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Evolution of an anthropogenic desert gully system

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Abstract Studies of an unusual quasi-stationary desert gully permit the description of a system attractor which may be inherent, if rarely physically expressed, in many gully systems. Measurements made in a desert gully system (controlled by the seepage of irrigation waters towards a stable base level) suggest a character for the attractor's equilibrium manifold. This is modelled mathematically as an hysteresis or limit cycle in phase space. Overall, the phenomenon is seen to belong to a kind of hierarchical process associated with the Scheidegger "principle of instability/saturation effect". It reflects a dynamic balance between two hierarchical levels inside a single system. The system includes a singularity which is an example of an hierarchical jump caused by the reintegration of a self-assertive holon.

INTRODUCTION

Greene's Canal, on the borders of Pima and Pinal Counties in southern Arizona, is an arroyo. It was created between 1908 and 1910 as a diversion ditch (6.1 m wide by 1.5 m deep) for an agricultural development project. This involved the redirection of flood waters from the Santa Cruz River to a shallow earth-dammed impoundment on the Lower Santa Cruz Plains (Fig. 1). Unfortunately, the dam was destroyed and the project wrecked by severe floods in the years 1914–1915. These floods, called "the worst for generations" (Peirce & Kresnan, 1984), cut the canal into a 3.7 m deep trench, which is twice the depth of the Santa Cruz river at the original point of diversion (Turner *et al.*, 1943; Cooke & Reeves, 1976). At the time of the commencement of this study in 1976, Greene's Canal was 6 m deep and around 80 m wide midway between the diversion and the old reservoir bed.

Greene's Canal is by no means a natural channel. It does not even occupy the topographic low ground. Instead it retains the alignment of its construction which cuts across the contours above the level of the now abandoned natural channel of the Santa Cruz. The arroyo flows north of westward across Quaternary alluvium that slopes gently towards the northeast and the centre of the desert basin. As a consequence, Greene's Canal arroyo has an upslope bank and a downslope bank where the ground falls away from the channel rim.

The cutting of the arroyo trench has steepened the hydraulic gradient above its upslope bank. Existing channels have been rejuvenated and new

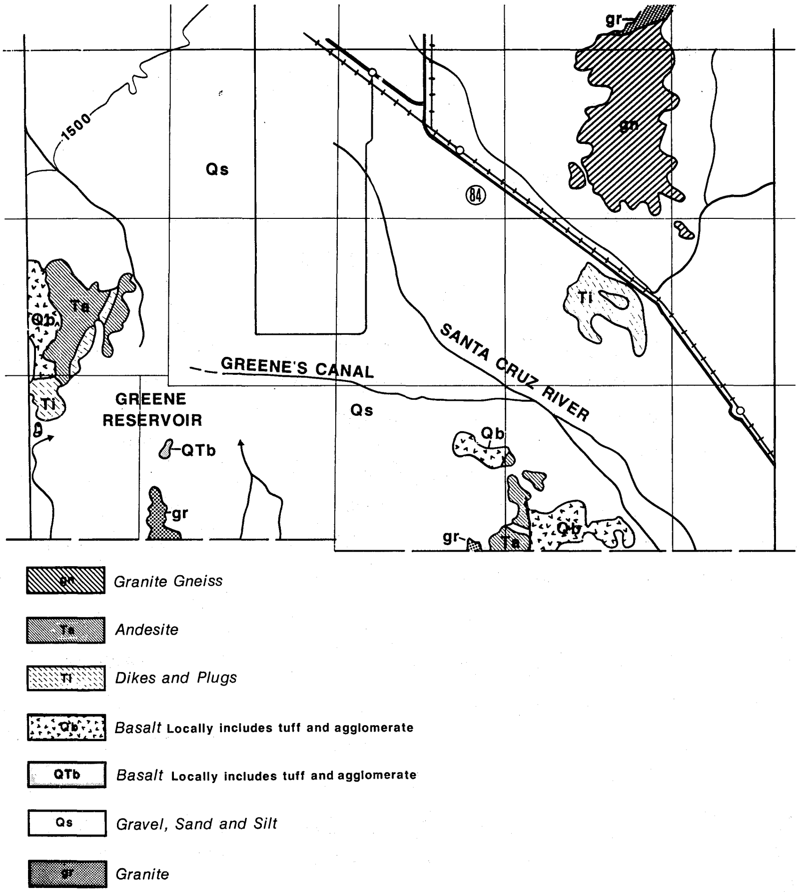

Fig. 1 ***Greene's Canal, South Arizona, USA.***

systems of soil pipes and gully channels have evolved. Comparisons of the USGS topographic survey sheets for 1946 and 1976 show gully displacement of the contours increasing from 500–1000 m to over 1500 m on the arroyo's southwestern flank. By contrast, there has been relatively little gully development on the channel's northeastern, downslope flank. However, some gully channels have evolved here cutting back 20–25 m from the arroyo wall, against the slope of the land.

