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Towards a dynamic model of gully growth

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Abstract Headward migration of gullies around the margins of drainage networks may be a major factor in the increased sediment yields observed in semi-arid southeast Spain. Although many processes have been cited, the precise nature of this gully extension and bifurcation is not known. A gully which is migrating into a hillslope may be considered by way of its morphological dynamics. A digital model has been formulated in which the governing process is erosion by overland flow. Using finite techniques, the continuous phenomena may be approximated by discrete functions. The model grid is set up by defining strip catchments which are bounded by orthogonal flowlines. Overland flow is then routed downslope kinematically, assuming gravity and friction as the controlling forces. Sediment detachment, transport, and deposition are estimated using a Musgrave-type approach which incorporates an interaction term. This compares energy required to carry sediment already in transport with the total capacity of the flow to do work. The consequent change in the slope surface is expressed by migration of the contours along the flowlines. In this manner, the effects of hillslope and gully geometry on gully development may be explored. The simulations indicate a critical balance between a linear propagation of the erosion headwards, and a diffusion laterally of this impulse. Within this balance is identified a possible mechanism for the bifurcation phenomenon. These results are being verified by laboratory experimentation.

INTRODUCTION

This paper reports an investigation into the possible mechanisms of gully head migration. More specifically, the role of gully head geometry and slope morphology in controlling the processes which lead to the extension and bifurcation of gullies is explored.

The objectives of this paper are to present:

a) a theoretical model of gully growth and bifurcation which offers a simple and easily formulated framework on which to base experimental studies; and
b) a two dimensional digital model which is being developed to simulate the dynamics of soil erosion, and the resultant change in the

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morphology of the hillslope and gully head.

The growth of rills and their development into gullies and even badlands erosion increase by orders of magnitude once rilling and gullying ensue, and any model which fails to take account of their development will be of limited value in many conservation situations. High sediment yields are often attributed to surface wash, yet evidence in southeast Spain indicates that the problem actually originates from the headward extension of gullies around the drainage net margins (Thornes, 1976, p.41, p.70; Thornes & Gilman, 1983, p.133).

MODELLING GULLY GROWTH

Sheet erosion can be directly related to the bottom shear stress produced by overland flow. Field evidence (Dunne, 1980; Dunne & Aubrey, 1986) and laboratory evidence (Moss *et al.*, 1982) indicate that sheet flow is inherently unstable and will split into small concentrated rivulets of flow. Rainfall, micro-topography and vegetation will have a considerable effect on this tendency.

In a general sense, the initiation and growth of rills and gullies is dependent on a sufficient concentration of this bottom shear stress to form a definable channel. Surrogates for the amount of discharge, such as the length or area of contributing slope, have been discussed by Horton (1945) and Schumm (1956), respectively. Schumm proposes a constant of channel maintenance, essentially the drainage area required to support a given length of channel. The underlying implications are of great significance for channel extension. In the case where full extension of a channel has been reached, such as in Fig. 1 for a hypothetical planar surface, if all other factors remain constant further growth can only occur if a length or area is obtained greater than already existing. A curvature of the contours must occur which is sufficient to induce a greater than critical component of length or area.

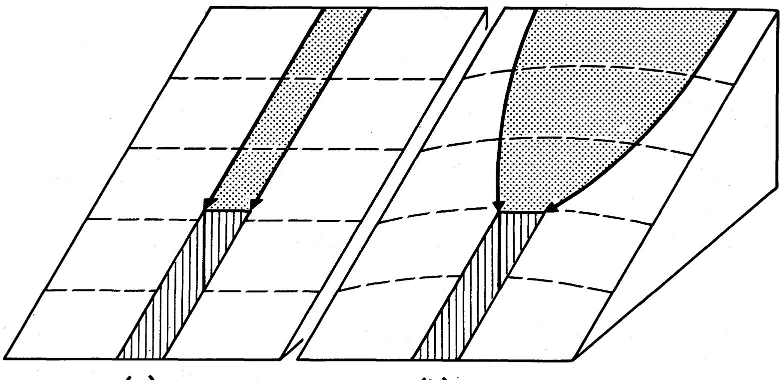

Fig. 1 Drainage area required for channel maintenance: where full extension of a channel has occurred (a), growth may be re-initiated if the contributing area is increased above a critical component by a curvature of the contours (b).

