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Improved model for bed-level changes in mountain rivers

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Abstract This study is concerned with a method to simulate one-dimensional bed-level change in mountainous rivers. A characteristic curve analysis on the basic equations indicates that bed disturbances should be treated as dynamic waves. Bases on this result a new model to calculate bed-level change in a small time interval is derived and its stability is clarified through the simulation of simple morphological processes. Furthermore, a flood-sediment routing model composed of the new model and an unsteady-flood routing model based on MacCormack's scheme is presented with a numerical example.

INTRODUCTION

The bed morphology in a mountainous river is frequently subject to floods in which subcritical flow and supercritical flow coexist. Consequently, as explained below, computation of one-dimensional bed-level change becomes very troublesome.

A computation model of bed-level change is composed of a model for flood routing and one for sediment routing. A steady-flow routing model has been usually preferred to an unsteady-flow routing model. In the case of mountain rivers, however, the former sometimes presents a nearly insurmountable difficulty in automatic search for sections of flow transition. This difficulty may be avoided by adopting a kind of unsteady-flow routing model. Because of the extreme increase in CPU time, however, unsteady-flow routing becomes impractical in the case of computation for a long duration of up to several decades. On the other hand, the following problem may be pointed out concerned with sediment routing. Generally, based on the kinematic wave model of bed-disturbance such as that derived by de Vries (1973), the spatial derivative in the sediment continuity equation has been discretized by backward- or forward-difference when the flow condition is subcritical or supercritical, respectively. In the case where both conditions of flow coexist, however, the above method yields a serious error in the sediment continuity, as already pointed out by Michiue *et al.* (1990).

In this paper, a new model for sediment routing is derived based on a dynamic wave model of bed disturbance. Moreover, this model is combined with an unsteady-flow routing model based on MacCormack's scheme introduced by Garcia & Kahwita (1986). Though the selective transport of mixed sediment is an important aspect of mountainous rivers, this paper is devoted to the case of uniform sediment.

BASIC EQUATIONS

Consider one-dimensional bed-level change in a wide rectangular river. On the assumption of gradually varied flow without lateral inflow, the equations of mass and energy conservative for water may be written as:

$$B\partial h/\partial t + \partial(Bvh)/\partial x + B\partial z/\partial t = 0 \quad (1)$$

$$\partial v/\partial t + v\partial v/\partial x + g\partial h/\partial x + g\partial z/\partial x = -gi_f \quad (2)$$

where t = time; x = streamwise coordinate along the river bed; z = bed-level; B = river width; h = flow depth; v = mean flow velocity; g = gravitational acceleration; i_f = energy slope. On the other hand, the continuity equation of sediment may be written as:

$$B(1-\lambda)\partial z/\partial t + \partial(Bq_s)/\partial x = 0 \quad (3)$$

where λ = porosity of bed; q_s = unit-width sediment transport rate.

For the solution of equations (1)-(3), expressions for i_f and q_s must be introduced. For i_f, Manning's formula is

$$v = h^{2/3}i_f^{1/2}/n = h^{1/6}u_s/(ng^{1/2}) \quad (4)$$

where n = roughness coefficient; u_s = shear velocity of flow. For q_s, the following Brown-type formula may be used:

$$\phi_s = 10\psi^{5/2} \quad (5)$$

where $\phi_s = q_s/(sgd^3)$; $\psi = u_s^2/(sgd)$; d = particle size of sediment; s = specific gravity of sediment in water.