This study concerns the evolution of one of the gully systems on Greene's Canal's downslope, northeastern flank. This is a very unusual gully in several respects. First, it cuts against the grain of the land. The height of the land at its mouth is almost 200 mm above that at its head cut some 15 m downslope. Secondly, the gully owes relatively little to rainwater. More important is the steady seepage of irrigation waters applied to neighbouring fields. In fact, the lower Santa Cruz Plains are one of the most conspicuous zones of groundwater mining in the USA. This has caused serious and accelerating hydraulic subsidence, amounting to more than 2 m at the point of study, and 5 m at the centre of the desert basin, during the last half century of Greene's Canal's existence (Laney *et al.*, 1978). Subsidence has opened up a network of fissures around the margins of the basin and it has increased the slope across which Greene's Canal flows. Coincidentally, it has back tilted the channel of the study gully, albeit very slightly.

Fortunately, these singularities make this tributary gully of Greene's Canal arroyo of very special research significance. Firstly, it demonstrates the pattern of evolution of a gully whose behaviour is entirely controlled by groundwater seepage. This gully has virtually no catchment for surface water beyond its own walls. Secondly, and in contrast to most desert gullies, this gully can be examined as a discrete system. From the emplacement of erosion monitoring equipment in September 1976 and until its destruction during the floods of October 1983, there was insufficient rainfall to cause incision in the main arroyo. The base level of the gully remained constant during 6.6 years of data collection. Meanwhile, gully evolution continued with the help of regular inputs of irrigation water seeping from nearby fields. Thirdly, therefore, this gully is a representative of a particular breed of desert gully, which is liable to evolve wherever unlined canals run close to irrigated fields.

The behaviour of this gully system also has special theoretical significance. This is because its pattern of evolution includes a stationary cyclic element or torus of the type described by Scheidegger (1983) as the geomorphological "instability principle/saturation effect" and by Bennett & Chorley (1978) as stability in the sense of Liapunov.

The cyclic element includes a long period of slow progressive evolution and a short-lived phase of hysteretic reversion, which suggests that it might be modelled through the medium of catastrophe theory. However, more important is the fact that this pattern is a rare demonstration of how an active gully system may exist in "dynamic equilibrium" with its environment. It is argued that this stability is possible because the system is operating close to an intrinsic system attractor. It is the character of this attractor which this paper seeks to identify.

THE TEST GULLY

The gully examined for this project is typical of those on the downslope bank of Greene's Canal. It is 15 m long from mouth to channel head cut. Its average slope is 6% if the most recent deep head cut is ignored and 17% if it is included. The channel long-profile is broken by two head cuts, a degraded 100 mm step just 2.2 m from its final head cut, and a deep active head cut near the mouth of the gully (Fig. 2).

Gully morphometry has been recorded by means of a 150 mm slope pantometer. Survey included the original long profile and cross profiles recorded at 2 m intervals between mouth and head cut (Fig. 3). One of the seven cross profiles coincided with a minor confluence and this was abbreviated at the mid point of the channel.

Erosion pins are established at regular intervals along each cross profile. A record of changes in the exposures of these erosion pins was kept for 6.7 years between September 1976 and April 1983. The results of these

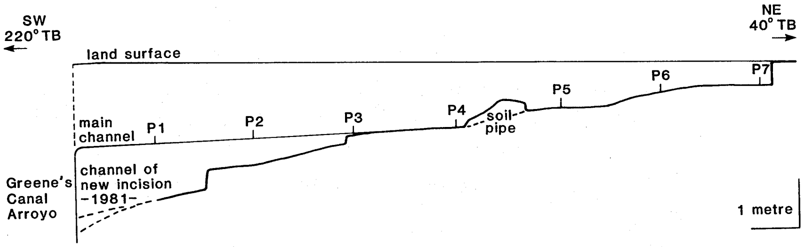

Fig. 2 Long profile of test gully showing locations of erosion pin gross profiles.

measurements of total ground loss in millimetres are recorded for individual sites on Fig. 3.