Small depressions or nicks may be initiated and enlarged as a result of some sort of perturbation, such as a local variation in vegetation, surface roughness or surface crusting. The growth of a hollow will increase the curvature of contours across a hillslope, and will therefore lead to a convergence of water and sediment. The hollow will grow if the increase in the rate of work is greater than the increase in sediment to be transported. Smith & Bretherton (1972) and Kirkby (1980a) formalize these relationships using the following instability criterion:

$$A \frac{dS}{dA} > S \qquad (1)$$

where A is contributing area
S is sediment in transport.

If the increase in the amount of sediment to be transported (right hand side of the equation) is greater than the increase in capacity to transport (left hand side), infilling of the hollow will occur. Growth can only occur if there is a relative increase in the transporting power.

Clearly, the influence of three-dimensional topography is of the greatest importance to drainage net development and reflects the self-generating nature of drainage channels. Where there is a flow convergence and an adequate concentration of erosional power, channel growth is possible.

Rills and gullies are dynamically similar to channels, but are characterized by ephemeral flow, and a close coupling to the hillslope. Within active gullies, near-vertical scarps can develop at the head of the channel. Once a headcut is initiated, it may retreat upslope into otherwise undisturbed hillslopes. Many examples have been observed in southeast Spain. Channel storage will be reduced where there is a concentration of water in the gully, and erosive power will increase in relation to the flow depth. Runoff over the headcut may contribute to gully growth by exerting stresses on the channel boundary, by removing accumulated soil debris from the channel and by eroding the gully banks through undercutting (Francis, 1985).

BIFURCATION OF GULLIES

The question then arises as to why a headward-extending gully should branch. Thornes (1984) has proposed an analytical model in which the geometry of the gully head controls the branching, or bifurcation, phenomenon. Once an initial perturbation has developed, in this context perceived as a headcut, or slope failure boundary, an erosional pulse or signal will pass up throughout the system. Its forward and lateral velocities will vary as a function of the imbalance between force and resistance.

Assume that the form of the pulse is analogous to a shock wave migrating through the hillslope. It is propagated linearly along the line of greatest erosive power, calculated here per unit width of bed slope at the shock boundary. Overland flow, taken as the generator of this erosive power, is assumed to be orthogonal to the contours. The contours themselves will migrate as erosion proceeds. The lateral dissipation of the erosional energy is controlled by the relative strength and cohesiveness of the bank material.

If the gully head is in a fixed location, it is assumed that a wave of erosion propagates outwards with a velocity V_s. This is mainly determined by processes such as creep and mass failure which are controlled by the properties of the materials. If migration is wash-controlled, then it is also moving upslope with a propagational velocity, V_x, as shown in Fig. 2. Where the ratio of the velocities is greater than unity:

$$\frac{V_x}{V_s} > 1 \tag{2}$$

then the shock travels ahead of the outwardly propagated waves, and the opportunity time for relative widening of the channel is less. The shock is 'supersonic'. If this criterion does not hold, the shock is 'subsonic', and growth is influenced by the outward movement of the propagating wave. In semi-arid environments the wash (forward) component occurs intermittently, and other processes, such as weathering, creep and local wash and splash, may diffuse the gully head outwards. This will be particularly the case on weak lithologies or on surfaces which do not generate much overland flow.

