Empirical relationships for the transport capacity of overland flow

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Abstract This paper reports the results of an experimental study on the transport capacity of overland flow. Experiments were carried out with five materials varying from silt to coarse sand, using a wide range of discharges and slopes, in a flume with a plane bed. The experiments provided the necessary data to establish highly correlated relationships between the sediment the flow and different hvdraulic transport capacity of parameters, which are a necessary element of all physicallybased erosion models. Some important characteristics of these relationships are identified. These include the limited range of validity of individual relationships and the apparent lack of connection between initiation of sediment motion and sediment transport. Some relationships can also be applied to rough surfaces and to surfaces with a vegetation cover. It is believed that the proposed set of empirical relationships can be of use in the study of many aspects of soil erosion and deposition.

INTRODUCTION

The sediment transporting capacity of overland flow is a parameter of fundamental importance in the physically-based description of soil erosion and deposition processes. The transport capacity of the flow at a given point equals the maximum net erosion potential upslope. Flow incision, for example in a tractor wheeling, will only occur when the transport capacity of the flow is sufficiently high to evacuate all the material that is transported into the flowpath from the inter-rill areas. Knowledge of the amount of sediment that can be transported over a concave, basal slope segment (where sedimentation takes place) will often be of crucial importance when the response of a first-order watercourse has to be understood and predicted. The variable transportability of grains of different size and density can cause size sorting during erosion and transport. Furthermore, some researchers state that the detachment capacity of the flow can be directly related to its transport capacity. Foster & Meyer (1972a) express this relationship as:

$$D_r/D_c + q_s/T_c = 1$$

(1)

where:

 D_c = the detachment rate for clear water flow;

 q_s^c = the unit solid discharge; and T_c = the transport capacity of the flow.

Foster (1982) transforms this relationship into:

$$D_r = \alpha (T_c - q_s)$$

(2)

where:

$$\alpha = D_c / T_c$$

Equation (2) describes the detachment rate in terms of the difference between transport capacity and actual transport rate. It implies that the detachment rate will decline as the transport capacity is approached.

Almost all physically-based erosion models that have been developed during the last two decades contain a sediment transport capacity equation. In most cases, a formula that has been developed for rivers is used, although the empirical coefficients are sometimes modified. Foster & Meyer (1972b) proposed the formula of Yalin (1963) as being the most applicable to shallow flow conditions. This formula has subsequently been used by several other modellers (Dillahah & Beasley, 1983; Kahnbilvardi et al., 1983; Park et al., 1982). Savat (1979) found that sediment concentrations measured during recirculating flume experiments on a loamy soil were generally only about 25% of those predicted by the Yalin formula. Alonso et al., (1981) concluded that the formula gave good results for sheet flow on concave surfaces with relatively low sediment loads. Preliminary results obtained by the author showed that the transport capacity of overland flow on steep slopes could be much higher than predicted by the expression of Yalin (Govers, 1985).

Other formulae that have been employed include the Ackers & White (1973) formula used by Morgan (1980); the Bagnold (1966) formula used by Rose et al. (1983) and Gilley et al. (1985); the Kalinske (1942) formula used by Komura (1976) and Mossaad & Wu (1984); and the Yang (1973) formula used by Wilson et al. (1984).

A different approach has been proposed by Tödten (1976) and Prasad & Singh (1982). They derive complex transport expressions from basic physical considerations of sediment detachment and movement. Other modellers relate transport capacity directly with a simple hydraulic parameter such as shear stress (David & Beer, 1975; Croley, 1982) or stream power (Kirkby, 1980).

The great variety of formulae that have been used in theoretical approaches is largely due to the fact that insufficient experimental data are available to test the validity of the proposed equations. Nevertheless, it cannot be expected that any transport formula can be successfully applied without suitable calibration over the whole range of field conditions. Even the more sophisticated river formulae (e.g. Yalin, 1963) contain one or more empirically determined constants. Calibration requires the availability of an experimental

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data set acquired under controlled conditions.