A detailed discussion of the empirical results has been published by Haigh & Rydout (1987). In summary, what happens is that the gully banks tend to retreat rapidly and parallel to themselves. In the process, a small, compacted slope foot segment and a wide flat depositional basin are created. This process is interrupted periodically, when the roof of a subterranean soil pipe breaches the gully floor creating a narrow, vertical sided slot in the gully basin. The channel at the base of this trench quickly becomes choked by the debris released from its own walls. As these walls retreat, the trench becomes wider and more shallow (Fig. 4).

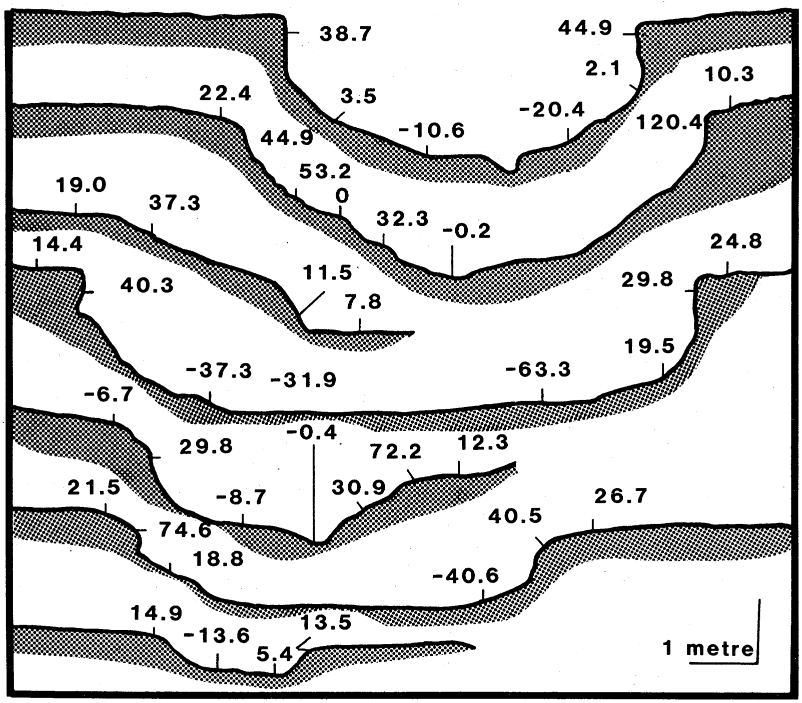

Fig. 3 Cross profiles of test gully showing data collected at each erosion pin (ground retreat in mm: total for 6.6 years of study).