It is suggested that the geometry of a gully head governs the distribution

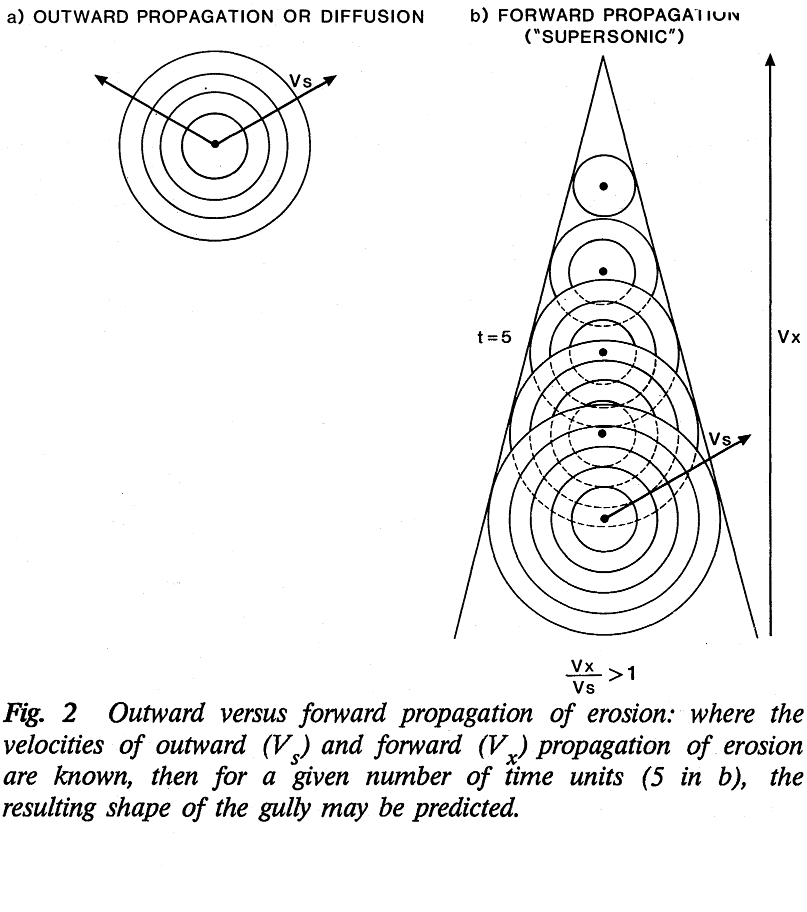

Fig. 2 Outward versus forward propagation of erosion: where the velocities of outward (V_s) and forward (V_x) propagation of erosion are known, then for a given number of time units (5 in b), the resulting shape of the gully may be predicted.

of shear stress around its boundary, through its spatial relation with the orthogonal flowlines from upslope. In Fig. 3, it is apparent that, as one moves around the gully head boundary away from the apex, or central axis of the gully head, the angle at which the flowlines intersect the gully boundary decreases. Each unit width of flow from upslope has to cross a progressively larger width of gully boundary. Thus the concentration of erosive power on the boundary decreases. If this concentration of erosive power is greatest at the apex, extension will continue linearly. If deflected to either side, then branching of the gully may occur.

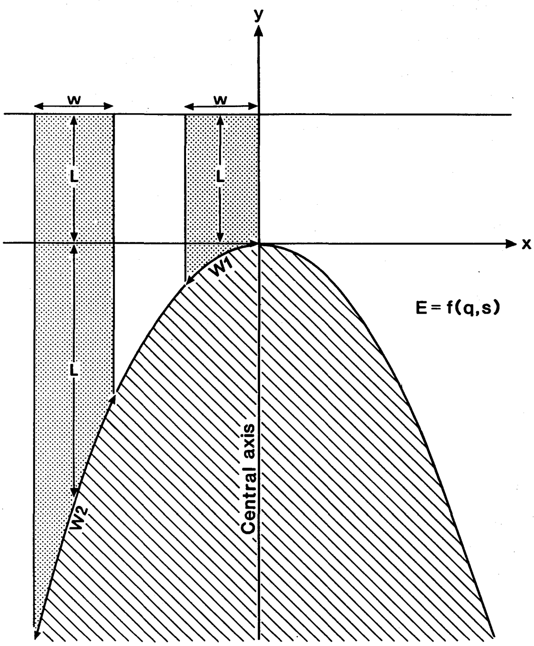

Fig. 3 Distribution of shear stress around a gully head boundary: with distance from the central axis of the gully head, a given width of flow from upslope will have to cross a greater width of gully boundary; this will have the effect of decreasing the erosive power of flow per unit width.

Thornes (1984) suggests that the relative magnitudes of forward and lateral migration of the gully walls may be a critical control of bifurcation. In southeast Spain, low density drainage nets may be observed in lithologies with a high shear strength. Supersonic migration should lead to the formation of long narrow channels, due to the limited opportunity for branching. Where subsonic conditions exist, the widening of channels by active slumping and other processes, such as creep, may increase the likelihood of branching. Higher density dendritic networks should develop. Thornes' model describes the conditions of hillslope and gully head geometry, in which bifurcation is likely to occur.

DIGITAL SIMULATION

Once the point of bifurcation is reached, the analytical problem becomes intractable, and an alternative lies in numerical solution through digital simulation. This is a powerful tool for exploring some of the implications of such a conceptual model, in that the model system may be completely controlled. The model which is devised needs to be simple enough to be easily manipulated and understood, whilst being sufficiently representative of the natural system under scrutiny to provide a meaningful evaluation.

