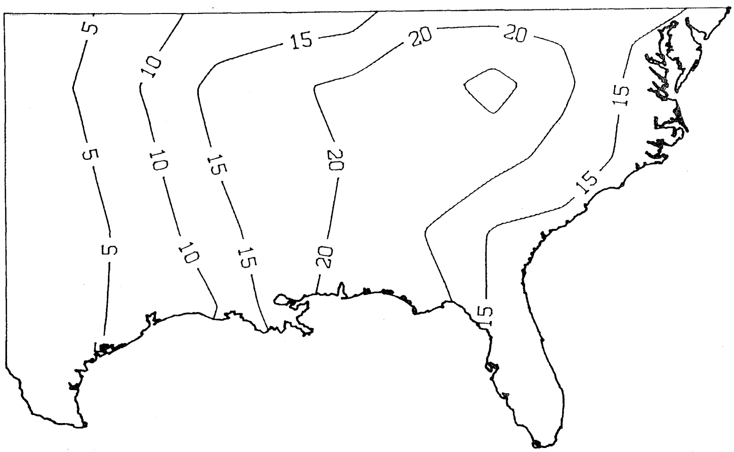

Fig. 1 Observed mean annual runoff (inches).

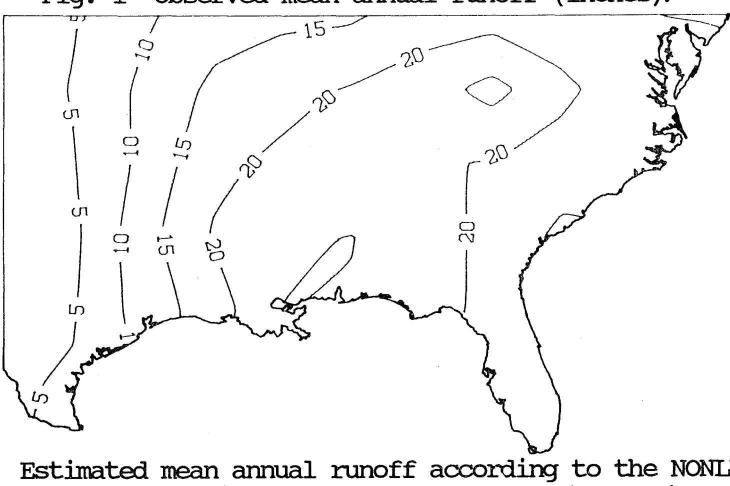

Fig. 2 Estimated mean annual runoff according to the NONLINEAR MODEL with fixed parameters over the entire region.

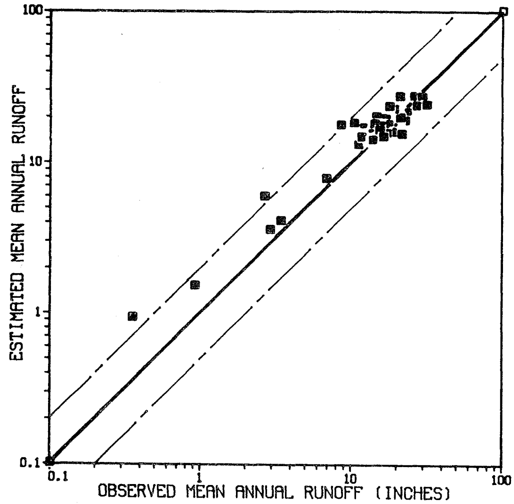

Fig. 3 Model estimated mean annual runoff vs observed for 52 river basins.

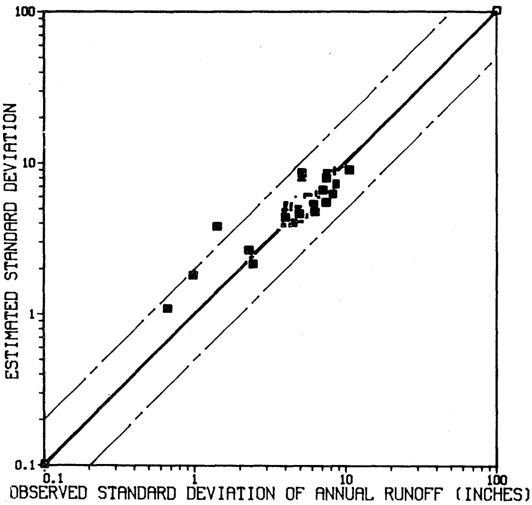

Fig. 4 Model estimated standard deviation of annual runoff vs observed for 52 river basins.

SENSITIVITY OF WATER RESOURCES TO CLIMATIC CHANGE

The sensitivity of mean annual runoff to a 10 percent change in either precipitation or potential evapotranspiration varies widely over the southeastern quadrant of the U.S. as shown in figures 5 and 6. A 10 percent change in precipitation will produce between 20 and 45 percent changes in mean annual runoff. The greatest changes are in the states from Nebraska to Texas. The elasticity, Φ_P, ranges from 1.9 to 4.5, exceeding the LINEAR MODEL estimate of 1.0. Elasticity increases as the difference of precipitation minus potential evapotranspiration decreases. These results are almost identical to results obtained by Nemec and Schaake (1982) from a more detailed simulation at 6-hour steps of two basins in this region.