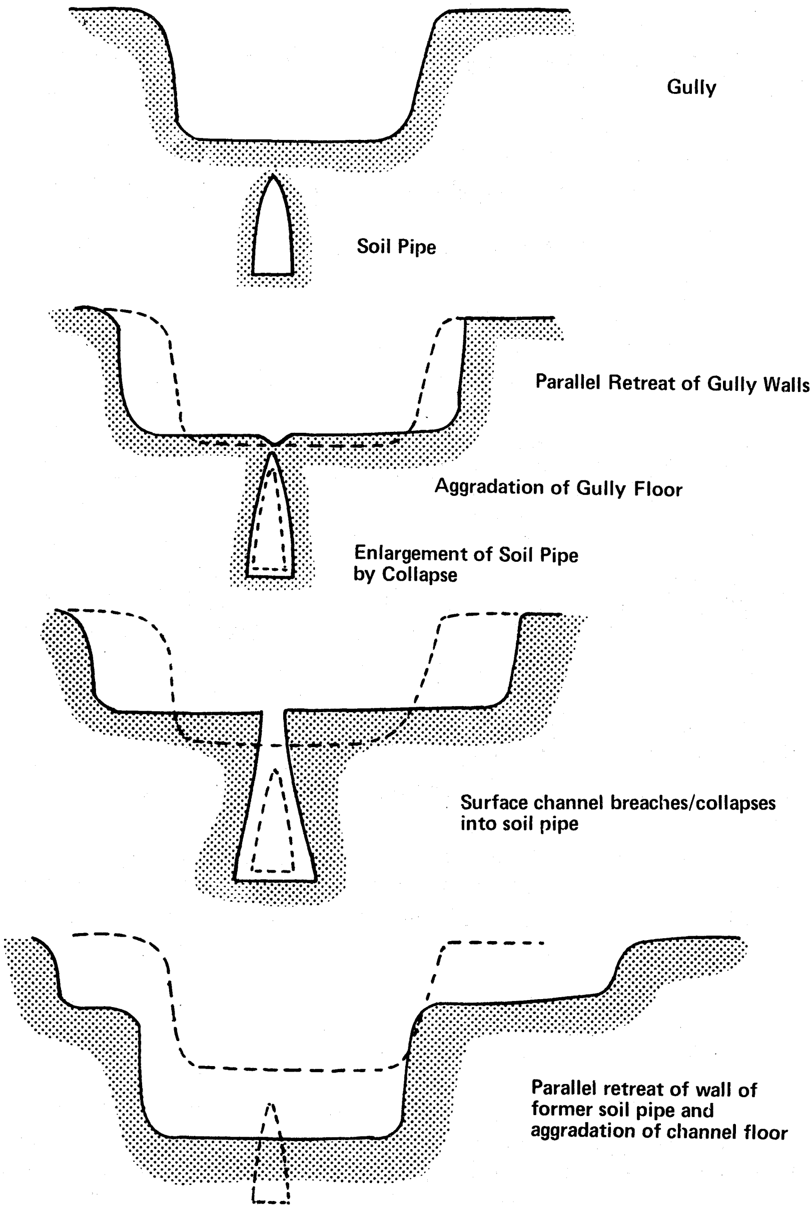

Fig. 4 Collapse/fill cycle of test gully basin.

SITE CHARACTERISTICS AND PROCESSES

The test gully is cut into soils which are Typic Torrifluvents, perhaps mixed with Aridic Cumulic Haplustolls. They are fine textured (90% passing the 63 μm sieve), calcareous (pH 7.9–8.4), slowly permeable (5–15 mm h^{-1}) loams of low density (1.35–1.80 g cm^{-3}) and high erodibility (K = 0.32–0.37). However, the soils include thin clayey layers of higher density and lower erodibility dispersed irregularly through the profile.

The ground beneath the gully's floor is cut by a network of large soil pipes. Where they break through into the walls of the main arroyo, these triangular pipes may be 2 m high and 0.30–0.5 m wide at their base. Collapse hollows are common even 5 and 10 m beyond the head cuts of the surface gully fringe to the arroyo, and subsidence ruts can be discovered in the arable fields beyond.

The test gully channel was unvegetated throughout the study except for a brief period in 1983 when a 30% grass cover developed on its floor and lower north wall. However, the channel is cut into a platform dominated by creosote bush (*Larrea tridenta*). At the mouth of the gully, the arroyo floor is colonized by a mature scrub of mesquite (*Prosopis sp.*) and saltbush (*Atriplex polycarpa*). Some 5 m beyond the gully head cut are pasture grasses on the margins of arable fields.

Irrigation is the key to agriculture in this region. Flood irrigation methods involve anything from 0.37 to 0.74 m year^{-1} of water application. By contrast, rainfall during the 6.7 years of measurement totalled only 1.9 m (277 mm year^{-1}), which was a little above average (248 mm year^{-1}). The erosivity of the rainfall is rated as low (R = 75), despite the fact that 62% falls as storms (> 12.5 mm day^{-1}) which are arguably severe enough to activate surface runoff (USDA 1966). However, Cable (1977) confirms that rainfall events in this area effect very little soil moisture recharge and the wetting front does not often penetrate far into the soil. In sum, the evidence indicates that the soil pipes which run from the irrigated fields to the arroyo and underneath the experimental site are primarily caused by the drainage of irrigation waters.

Light frosts affect the area on 11 days year^{-1}, but there is very little soil moisture to freeze even in winter. Despite this, steep soil faces suffer considerable erosion due to shrink/swell and slaking processes, and also to burrowing and trampling (Haigh & Rydout, 1987).

ANALYSIS

The aim of this paper is to explore the theoretical implications of the pattern of morphological evolution indicated by the empirical field study. The following aspects of that development seem significant. The pattern of gully evolution implies the existence of a cycle. During this cycle, the gully exhibits two mutually exclusive models of behaviour. Mode 1 is where morphogenesis is led by gully side wall retreat and aggradation of the depositional gully basin floor. Mode 2 is where morphogenesis is led by the exposure, mainly if not exclusively by collapse, of the soil pipe growing within the deposited sediments of the gully basin. Mode 2 morphogenesis operates through a much shorter time scale then mode 1. However, both can be occurring concurrently at different points within the same gully basin. Furthermore, the possibility exists for the two processes to hold each other in check as a kind of limit cycle. Mapped in phase space, the relationship can be represented as a torus (Kaneko, 1986).