The key questions to be addressed are:

(a) how will the form of the gully change through time and space; and
(b) under what circumstances will the gully branch?

The modelling of overland flow, in the context of gully growth, is relatively straightforward, and is therefore an appropriate process to adopt in the initial assessment of this concept. To study gully development in this paper, a hydrodynamic model and a sediment transport model are coupled in order to account for the hydraulics of overland flow, sediment transport, and morphological considerations (cf. Cordova *et al.*, 1983). By routing sediment across the slope, it is then possible to calculate the migration of the contours and gully head. The dynamics of the processes, and the changing morphology may thereby be simulated.

PROGRAM OUTLINE

There are three major elements to the program:

(a) the morphological response surface;
(b) the overland flow; and
(c) the erosion, and subsequent migration of the contours.

Morphology

In developing a numerical model of this phenomenon, a generalized geometrical framework is required which is flexible and which can take account of the topographic variations of the simulated hillslope. Hypothetical initial surfaces are constructed by varying the curvature and interval of the contours. Slopes may be planar, converging, diverging, convex or concave. The two-dimensional hillslope is represented (Fig. 4) by an assembly of one-dimensional strip catchments which are assumed to be independent of each other. The strips are defined on the basis that the water flows in directions orthogonal to the topographic contours, the steepest route available. Within each strip, a number of cells are defined by the intersection of the orthogonal flow lines and contours. The average length, width, slope and area are calculated for each cell, and a roughness parameter is established. These cells are the basis for the numerical solution of the erosive process operating over the hillslope.

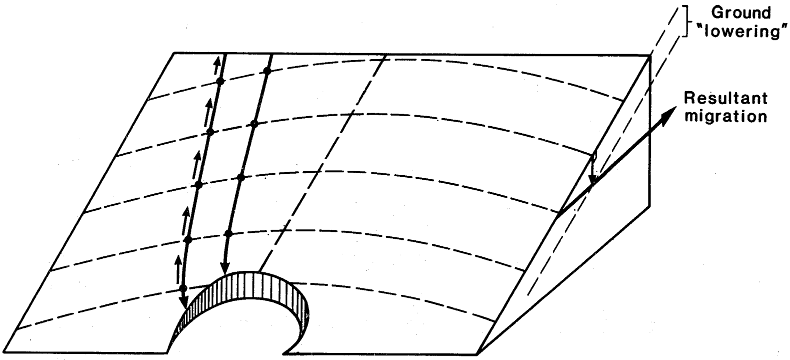

Fig. 4 Representation of two dimensional slope by an assembly of one dimensional strip catchments: strip catchments are defined by flow lines orthogonal to the contours, and migration of the contours is calculated along the flowlines, as a function of the perpendicular ground lowering.

Overland flow

The erosion-deposition algorithm is a second stage component of a runoff generating model. In the kinematic approximation of the hydrodynamic equations, which is used here, the controlling forces are the gravity and the friction terms. The assumptions of this approximation are that the slope is between 2° and 25° and is varied gradually. The final form of the equation, after Kibler & Woolhiser (1970), is derived from a combination of the equations of continuity and momentum:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \tag{3}$$

where $A = w\, h$ is area;
w is width;
h is depth;
$Q = A\, u$ is discharge;
u is velocity;
q is input per unit area;
t is time; and
x is distance.

A stage equation is adopted to eliminate one of the two dependent variables:

$$u = n\, h^{m\text{-}1} \tag{4}$$

where n is a roughness and slope coefficient;
m is an exponent.

This is substituted into equation (1) and the terms expanded, thus:

$$\frac{\partial w h}{\partial t} + \frac{\partial w n h^m}{\partial x} = q \tag{5}$$

The model assumes that overland flow occurs as turbulent sheet flow.

A subroutine based on the method of characteristics has been implemented for solution of the kinematic approximation. This allows a more accurate simulation of the hydrograph than has been achieved by finite difference methods, and it calculates the necessary time increment for stability. The equations are solved in a sequential manner, such that the dependent variable is determined for each node of the solution mesh. This is achieved by solving along the characteristics, and extrapolating back to the specified nodes. This numerical procedure will be described in greater detail in a later paper.