In this paper experimental results on the transport capacity of overland flow are presented. Experiments were carried out on slopes ranging from 1 to 12° and with unit discharges between 2 and $100 \text{ cm}^3 \text{ cm}^{-1} \text{ s}^{-1}$. The results were analysed, not by comparing them with predictions based on existing formulae, but by relating them to hydraulic parameters which are commonly considered to be relevant to the transport capacity of river flow. This approach has the advantage of leading to simple expressions, which are easy to manipulate, whilst the best correlation that can be obtained might be better than that associated with an existing formula.

EXPERIMENTAL PROCEDURE

All experiments were carried out in a flume 12 m long and 0.117 m wide of which only 6 m was effectively used, in order to avoid water surface instability (Fig. 1). The bottom of the flume was covered with a 1 cm thick layer of sediment that was carefully smoothed. Water was then applied and, when equilibrium was established, water and sediment were collected at the lower end of the flume during a short time interval.

This experimental procedure is clearly different from those used to determine the transport capacity of river flows. These involve either recirculating sediment and water over a movable bed until equilibrium is established (e.g. Rathbun & Guy, 1967), or adding sediment to clear water flowing over a moveable bed and varying the sediment supply rate until stable conditions are obtained (e.g. Luque & Van Beek, 1976). These procedures were not feasible because of the high sediment concentrations that can be transported by overland flow on steeper slopes. The only disadvantage of the method described above is that in some cases it might be possible that sediment transport capacity is not reached within a distance of 6 m. However, experiments carried out with an effective length of only 3 m yielded comparable results (Fig. 2). It can therefore be concluded that a length of 6 m is sufficient to reach full transport capacity.

Experiments were carried out using five well-sorted quartz materials with a median grain size varying from silt to coarse sand (Table 1). Slopes were 1, 2, 5, 8 and 12° and unit discharges from 2 to 100 cm³ cm⁻¹ s⁻¹. In total, 436 measuring runs were undertaken.

The hydraulic characteristics of such flows can be accurately calculated using an algorithm developed by Savat (1980), providing no sediment is present. In his paper, Savat (1980) takes account of sediment load by increasing water viscosity, which leads to a lower Reynolds flow number and therefore to a reduction of mean velocity and to an increase of water depth.

A number of experiments were carried out to investigate the validity of this approach. Comparison of measured with predicted velocities revealed that, instead of a velocity decrease, a significant velocity increase (up to 40%) occurs (Fig. 3). The fact that the presence of suspended sediment increases the flow velocity in rivers has been known for a long time (e.g. Vanoni & Nomicos, 1959). Two mechanisms are considered to be responsible for this. The presence of sediment adds momentum to the flow and alters the turbulence structure. In



Fig. 1 View of the experimental flume.

earlier studies it was believed that the von Karman constant was reduced, but recent research stresses the impact of the sediment load on the wake term of the turbulent flow equation, such that the von Karman constant remains basically unaffected (Coleman, 1981, Parker & Coleman, 1986).

The models developed to describe turbulent flow carrying suspended sediment are not directly applicable to the situation considered here. This is especially the case for the coarsest material where the water film is often only a few grain diameters thick, so that one cannot speak of true suspension. Furthermore, sediment concentrations are so high that grain interactions become important, leading to the development of significant dispersive and tangential stresses (Bagnold, 1954). It can therefore be expected that the influence of grain size is less than when true suspension is being considered (Parker & Coleman, 1986).

In order to estimate mean velocity for our experimental results, the mean velocity was calculated according to the procedure of Savat (without taking into account the influence of sediment on viscosity). This estimate was



Fig. 2 Comparison of observed and predicted sediment concentrations. Measurements were made with an effective flume length of 3 m using material C. Predictions are based on the empirical S \overline{u} vs C and q_s vs Ω relationships, developed from experiments with material C on a 6 m long flume (see Figs. 6(a) and 7(c)).