Increases in potential evapotranspiration cause changes in annual runoff as mapped in Figure 6. As the LINEAR MODEL predicted,

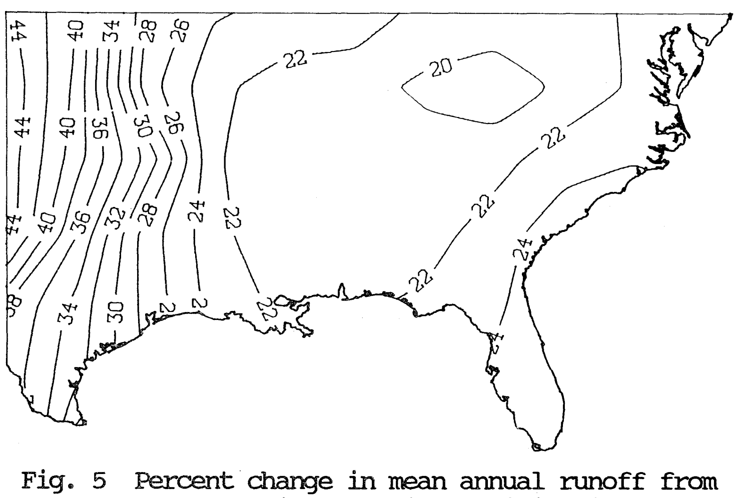

Fig. 5 Percent change in mean annual runoff from a 10 percent increase in precipitation.

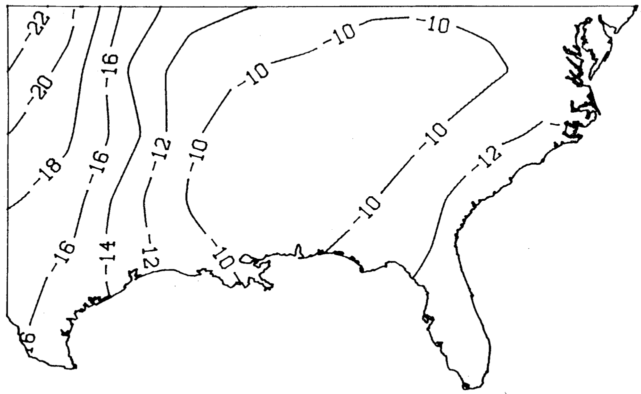

Fig. 6 Percent change in mean annual runoff from a 10 percent increase in potential evapotranspiration

the elasticities, Φ_E of about 1 to 3 are smaller than the Φ_P for precipitation change, and the Φ_E increase from moist east to arid west with increasing β.

Sensitivity of the standard deviation of annual runoff to precipitation change ranges from Φ_P of 1.5 in the east to 3 in the west. This means the variability of annual runoff, as measured by the standard deviation, is less sensitive to precipitation change than the mean annual runoff.

Sensitivity of the standard deviation of annual runoff to potential evapotranspiration change ranges from Φ_E of 0.4 in the east to 1.2 in the west. This also is less than the sensitivity of the mean annual runoff.

Sensitivity Analysis of Floods

A monthly simulation model only generally indicates how floods might be affected by climatic change. The best that can be done is use the maximum monthly runoff in a year as a surrogate for the annual flood. The spatial pattern of sensitivity of the mean annual flood to climatic change resembles the sensitivity of the mean annual runoff, except the mean annual flood is less sensitive.

The standard deviation of the annual flood measures variability from year to year. If this statistic should increase, floods now considered rare would occur more frequently. Our results show the variability of the annual flood is less sensitive to climatic change than the variability of annual runoff. Change in potential evapotranspiration affects variability of the annual flood little. This seems reasonable because change in evapotranspiration would only change the initial conditions of the basin before the flood.

Sensitivity Analysis of Drought

No single statistic measures drought. A measure important to water quality is minimum monthly runoff because water quality standards are keyed to low flow statistics such as the minimum average 7-day flow to be expected once in 10 years. But in arid parts, streams dry up part of the year; climatic changes will increase or decrease the duration of dry streams. To sense how low flows are affected by climatic change, the average July - December runoff volume was analyzed. Less than half the annual runoff occurs during this period everywhere in the region except in Southern Florida. This drought statistic was found to be more sensitive to climatic change than mean annual runoff. The greatest change in sensitivity occurred in the humid parts, suggesting the relative effects of climatic change on water quality might be greatest in humid areas.

Sensitivity Analysis of Reservoir Yield

Reliable supply of water from reservoirs depends on their size, operating rules, watershed, and climate. A measure of reliable supply is the "safe yield" they can provide with some limited chance of imposing conservation. A hypothetical reservoir with a capacity of a quarter of the mean annual flow was assumed in each of the 52 basins and operated to supply the safe yield constantly. The safe yields, as percent of annual streamflow, were calculated by simulating reservoir operation with both the observed historical monthly streamflow and with estimated historical streamflow computed by the NONLINEAR MODEL from historical climate data. Most basins had more than 40 years of historical data so a failure probability of 10 percent was used to assure stable computational results. Safe Yields from the observed and estimated streamflow for the 52 basins agreed, except for two basins, within 20 percent error limits, verifying the NONLINEAR MODEL represents hydrologic processes important for reservoir operation.

The elasticity of safe yield to changes in precipitation rises from about 1.5 in the east to 4 in the west. Safe yield elasticity to changes in potential evapotranspiration, however, was uniformly near 1.

ACKNOWLEDGEMENT

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