Formulation of the erosion algorithm

The modelling of overland flow generation across a hillslope is well-established in the literature. The formulation of an erosional-depositional algorithm is more problematic.

Ellison (1946) cites four critical features of the entrainment-transport situation:

(a) the detaching capacity of the erosive agent;
(b) the transport capacity of the erosive agent;
(c) the detachability of the soil; and
(d) the transportability of the soil.

Flow at a point in time and space has a given energy available for detaching and transporting soil. Foster & Meyer (1975) propose that the ratio of the sediment load to the transport capacity, a relative term of energy for transport, plus the ratio of the detachment rate to detachment capacity, a relative term of energy for detachment, equals unity, the total available energy, i.e.

$$\frac{D_f}{D_c} + \frac{G_f}{T_c} = 1 \tag{6}$$

where D_c is the detachment capacity;
D_f is the actual detachment rate;
T_c is the transport capacity; and
G_f is the sediment in transport.

The detaching and transporting capacity of the flow varies between inter-rill and rill areas. The dynamic nature of rilling is a difficult feature to model, not least because rainfall affects the presence of small scale channelized flow (Dunne & Aubrey, 1986). It is assumed firstly, that the erosive agent is overland flow, and that raindrop effects can be ignored, and secondly, that the estimation of detaching and transporting capacity can be assessed across the general region of the hillslope, for each cell of the spatial mesh.

The basis for most existing erosion algorithms was established by Zingg & Musgrave in the 1940s (Zingg, 1940; Musgrave, 1947) in which erosion rates are a function of surrogates for the shear stress of overland flow:

$$Y = Q^{1.66} S^{1.5} \tag{7}$$

where Y is sediment yield in $cm^3\ cm^{-1}\ year^{-1}$;
Q is water discharge; and
S is slope angle.

In the model, this equation is used to estimate the transporting and detaching capacities. The actual transport and detachment rates are determined by the interaction between transport and detaching capacities. The transport deficit approach discussed by Kirkby (1980a), is adopted so that the actual erosional rate approaches the detachment capacity when the sediment load is very much smaller than the transport capacity:

$$D_f = \frac{T_c - G_f}{T_c/D_c} \tag{8}$$

where: $T_c = a\ Q^b\ S^c$ (9)

$D_c = k\ Q^m\ S^n$ (10)

a, k are coefficients dependent on soil resistance; and
b, c, m, n are exponents.

The greater the difference between the transporting capacity and the detaching capacity, the slower the rate of uptake. As a result, there is a gradual rather than sudden transition from detachment-limited to transport-limited removal. This inter-relationship between detachment and sediment load explains the changes in the sediment yield even when other variables such as depth and energy grade-line remain constant.

The erosion-deposition algorithm uses the discharge values of each cell generated by the kinematic overland flow routine, the slope and soil parameters and the sediment concentration of the current time step. This enables sediment to be routed down the slope. The net change in surface throughout the storm is stored for every node on the surface.

At the end of each storm event, the change in topography is assessed. Ground lowering is reflected, not by changing the height values of each node, but by calculating the resultant migration of each node along the orthogonal flowline. This is illustrated in Fig. 4. By plotting the new positions of the contours, the change in the morphology is easily perceived through the deformation of the contour lines. Their relative velocities will alter the steepness of the slope and deformation will change the direction of the flow lines.

SIMULATIONS

The slope profile has an important effect on the distribution of erosion across the surface. The output of three runs is given in Fig. 5, where the three-dimensional plots show the total amount of ground lowering at a point, across the entire surface. For all these surfaces, overall erosion rates increase downslope, but the trends are significantly different. Here, the slope profile is respectively convex, planar, and concave. The average gradient is 6.3°, and the slope length is 2 m. On the convex slope, discharge and slope are increasing in a downslope direction, leading to a progressive increase in the total amount of ground lowering. On the planar slope, discharge only is increasing downslope. Once the increase in the transporting capacity no longer compensates the additional sediment to transport, the increase in ground lowering downslope declines. On the concave slope, there is an initially rapid increase in erosion downslope, but this levels out as the slope declines, and at the base of the slope, the rate of ground lowering begins to decrease. On a longer slope, deposition may occur, and a scanning routine is necessary to detect reverse slopes.