MATHEMATICAL MODEL

The two process systems involved must first be considered separately. First, is the normal, at least in terms of duration, pattern of gully evolution.

Under "normal" conditions, the elevation of the gully floor at the instant of surface collapse ($R = 0$) depends upon the depth at which the soil pipe channel floor forms. This seems to be close to the upper surface of the water table created by the seeping irrigation waters. Measurements made in pipe collapse hollows indicate that this ranges from around 1.5 m close to the arroyo wall to around 0.5 m at the outer fringe of the gullied area.

Gully depth begins as the difference between base level ($R = 0$) and the height of the original ground surface (K). However, as time passes, gully depth ($K - R$) declines as the channel floor is aggraded by the transfer of sediments released from the pipe/gully side walls. The morphological consequence of this is that the side walls shrink and retreat as the gully basin expands. As the gully becomes more shallow and wider towards the pre-gully ground surface (K), the rate of aggradation (x) declines asymptotically.

$$\frac{d\,(K - R)}{dt} = (K - R)^{x} \qquad (1)$$

where: x is the exponent representing the rate of channel aggradation;
K is the elevation of the original ground surface;
R is the initial depth of the gully channel; and
t is time.

The rate of aggradation (x) is a function of the rate of sediment release from the channel banks (ca. 8 mm year^{-1}), scaled by changes in packing density (the ratio between the bulk densities of the channel bank and channel floor sediments), and controlled by the progressively declining ratio between gully floor and gully side.

However, underground, the soil pipe is a water scoured channel. On exposure, this channel quickly becomes buried due to the massive release of sediments from the gully walls. As it becomes buried, it requires more and more energy to generate surface water flows. The frequency of flows and the volume of sediments cleared declines. Scour decreases further as wash due to rainwater becomes spread across the floor of an increasingly large depositional basin with progressively reduced local relief. Eventually, surface aggradation in the gully comes to depend only on the rate of side wall retreat counterbalanced by the rate of scour due to surface wash processes.

Adding these complications, transforms the pattern of "normal" morphogenesis from a negative exponential curve to a sigmoid curve. The equation, then, may be rewritten thus:

$$A = (K - R)^{x} - S - P \qquad (2)$$

where: A is channel shallowing (mm year^{-1})

$(K - R)$ is gully depth (mm);
x is an empirical exponent with a value of 0.26;
S is channel deepening due to scour, which in this environment, witness the crest segment retreat rates, is at least 2.7 mm year^{-1};
R is current elevation of the gully floor (mm);
K is the elevation of the ground surface (mm); and
P represents scour associated with soil pipe flows (mm).

Naturally, the values of x, P, and S may be expected to vary with soil type, climate, irrigation levels, and variations in the initial width of the gully channel at the start of a new cycle. The values cited here are calculated from the results of this single case study. This includes few good measurements in the soil pipe scour zone, so this term is not quantified. However, the influence of P appears to be restricted to the zone below R = 150 mm. The field data, however, do permit the quantification of the balance of the process. So, solving Equation (2), it can be seen that at 2 m depth, the maximum rate of aggradation would be 7.2 mm minus S = 2.7, which leaves a net channel aggradation of 4.5 mm year^{-1}. At a depth of 200 mm the rate of aggradation would be 4.0 mm minus 2.7, leaving 1.3 mm year^{-1} net aggradation (cf. Fig. 4).

However, the numbers gleaned from this case study are much less important than the pattern they suggest. It now becomes possible to represent this gully collapse and fill cycle as a graph (Fig. 5), and the problem reduces to finding a term for the collapse phase of the system. This is the period during which the surface gully collapses into the soil pipe evolving within its own deposits.

Several aspects of the process affect any mathematical model. First, once the initial stages of channel infilling bury the groundwaters, the two systems operate completely independently one of the other. The evolution of the soil pipe has no impact on the surface until the commencement of collapse. Second, nothing is known about the character of the expansion of the soil pipe prior to its exposure at the ground surface. However, because this pipe is developing within a relatively stable energy stream, the throughput of irrigation waters, it might be suggested that for a stable pipe/gully system, the phase change might be identified as a threshold depth of deposited sediments (T).