Table 1 Characteristics of the materials used in the experiments $(CSF = Corey shape factor = c(ab)^{-1/2}$, where c = the shortest axis, b = the intermediate axis and a = the longest axis of the grain)

Material	D ₅₀ (µm)	CSF		
A	58	0.59		
B	127	0.79		
C	218	0.71		
D	414	0.64		
Ε	1098	0.66		



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flow velocity (\overline{u}_c : mean velocity as calculated using the algorithm of Savat for clear water, \overline{u}_s : measured mean velocity at a given sediment concentration).

then corrected using:

 $\overline{u}_s = \overline{u}_c / (1 - C)$ (3)

where: C = the volumetric sediment concentration;

 \overline{u}_c = the velocity calculated for clear water flow; and \overline{u}_s = the actual velocity of the sediment-laden water flow.

Hydraulic parameters were then calculated without taking into account the influence of the sediment on the specific weight of the fluid and fluid depth, because no information is at present available on grain velocities in overland flow. As a consequence the amount of sediment present per unit surface of the bed and therefore total shear stress and energy dissipation cannot be calculated.

ANALYSIS OF RESULTS

Relevant hydraulic parameters

Most of the older river transport formulae rely on the concept of excess

shear. Sediment transport capacity (expressed as a solid discharge) is then related to the excess shear stress, being the difference between the actual shear stress and the critical shear stress necessary to initiate movement. The shear stress is calculated as:

$$\Gamma = \rho g R S$$

where :

 ρ = the density of the fluid;

g = the gravitational acceleration;

R = the fluid depth; and

S = the slope.

Bagnold (1966) was the first to introduce an equation which was no longer based on a balance of forces, but on a balance of energy. He introduced the concept of stream power, which represents the amount of energy dissipated per unit of time and per unit of bed surface. The stream power can therefore be expressed as the product of the shear stress and the mean flow velocity. Later, he stated that there is only a unique relationship between sediment transport capacity and stream power if the depth of the flow is constant. Finally he proposed the following relationship (Bagnold, 1977, 1980):

$$q_s \sim (w - w_{cr})^{1.5} / (R^{2/3} D^{1/2})$$

where :

w = the stream power; and

 w_{cr} = the critical stream power value at which sediment movement starts.

The expression $(w - w_{cr})^{1.5}/R^{2/3}$ might be considered to be an effective stream power (Ω) , corrected for the influence of depth.

Yang (1972) introduced the concept of unit stream power, which is the amount of energy dissipated per unit time and per unit weight of the flow and which is equal to the product of slope and mean velocity. This quantity should not be related to solid discharge, but to sediment concentration, so:

 $\log C = A + B \log(S \overline{u})$

where :

 \overline{u} = the mean flow velocity; and

C = the concentration by weight, expressed in ppm.

Later, he developed more complex dimensionless equations, which are based on four dimensionless groups describing the flow and the sediment: one equation does take into account the existence of a critical stream power value required to initiate motion, while the other does not (Yang, 1973). The most important parameter is, according to Yang, the dimensionless unit stream power, which equals the stream power divided by the fall velocity of the particles. Recently, Moore & Burch (1986) concluded that the formula of Yang (1973) gave promising results with respect to the transport capacity

(4)

(6)

(5)

of sheet and rill flow.

In the following sections, our results are related to the parameters described above. Logarithms were used in the relationships involving solid discharge because of the wide range of absolute values.

Shear stress

A reasonable relationship between solid discharge and shear stress was found for all tested grain sizes (Fig. 4). The curves for materials A and B show a well defined break at a shear stress of about 20 g cm⁻¹ s⁻². Apart from this observation, all relationships are more or less linear. No clear tendency towards a vertical asymptote is present, which indicates that it is not necessary to introduce a critical shear stress into the relationship.

The slope of the regression equation decreases with increasing grain size. For the finest materials, the shear stress coefficient is greater than 4.0, while it approaches 2.5 for material E. More surprising is the fact that the intercept increases with grain size, so that at low shear stresses, the transport capacity of overland flow is higher for coarse than for fine sediment (Fig. 5).