PROPAGATION OF DISTURBANCE ON BED

Combining equations (1)-(3), the following non-dimensional form of characteristic curve equation can be obtained, i.e. denoting the characteristic velocity by ω,

$$w^3 - 2w^2 + w = (\xi/6)w^2 + (1/F_r^2 - \xi/6)w - (7/6)\xi/F_r^2 \quad (6)$$

where $w = \omega/v$ = relative characteristic velocity; $F_r = v/(gh)^{1/2}$; $\xi = [1/(1-\lambda)](\partial q_s/\partial v)/h$. Using equations (4) and (5), ξ is expressed in the following form:

$$\xi = [50/(1-\lambda)](u_s/v)(d/h)\psi^2 \tag{7}$$

Equation (6) provides three roots for a couple of values of F_r and ξ, as shown in Fig. 1, where the curves $f(w)$ and $g(w)$ represent the left-hand and right-hand sides of equation (6), respectively. When $\xi = 0$, the three roots become $w_{01} = 1 - 1/F_r$, $w_{02} = 0$ and $w_{03} = 1 + 1/F_r$; w_{01} and w_{03} are the relative velocities of the water surface waves. The assumption of steady flow gives single root w_z which can be regarded as the relative propagation velocity of bed disturbance.

$$w_z = (7/6)\xi/(F_r^2 - 1) \tag{8}$$

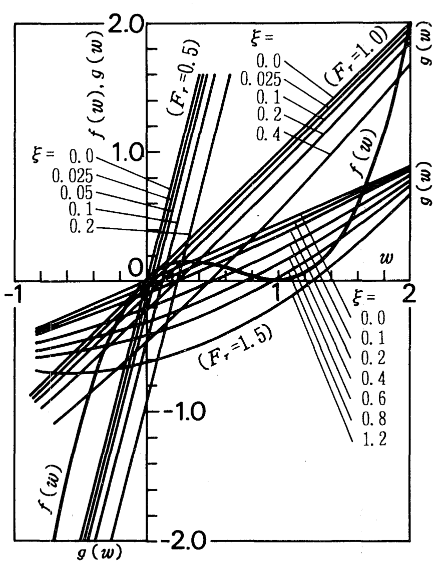

Fig. 1 The characteristics of the three roots of equation (6).

Let the three roots of equation (6) be w_1, w_2 and w_3 in the order of their numerical values. Comparing the characteristics of w_1, w_2 and w_3 with those of w_z, w_{01} and w_{03}, it may be suggested that w_1 and w_2 correspond to the propagation velocities of bed disturbance in the case of $F_r \geq 1$ and $F_r < 1$, respectively, while the others correspond to those of a water surface wave. This interpretation coincides with the kinematic model of bed disturbance. On the other hand, based on the concept of the characteristic curve method, all unknowns of the basic equations are to be affected by every one of the three roots. From the physical point of view, however, only w_1 and w_2 seem to have some substantial relation to the propagation of the bed disturbance because $w_3 > 1$. Thus, it may be concluded that the bed disturbance should be treated as dynamic waves. This conclusion coincides with the hyperbolic model of bed transient which was derived by Vreugdenhill & de Vries (1973).

A NEW MODEL FOR SEDIMENT ROUTING

Model formation

Consider the bed-level changes in a time interval Δt at sections in a river reach. As shown in Fig. 2, let $x_{(i)}$, $qs_{(i)}$ and $\Delta z_{(i)}$) be the distance, the unit-width sediment transport rate and the bed-level change in Δt, respectively, at the section i. Hereafter, the region bounded by the section i and $i + 1$ is called division i and the interval between the two sections is denoted by $\Delta x_{(i)}$.

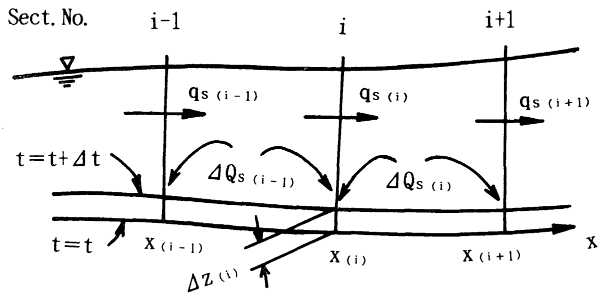

Fig. 2 Definition sketch of the new model.