Naturally, any such threshold is only identifiable as a statistical mode

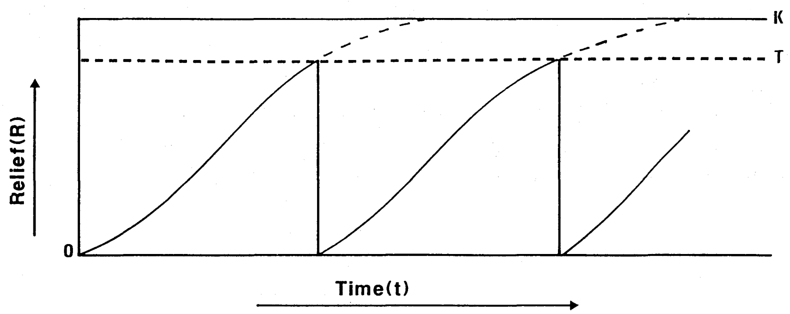

Fig. 5 Cyclic fluctuation of gully basin floor as a time series.

or mean. However, there are few head cuts in this gully system or its neighbours which are smaller than 0.5 m and very few which are deeper than 2.5 m. Most fresh collapse hollows lie in the range 0.5–1.5 m. Indeed, the fact that collapse hollows exist apart from the major gully systems indicates that ground surface and soil pipe can co-exist through quite a large range of intervening soil depths.

The second important aspect of the collapse part of the cycle is that it occurs very abruptly in the terms of the normal operating time scales of the system. The elimination of the work accomplished by the "normal" operations of the surface system occurs within days rather than the decades of the full cycle.

In mathematical terms, the operations of this unrecordable subterranean control of the gully channel can be conceived as the equivalent of a modulus. The system is regulated by modulo T, which may be approximated in a stable system as a critical depth of burial of the water percoline at $R = 0$.

$$\frac{d(K - R)}{dt} = (K - R)^{x} - S - P : {}^{*}T \text{ (modulo } T) \qquad (3)$$

where:

T is the threshold depth of sediment infilling where collapse into the underlying soil pipe usually occurs (500 mm < 1500 mm);
R is the height of the gully floor above its base level (R_o = 0 mm);
K is the altitude of the original surface above R_o (mm);
x is the exponent for gully infilling;
S is channel deepening due to scour (mm); and
P represents scour associated with soil pipe flows (mm).

CATASTROPHE THEORY

The problem with the foregoing analysis is, of course, the fact that the phenomenon of surface collapse is not really controlled by a modulus. Between depths of burial of 0.5 to 1.5 m, surface gully and soil pipe can co-exist independently, or because of local circumstances, they may interact through collapse. The system can exist quite easily in either of two distinct conditions through a large part of its range of operations. It can, abruptly, move from one state to the other unpredictably at any moment during the period when these two possibilities exist. In mathematical terms, the proper description for this condition is a catastrophe (Saunders, 1980).

Catastrophe theory is NOT a theory. It is merely a mathematical language for describing some types of system which contain discontinuities or abrupt changes of a particular character. It was developed by Thom (1972) as an alternative to calculus, which is a language for examining continuous or smooth changes. Catastrophe theory is a branch of the mathematical science of topology which deals with phenomena both numerically and as geometrical forms. The catastrophes which it describes are not necessarily "disasters". They are merely species of phenomena which jump abruptly from one mode of behaviour to another because of the co-existence of two basins of attraction in the system.

Since its creation, catastrophe theory has been both in and back out of fashion. It has been both used and abused (Zahler & Sussman, 1977; Thompson, 1982). It has been applied to geomorphology and geology (Henley, 1976; Wilson, 1981). The truth is that the language of catastrophes has major limitations. First, this language applies only to phenomena which can be described by a potential function - where system behaviour is a consequence of something being maximized or minimized across a mappable gradient. Second, its models only hold locally, in the vicinity of the critical state. Finally, the language is entirely descriptive and not at all predictive (Wilson, 1984).

There are several major classes of catastrophe. Each can be defined in terms of a number of control functions and behaviour factors. The simplest catastrophe has a single control factor and a single behaviour function which describes a folded graph with a single maximum and a single minimum (Woodcock & Poston, 1974). The cusp catastrophe includes the fold catastrophe and an extra control factor or dimension along which the degree of the fold varies. The result can be pictured in three dimensions and its behaviour function has two minima and one maximum. In effect, the cusp catastrophe organizes two one parameter families of fold.

Higher order catastrophes have larger numbers of control and behavioural dimensions. Their full geometry is multidimensional and hence rather difficult to portray and conceive (Woodcock & Post 1974). To date, there have been few attempts to apply catastrophe models above the level of the simple cusp.