The effective stream power

For materials C, D and E, solid discharge is well related to the effective stream power as derived by Bagnold (1980) (Fig. 6). For the finest materials, the relationship was much less satisfactory. Again, the relationships are more or less linear over the whole range. The influence of grain size is thus clearly different from that supposed by Bagnold (1980), who stated that sediment transport capacity should be inversely related to the square root of the grain diameter. The effective stream power coefficient is not constant but decreases with grain size. Furthermore, at low effective stream power values, sediment transport capacity increases with grain size.

The unit stream power

For the finest sediments a very good relationship exists between volumetric sediment concentration and unit stream power, at least if the data obtained on a 12° slope are excluded (Fig. 7). On this slope, sediment concentrations were considerably higher than on lower slopes at comparable unit stream power values, especially for low discharges. It should also be noted that a limiting volumetric concentration of about 0.32 was reached at high unit stream power values. A further increase of unit stream power did not cause an increase of sediment concentration. The unit stream power exponent increases with increasing grain size. Also, a critical value had to be introduced into the relationships, which appeared to be more or less independent of the grain size (S $\overline{u}_{cr} = 0.4 \text{ cm}^{-1}$).

Over the whole range covered by the equations, there is an increase of



Fig. 4 Fluid shear stress vs unit solid discharge relationships for all materials A-E.



Comparison of regression equations Fig. 5 relating unit solid discharge with fluid shear stress obtained for various materials (A-E).

transport capacity with increasing grain size. An attempt was made to include the influence of grain size directly, by using the dimensionless unit stream power, but no good result was obtained.

Validation

Only few data are available in the literature which can be readily compared with our own results. Probably the data which are most suitable for a first verification of the proposed relationships are those collected by Meyer & Monke (1965), Bubenzer et al. (1966) and Kramer & Meyer (1969), who all used the same experimental facility. Their papers discuss sediment loads measured at the basal end of a 2 m by 0.6 m flume at various slopes and using various total discharges. During the experiments, the bed of the flume was kept in dynamic equilibrium by adding sediment at the top of the flume using a sediment hopper. Meyer & Monke (1965), using glass beads, paid special attention to the influence of particle size and rainfall. Bubenzer et al. (1966) studied the effect of particle size and roughness, while Kramer & Meyer (1969) emphasised the effect of a vegetation cover, using glass beads of 33 and 121 µm.

As the applied water was allowed to move freely over the whole width of the flume, channelling of the flow occurred to various degrees depending on slope and unit discharge (Meyer & Monke, 1965). Unit discharge of the flow is therefore not known exactly and mean flow velocity and depth cannot be accurately calculated. However, Kramer & Meyer (1969) report mean flow velocities, so that for their data mean unit discharge and flow depth could be



Fig. 6 Effective stream power vs unit solid discharge relationships for materials C, D and E.

calculated using the procedure of Savat (1980) (see Govers & Rauws, 1986). Correction factors were then calculated from their data, allowing the estimation of unit discharge and the calculation of estimated mean velocity and depth for the data of Meyer & Monke (1965) and Bubenzer *et al.* (1966). Total discharges were always low, so that the flow was in all cases laminar or nearly laminar. Predicted sediment loads were then calculated using these estimates and deriving appropriate constants for the various equations from Fig. 8.

It appeared that the sediment loads of very low energy flows ($S \ \overline{u} < 0.7$ cm s⁻¹) measured by these authors were considerably lower than the sediment transport capacity predicted by the various empirical equations presented in this paper. This was especially true for coarse materials ($D_{50} > 450 \ \mu m$). However, if these data are excluded, there appears to be a good agreement (taking into account the rather approximate estimates of hydraulic charac-



Fig. 7 Unit stream power vs volumetric concentration relationships for materials A, B and C.

teristics) between the predicted and the actually measured sediment load (Fig. 9). The effective stream power gives the best results, whilst scatter is greater for the shear stress and unit stream power relationships. A possible explanation for the discrepancy at low energy values will be discussed later. It should also be stressed that the transport capacity of overland flow in this range is always rather low.