According to the result in the preceding paragraph, $\Delta z_{(i)}$ is affected significantly by the bed disturbances propagated from the both sides of section i. If Δt is taken as sufficiently small, $\Delta z_{(i)}$ may be evaluated by superimposing the contributions of the bed disturbances in the division $i - 1$ and i:

$$\Delta z_{(i)} = \Delta z'_{(i,i-1)} + \Delta z'_{(i,i)} \tag{9}$$

where $\Delta z'_{(i,i)}$ = contribution of the bed disturbance in division i to $\Delta z_{(i)}$; the first and the second subscript indicate the number of the section and that of the division, respectively.

Let $\Delta Q_{s,(i)}$ be the residual of sediment transport in division i:

$$\Delta Q_{s,(i)} = B_{(i)} q_{B,(i)} - B_{(i+1)} q_{B,(i+1)} \tag{10}$$

where $B_{(i)}$ = bed-width at section i. Now, on the analogy of CFL criterion for finite difference methods, it may be assumed that the greater is the propagation velocity of the bed disturbance toward section i or $i + 1$, the smaller becomes its contribution to $\Delta z'_{(i,i)}$ or $\Delta z'_{(i+1,i)}$, respectively. Furthermore, for simplicity, assume that the contribution is proportional to the reciprocal of the propagation velocity concerned. Then, $\Delta z'_{(i,i)}$ and $\Delta z'_{(i+1,i)}$ may be written as:

$$\Delta z'_{(i,i)} \Delta x'_{(i)} B_{(i)} = \frac{w_{2,(i)}}{(w_{1,(i)} + w_{2,(i)})} \cdot \frac{\Delta Q_{s,(i)} \Delta t}{(1 - \lambda)} \tag{11}$$

$$\Delta z'_{(i+1,i)}\Delta x'_{(i+1)}B_{(i+1)} = \frac{w_{1,(i)}}{(w_{1,(i)} + w_{2,(i)})} \cdot \frac{\Delta Q_{s,(i)}\Delta t}{(1-\lambda)} \qquad (12)$$

where $\Delta' x_{(i)} = (\Delta x_{(i-1)} + \Delta x_{(i)})/2$; $w_{1,(i)}$ and $w_{2,(i)}$ = relative propagation velocities to be approximately estimated by:

$$w_{1,(i)} = (|w_{1,(i)}| + |w_{1,(i+1)}|)/2 \qquad (13)$$

and

$$w_{2,(i)} = (w_{2,(i)} + w_{2,(i+1)})/2 \qquad (14)$$

Numerical investigation of stability

The numerical stability of the new model proposed above was tested through the simulation of ideal morphological processes as shown in Fig. 3. It was assumed that a gently-sloping mound on a sediment bed with a uniform width and also a uniform slope i_0 is distorted under the condition of steady discharge.

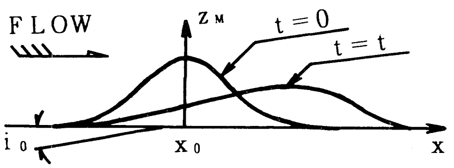

Fig. 3 Schematic representation of the ideal morphological process.

In the test, i_0 and d were varied widely whereas the unit-width discharge q, s and n were kept constant at 1.0 $m^2 s^{-1}$, 1.65 and 0.03 s $m^{-1/3}$, respectively, and also the initial shape of the mound was uniformly given in m-unit as:

$$z_m = 0.05 \exp[-(x-x_0)^2/250] \qquad (15)$$

where z_m = height of the mound; x_0 = location of the top of the mound. The flood routing was performed by a steady-flow routing model. The value of $\Delta x_{(i)}$ was uniformly set equal to 5.0 m for all cases, while that of Δt was determined in every flood-sediment routing sequence so as to satisfy CFL criterion in all divisions.

The stability of the computation depends on the reference velocity for CFL criterion and also on Courant number σ. As a result, it was found that the smaller of $w_{1,(i)}$ and $w_{2,(i)}$ may be adopted as the relative reference velocity and that σ may be set as large as 3/4.