A CATASTROPHE MODEL FOR THE DESERT GULLY SYSTEM

Examined as the graph of Fig. 5, an individual cycle of system behaviour is explained in terms of one control dimension, gully depth, and one behaviour dimension, aggradation/collapse. So the graph portrays a simple fold catastrophe which is repeated as a regular cycle bounded by threshold (T) and base level ($R_o = 0$).

The location of the fold is determined by the soil strength variable which defines the location of the collapse threshold ($K - R = 500 < 1500$ mm approximately) and hence both the point of bifurcation in the system and the orbit of the torus. This control of the bifurcation may be added to the model in terms of a second control dimension. This variable is ($K - R_o$) which varies with the depth of the soil pipe and hence controls the expression of the catastrophe as well as the pattern of aggradation. The behavioural dimension (A = solution of Equation (3)) can then be mapped across the two control dimensions: ($K - R_o$) and time (t). This additional control organizes the family of fold catastrophes as a cusp catastrophe.

This surface requires an equation of the form

$$V\,(A,\ (K - R_o),\ t) = A^4 + A^2 \cdot (K - R_o) + A \cdot t \qquad (4)$$

where V is the system potential (Thompson, 1982).

DISCUSSION

In geomorphological theory, Scheidegger (1983) illustrates his principle of instability by reference to the behaviour of river meanders. Every river meander tends to divert its flow of water to the outside of the meander bend and so increase its curvature. River meanders belong to that large class of landforms which do not tend to a steady state (Bull, 1975). However, river meanders are also subsystems (holons) within the larger system of the river channel (Haigh, 1987). This has work to do as is witnessed by the way that rivers' long profiles hold to grade (Bull, 1979). The self-assertion of the meander holons reduces the efficiency of the operations of the larger system. Eventually, that larger system is forced to re-integrate its member holons. A surge of energy in the system causes the re-establishment of the original energy line (channel way), the development of meander cutoffs and so the restructuring of the meander holon.

The behaviour of the Greene's Canal tributary gully system is not very different in character. The gully is part of a larger system, the drainage of irrigation waters from arable fields to neighbouring arroyo. The gully emerges because the interactions between the seeping waters and the desert soils lead to the development of soil pipes. These pipes expand upwards by roof collapse until they become exposed at the soil surface. At this point, the gully/soil pipe may be performing its role as a sub-system (holon) efficiently. However, sub-aerial processes tend to the elimination of surface gullies. The seepage water flow-away becomes buried and the gully begins to heal. However, the original energy line still lies buried in the soil, so eventually the soil pipes reform and the whole cycle goes through another loop.

The significance of the relationship is, however, best understood in the terms of hierarchy theory. The gully can be examined as a holon within the larger system of relationships linking the irrigated fields to the nearby arroyo. Normally, morphogenesis in the gully continues independently of higher level control. However, these processes decrease the efficiency of operations at the higher level. Eventually, this higher level reasserts its control over the sub-system. It is this hierarchical restructuring which is the system catastrophe. It is this relationship between two hierarchical levels in the system which the mathematical models really describe.

The relationship, then, is an example of interaction between two hierarchical levels in a single system. The catastrophe is an example of hierarchical restructuring (Haigh, 1987; Platt, 1970). The cusp model control factors represent the antagonistic agenda of the two holonic levels involved.

Similarly, the linear model of sigmoid growth regulated by a modulus, is a first cousin to the predator-prey equations which describe the interactions between two hierarchical levels in any ecosystem (Thompson, 1982). The predator-prey relationship with its clear pattern of stable oscillation is widely recognized as a significant system attractor in ecological systems. The status of the gully system attractor demonstrated here is rather less certain. Certainly, rill systems often show a similar dynamic balance between incision due to flowing water and healing due to other sub-aerial processes. Blong *et al.* (1982) have argued that the role of side wall processes is frequently understated in studies of desert gullies. Furthermore, the fact that some gully systems can remain stable over many centuries is now well established. It may well be that the character of the "dynamic equilibrium" described at Greene's Canal may not be so unusual, and that the system attractor here described is merely underrecognized.