If it is assumed that the migration of the gully head is related to the erosive power of the flow, the position of the gully head in relation to the above trends becomes important. In the second set of simulations (Fig. 6), the same shape of gully head is considered at two different

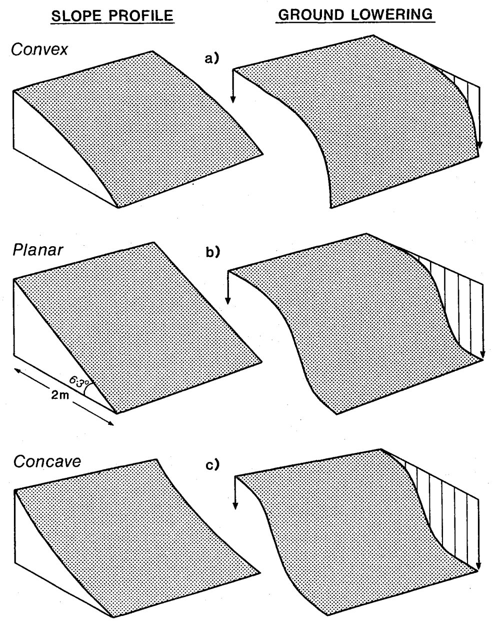

***Fig. 5** Three dimensional plots of ground lowering across the entire hillslope at the end of the storm; the slope morphology is shown on the left; actual ground lowering, shown on the right, is increasing downslope.*

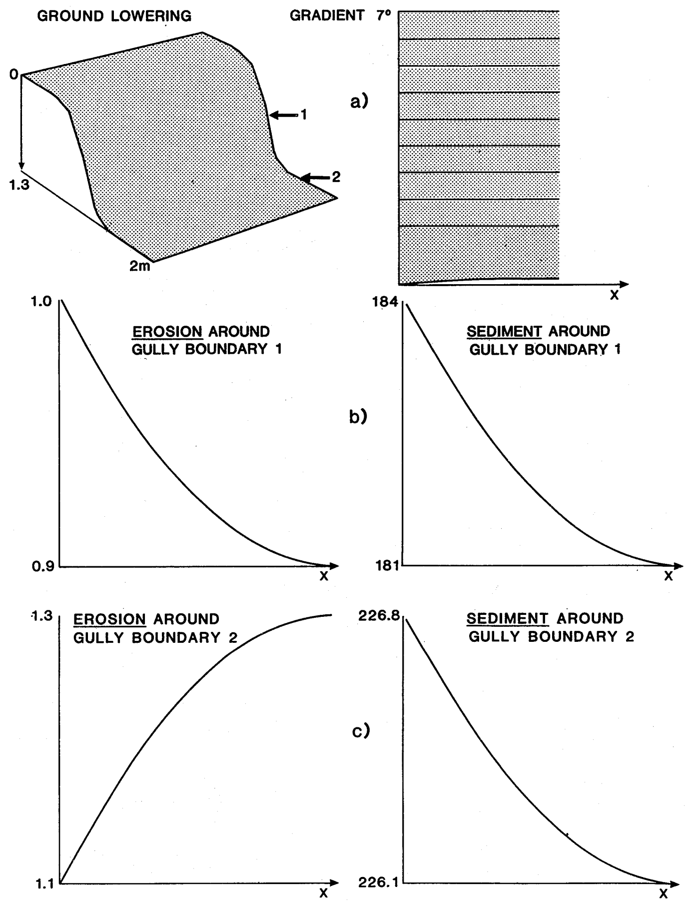

Fig. 6 Gully head migration from two different positions on a slope: (a) the distribution of erosion (left), the plan form of the slope (right); (b) amount of erosion (left) and sediment yield (right) around the gully head for position 1 on slope; and (c) amount of erosion (left) and sediment yield (right) around the gully head for position 2 on slope.

positions on a planar slope; firstly where the erosion rate is still increasing in the downslope direction; secondly where it is not. In the first case, the erosive power of the water increases as it flows downslope, over and above the increase in sediment load, and subsequently the curvature of the bottom boundary is obliterated. In the second case, the water's erosive power is no longer increasing downslope, and an increasing proportion is expended on transporting sediment. As a consequence, there is a concentration of erosion at the apex of the gully and it migrates forward with time.