Some of the simulation results obtained by setting σ equal to 2/3 are shown in Fig. 4, where h_c = critical flow depth; h_0 = uniform flow depth on the bed with slope i_0. In Fig. 4 the representative Froude number increases in the order of parts (a)-(d), added to the separated figures. The distortion processes of the mound shown in Fig. 4 seem quite reasonable. Thus, the stability of the new model has been clarified, while the problem of accuracy is currently under investigation.

FLOOD AND SEDIMENT ROUTING UTILIZING THE MACCORMACK SCHEME

The MacCormack scheme, composed of a predictor-corrector sequence, has been successfully applied to rapidly varying flows including shocks or discontinuities. The new model for sediment routing and a flood routing model based on the MacCormack scheme may be combined as follows.

Equations (1) and (2) can be rewritten as:

$$\partial A/\partial t + \partial Q/\partial x = Z \tag{16}$$

$$\partial Q/\partial t + \partial F/\partial x = S \tag{17}$$

where $A = Bh$; $Q = Bvh$; $Z = -B\partial z/\partial t$; $F = Bv^2h + gBh^2/2$; $S = -gBh(\partial z/\partial x + i_f)$. In the case of equation (16), the MacCormack scheme for the predictor sequence and the corrector sequence may be written as:

$$A_i^p = A_i^0 - (\Delta t/\Delta x_i)(Q_{i+1}^0 - Q_i^0 + D_{i+1}^0 - D_i^0) + \Delta t Z_i^0 \tag{18}$$

$$A_i^c = (A_i^0 + A_i^p)/2 - (\Delta t/\Delta x_{i-1}/2)(Q_i^p - Q_{i-1}^p - D_i^p + D_{i-1}^p) + (\Delta t/2)Z_i^p \tag{19}$$

where the superscript "0" indicates the solutions for time t and the superscripts "p" and "c" indicate the predicted and corrected solutions, respectively, while the subscripts indicate the section number. The term D is the artificial viscosity and, in this study, it was expressed as

$$D_i = K_D h_i u_{si} (A_{i+1} - 2A_i + A_{i-1})/\Delta x \tag{20}$$

where K_D = an empirical coefficient. The forms of the MacCormack scheme for equation (17) can be obtained only by replacing A, Q and Z in equations (18)-(20) with Q, F and S, respectively.

The values of Z_i^0, Z_i^p and $\partial z/\partial x$ included in S_i^p are to be calculated from the result of the sediment routing by the new model; this calculation is able to precede the calculation of flow in each sequence. Furthermore, referring to the expression of equation (19), the results of sediment routing in the two sequences may combined in the following manner to evaluate the bed-level at the time $t + \Delta t$, i.e. denoting it by z_i^c,