DISCUSSION

The results obtained clearly indicate that it is indeed possible to predict the transport capacity of overland flow using simple hydraulic parameters. However, the effective stream power and the unit stream power can only successfully be applied within a given grain size range.

Relationships other than those presented in this paper may also have a good predictive capacity. Sediment transport capacity, expressed as a concen-



Fig. 8 Nomographs to determine the coefficients of transport capacity relationships as a function of grain size: (a) shear stress, (b) effective stream power, (c) unit stream power.

tration, is, in the laminar to transitional flow range, also well related to the shear velocity of the flow, due to the fact that in the laminar flow range unit stream power and shear velocity are uniquely related (Govers & Rauws, 1986). These relationships always indicate a sharp rise in sediment transport capacity from a shear velocity of about 3 cm s⁻¹ (Fig. 10). This shear velocity value is therefore a valuable threshold for rill initiation, providing soil mechanical resistance is not too important (Govers, 1985; Rauws, 1987).

The shear stress, effective stream power and unit stream power relationships can, for a given grain size, be used in the turbulent as well as in the laminar flow range, despite important differences in flow structure and velocity profile. This contrasts with the findings on initial bed instability in overland flow. Critical shear stresses to initiate motion are clearly higher in Gerard Govers



Fig. 9 Comparison of predicted and measured sediment loads: (a) predictions based on shear stress relationships; (b) predictions based on effective stream power relationships; (c) predictions based on unit stream power relationships; data of Meyer & Monke (1965), Bubenzer et al. (1966) and Kramer & Meyer (1969); low intensity flows (Su < 0.7 cm s⁻¹) excluded.

laminar flows and increase with increasing grain size, at least when loose, homogeneous beds are compared (Govers, 1987). However, transport capacity increases with grain size at low flow intensity values. Furthermore, it appears that measurable amounts of coarse sediment are transported at shear stress values below the experimentally determined threshold value for laminar flows.

The latter can be explained by the influence of flume length. During the experiments on the initiation of motion it could be observed that, when coarse particles are set in motion in a laminar flow, they keep moving steadily and mobilize other grains by collision. This will result in a net increase of transport rate with distance from the upper flume end up to an unknown length. This phenomenon becomes more and more important with increasing grain size. A considerable amount of sediment can then be expected to be transported, although the number of primary detachments caused solely by the



sediment concentration (material A, Reynolds flow number < 1800).

fluid is very low. This process might also explain the discrepancies between our results and those of the American researchers in the low energy range. They used a flume only 2.0 m in length, so that, when grain collision is the primary agent of detachment, full transport capacity might not have been reached.

Another factor that needs to be considered is grain velocity. From theoretical considerations it can readily be shown that the velocity at the center of a grain top in contact with the bed in a laminar flow is proportional to the grain size. It can therefore be expected that, as long as grains are transported near the bed, the coarsest grains will move the fastest. This was experimentally verified by Parsons (1972) for glass beads and sand grains moving over a smooth bed. As the unit solid discharge equals the product of the mass of sediment in motion per unit surface of the bed and the mean grain velocity, it is then acceptable that transporting capacity increases with grain size, as long as sediment transport takes place in laminar flow or in a laminar sublayer.

The fact that sediment transport in overland flow conditions can be predicted by the same hydraulic parameters which are also considered to be of fundamental importance with respect to sediment transport in rivers, does not mean that river formulae can be directly applied to overland flow conditions. This is already evident from the ambivalent effect of grain size. As another example, it may be mentioned that the unmodified equation of Yang, which does not take into consideration a critical unit stream power, only yields a correlation coefficient of 0.38, when it is applied to all data. If

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a multiple regression is carried out, so that the basic form of the Yang equation is maintained, while the empirical constants are allowed to vary, a correlation coefficient of 0.90 is achieved.