CONCLUSION

The results of a 6.6 year period of erosion pin measurement in a desert gully indicate that for most of the time morphogenesis is controlled by the retreat of the gully side walls and deposition in the gully basin. However, this is interrupted occasionally by the collapse of the gully floor into underlying soil pipes. These pipes are created by the seepage of irrigation waters between arable fields and the Greene's Canal arroyo to which the gully is tributary. The end product of this control is a gully system which, to all intents and purposes, is held in a stationary limit cycle regulated by a catastrophe (hierarchical jump). This cycle may provide an attractor for many arid gully systems and may help explain some of the morphological complexity of relatively stable gullied landscapes.

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REFERENCES

Bennet, R. J. & Chorley, R. J. (1978) *Environmental Systems: Philosophy, Analysis and Control.* Methuen, London.

Blong, R. J., Graham, P. O. & Veness, J. A. (1982) The role of sidewall processes in gully development, some N.S.W. examples. *Earth Surf. Proc. Landforms* 7(6), 381-386.

Bull, W. B. (1975) Allometric change of landforms. *Geol. Soc. Am. Bull.* **86**, 1489-1498.

Bull, W. B. (1979) The threshold of critical power in streams. *Geol. Soc. Am. Bull.* **90**, 453-464.

Cable, D. R. (1977) Soil water changes in creosote bush and bursage during a dry period in southern Arizona. *Arizona Acad. Sci. J.* **12**(1), 15-20.

Cooke, R. U. & Reeves, R. W. (1976) *Arroyos and Environmental Change in The American Southwest.* Oxford Research Studies in Geography, Clarendon Press, Oxford, UK.

Graf, W. L. (1979) Catastrophe theory as a model for change in fluvial systems. In: *Adjustments of the Fluvial System, Binghamton Geomorphology Symp.* **10**, (ed. D. D. Rhodes & G. P. Williams), 13-32, Kendall Hunt, Dubuque, Iowa, USA.

Haigh, M. J. (1987) The holon, hierarchy theory and landscape research. *Catena, Suppl.* **10**, 181-192.

Haigh, M. J. & Rydout, G. B. (1987) Erosion pin measurements in a desert gully, Greene's Canal, Arizona, in: *International Geomorphology 1986* (ed. by V. Gardiner), 419-436, Wiley, Chichester, UK.

Henley, S. (1976) Catastrophe theory models in geology. *Math. Geol.* **8**(6), 649-662.

Kaneko, K. (1986) *Collapse of Tori and the Genesis of Chaos in Dissipative Systems.* World Scientific, Singapore.

Laney, R. L., Raymond, R. H., & Winikka, C. C. (1978) Maps showing water-level declines, land subsidence and earth-fissures in south-central Arizona. *USGS Wat. Resour. Invest.* 78-83,

Open File Report.
Saunders, P. T. (1980) *An Introduction to Catastrophe Theory.* Cambridge University Press, Cambridge, UK.
Peirce, H. W. & Kresnan, P. L. (1984) The floods of October 1983. *Arizona Bureau of Geology and Mineral Technology, Field Notes* **14**(2), 1-7.
Platt, J. (1970) Hierarchical restructuring. *General Systems* **15**, 49-54.
Scheidegger, A. E. (1983) Instability principle in geomorphic equilibrium. *Z. Geomorphol. N.F.* **27**(1), 1-19.
Thom, R. (1972) *Structural Stability and Morphogenesis.* (trans: Fowler, D. (1975)) Benjamin, Reading (in Thompson, 1982 op cit).
Thompson, J. M. T. (1982) *Instabilities and Catastrophes in Science and Engineering*. Wiley, Chichester, UK.
Thornes, J. (1983) Evolutionary geomorphology. *Geography* **68**(3), 225-235.
Turner, S. G. *et al*, (1943) Groundwater resources of the Santa Cruz Basin, Arizona. *USGS Open File Report.*
USDA (1966) Procedures for determining rates of land damage and depreciation and volume of sediment production by gully erosion. *US Department of Agriculture, Soil Conservation Service, Technical Release (Geology)* **32**.
Wilson, A. G. (1981) *Geography and the Environment, Systems Analytical Methods.* J. Wiley, Chichester, UK.
Wilson, A. G. (1984) Catastrophe theory. *Times Higher Education Supplement* 25.5.84, 15.
Woodcock, A. E. R. & Poston, T. (1974) *A Geometrical Study of the Elementary Catastrophes.* Lecture Notes in Mathematics 373, Springer-Verlag, Berlin.
Zahler, R. S. & Suusman, H. J. (1977) Claims and accomplishments of applied catastrophe theory. *Nature* **269**, 759-763.
Zeeman, E. C. (1976) Catastrophe theory. *Sci. Am.* **234**, 65-83.