$$z_i^c = (z_i + z_i^p)/2 + \Delta z_i^p/2 \tag{21}$$

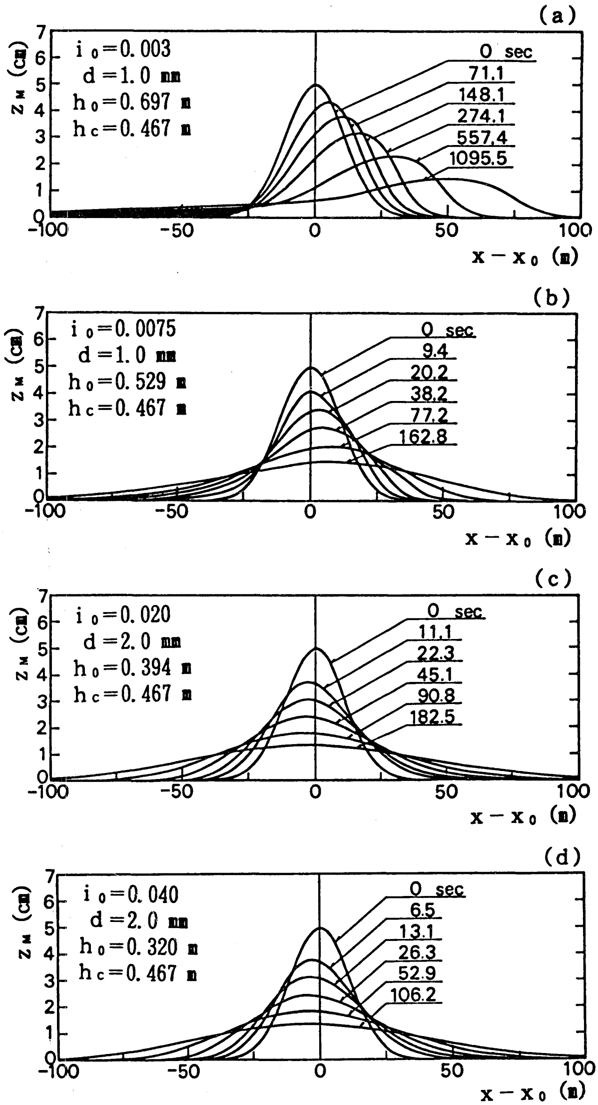

Fig. 4 Some examples of the simulation for the stability test.

where z_i = bed-level at time t; z_i^P = bed-level at time $t + \Delta t$ obtained in the prediction sequence; Δz_i^P = bed-level change in Δt, to be calculated by using the result of flood routing in the prediction sequence.

The combined model was applied to the simulation of a morphological process in an imaginary rectangular channel. The variation of the channel width is shown together with the simulation result in Fig. 5. The particle size was assumed to be 3.0 mm and the discharge at the upstream end was kept constant at 1.0 $m^3\ s^{-1}$. The water surface level at the downstream end was reduced from 3.2 to 1.0 m for the first 500 s and kept constant at the latter level. Thus, a morphological process accompanied by the transition of flow was simulated.

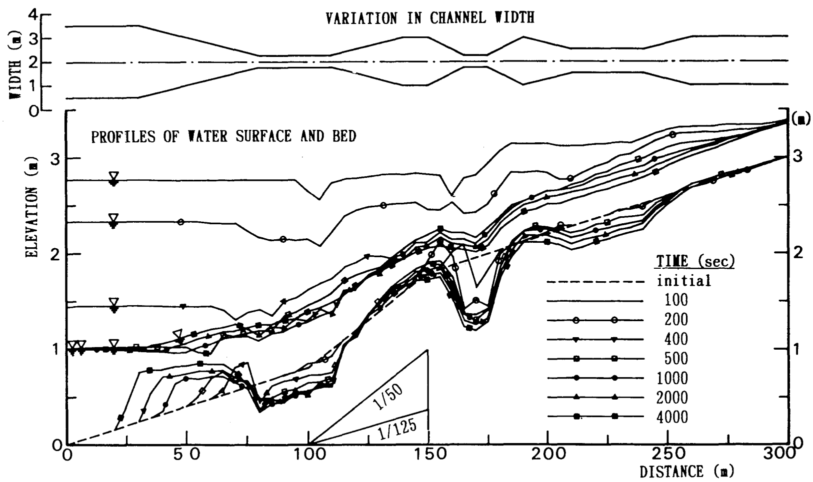

Fig. 5 A numerical example of the combined model.

The simulated process seems not to contradict what is expected from the physical point of view. However, wavy fluctuations were generated in the results for an elapsed time greater than 1000 s, as is found in the result for 2000 s. The fluctuation could be diminished to some degree by a simple moving average procedure, as is seen from the result for 4000 s.

CONCLUSION

In this paper, a framework of the one-dimensional flood-sediment routing model has been proposed. It seems useful for the computation of the morphological processes in mountain rivers where subcritical and supercritical flows frequently coexist. In order to solve the field problems, however, further investigations need to be carried out with regard to the effect of selective transport of mixed sediment.

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