The above described observations imply that in a relationship of the form:

$$Tc = A q^b S^c$$

where: q = the unit discharge (cm³ cm⁻¹s⁻¹), the proportionality factor as well as the slope and discharge exponent will vary with grain size and flow type. Coefficients resulting from regression analysis (in logarithmic form) on our data are presented in Table 2. In the laminar range the discharge

 Table 2
 Regression coefficients of the equation:

 $\log q_s = A + B\log q + C\log S$; obtained for various grain sizes and distinguishing between laminar and turbulent flow

	Laminar			Turbulent				-		
Material	A	B	С	r^2	n	A	B	С	r^2	n
A	1.56	1.65	2.62	0.98	29	0.24	1.66	1.44	0.87	61
B	1.58	1.55	2.76	0.98	28	0.17	1.80	1.69	0.95	57
C	1.21	1.70	2.50	0.98	31	0.74	1.50	1.96	0.98	- 56
D	0.79	1.53	1.97	0.96	20	0.76	1.24	1.71	0.99	69
Ε	0.51	1.73	1.76	0.98	21	0.85	1.04	1.47	0.97	62

coefficient appears to be more or less constant, while the slope coefficient decreases with increasing grain size. In contrast, the discharge coefficient decreases with increasing grain size for turbulent flow, while the slope coefficient reaches a maximum value for material C. It is interesting to note that the discharge coefficient always exceeds one. If it is assumed that unit discharge increases linearly with the distance from the divide and that the detachment rate at a given point is proportional to the increase in transport capacity, then detachment rate will always increase with distance from the divide. Dynamic equilibrium will therefore only be possible on a concave slope (Carson & Kirkby, 1972). The q_s versus S, q relationships discussed above are only valid on plane beds. The use of more relevant hydraulic parameters has the advantage that the validity of the relationships can be extended to irregular beds, which are much more common in nature.

It was shown in a former paper that the relationship between the sediment concentration that can be transported and the unit stream power does not appear to be fundamentally modified when the velocity of the water is reduced due to additional friction caused by bed surface irregularities or vegetation elements (Govers & Rauws, 1986). The reduction of sediment transport capacity seems to be directly related to the reduction in energy dissipation.

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The effective stream power is an empirical parameter, for which it is difficult to assess the appropriate value on irregular surfaces. However, for material B a reduction of the flow velocity to 50% of the original value will, according to the unit stream power equation, result in a reduction of transport capacity to about 53% of the original value (if the unit stream power is much greater than the critical value). When the effective stream power relation is applied, the same velocity reduction yields a transport capacity of about 56% of the original value (assuming an infinitely wide flow). This may be an indication that the effective stream power relationships are not fundamentally changed by bed irregularities.

With respect to the shear stress, computational procedures exist to split up total shear stress into a grain component, representing that part of the shear stress which can effectively be used for sediment transport, and a form component, which is that part of the shear stress that is dissipated on major surface irregularities. A modification of the procedure of Einstein & Barbarossa (1952) has already been used to predict sediment transport capacity and rill generation (Govers & Rauws 1986, Rauws & Govers, 1988). This computational method might yield considerably higher reductions in sediment load, as the grain shear stress is proportional to the square of the mean velocity, although this is partly compensated by the introduction of an appropriate grain roughness.

CONCLUSIONS

The transport capacity of overland flow was investigated experimentally and it has been shown that it is possible to predict sediment transport capacity using simple empirical relationships, which show considerable variation with grain size, but not with flow regime. Calculation of mean flow velocity and depth should take into account the presence of sediment load which can increase the velocity by up to 40%. The proposed relationships were validated using the results of US researchers, who used a different experimental set-up. It appeared that there is a good agreement between predicted and measured load, at least when very low energy flows were excluded.

It is believed that the proposed relationships can contribute significantly to the operationalization of physically-based erosion models. However, more information should be collected concerning the influence of sediment specific density and of bed surface irregularities. Furthermore, the applicability of the proposed relationships could be considerably extended, when data concerning the interaction between transport capacity, actual load and detachment rate become available.

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