In the final set of simulations presented here (Fig. 7), two different gully head shapes are considered on identical planar slopes, at a position where the erosion rates are still increasing downslope. In the first case, the

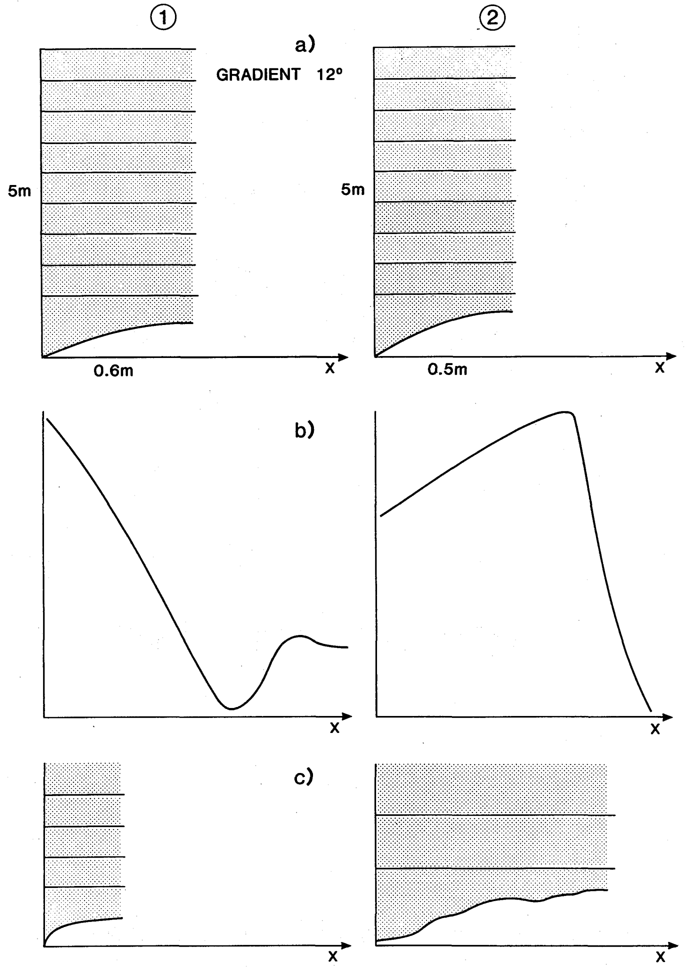

Fig. 7 Two different gully heads migrating into identical planar surfaces: (a) plan view of slopes 1 and 2; (b) amount of erosion around gully heads 1 and 2; and (c) resulting migration of gully heads 1 and 2.

flow has a greater erosive power at a distance from the apex of the gully head, but at the same time the same width of flow from upslope has to cross a greater width of gully boundary. There is a point near to the apex where the concentration of the flow leads to a local maxima of erosion. However, the trend is not strong and the gully head diminishes with time.

In the second case, however, the curvature of the gully head is greater, and the angle of the flowlines across the gully boundary changes more rapidly away from the apex. This has the effect of diminishing the erosive impact of the flow with distance from the central axis. At the same time, the erosive power of the flow is still increasing with distance downslope, and as a result, the focus of erosive power is strongly deflected away from the gully apex. A distinct bifurcation is observed at the end of the storm.

CONCLUSIONS

The implications of these simulations are that the balance between the erosional power in the flow and the relative concentration of this across the gully boundary may have a considerable influence on the way in which the gully will develop and branch. However, it is clear that the dynamics of sediment transport and the influence of slope profile may offset the effect of contributing area.

By modelling the dynamics of overland flow, as against using surrogates such as slope length or contributing area, certain characteristics of the erosive process can be explored. In shallow flows, with constant detachment and deposition, and where infiltration losses lead to a complex hydrology, there is no reason to expect that erosion rates are dependent on the distance from divide.

The investigation continues to focus on the influence of gully head geometry as the head migrates into planar, concave, and convex, converging and diverging slope forms. Two areas in particular are proposed for further development of the model:

(a) subsurface processes, in the context of both sapping and piping (e.g. Dunne, 1980); and

(b) gully head retreat as a function of slope instability along the gully boundary. The scale of these may vary from soil creep to slab or arcuate failures.

In the form presented here, this model provides a clear illustration of how the growth and bifurcation of a gully may be critically controlled by the way in which the gully head geometry and slope topography influence particular processes. It is proposed that this concept provides a useful framework for conducting digital and hardware simulations.

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