Modelling sediment transport in arid upland basins in India

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Abstract A model to predict sediment transport in arid upland basins associated with individual storms can be derived from closed form solutions to the governing erosion equations for steady-state conditions viz. $\ln(T_c - Q_s) = -G X + \ln C$, where T_c is transport capacity of overland flow, Q_s is actual transport rate, G is a first-order reaction coefficient, X is downslope distance and C is the integration constant. Values of G and C were determined at each stage of the hydrograph and with three calibration methods (the reasons for their variability were discussed). The model predicts the sediment transport rates within $\pm 10\%$ accuracy.

INTRODUCTION

Arid regions have a potential for generating and transporting large quantities of sediment (Schick, 1970). Contributing factors vary depending upon the situation, and include the high intensity of the episodic rainfall (Bell, 1972), the presence of excessively weathered surface material (Goudie & Wilkinson, 1977), the sparse vegetation cover (Pilgrim *et al.*, 1988), erodible aeolian surficial deposits within the drainage basin (Jones, 1981) and increased biotic interference (FAO, 1973).

In the Indian arid zone sediment yields vary from 1.6 to 445.7 t km^{-2} $event^{-1}$ (0.2 to 53.5 g l^{-1}) in the hilly terrain, and 90% of the sediment is fine sand and silt (Sharma *et al.*, 1984). These values are considerably higher than the sediment concentrations of 1.0 to 12.0 g l^{-1} reported for central Australia and of 5.0 g l^{-1} reported for the western USA (Mabbutt, 1977). In the present study the upland basins are representative of areas where the runoff is related only to rainfall onto the drainage basin surface viz. the hilly/mountainous region.

THEORY

Many models of sediment transport by water in upland basins dynamically route sediment by solving the continuity equation for sediment transport (Bennett, 1974). The solution of this equation is generally accomplished using numerical methods which are not only unstable, but also uncertain due to the friction losses. In the present study we have attempted a closed form solution to the governing differential equation under steady state conditions which not only reduced the number of computations but also reduced the instabilities associated with the numerical solutions.

Governing equations

Sediment movement downslope obeys the principle of continuity of mass expressed by (Nearing *et al.*, 1989):

$$\frac{\partial Q_s}{\partial X} = D_F + R_{DT} \tag{1}$$

where Q_s (kg s^{-1} m^{-1}) is mass transport rate per unit of width, X (m) is downslope distance, D_F (kg s^{-1} m^{-2}) is the net flow detachment rate and R_{DT} (kg s^{-1} m^{-2}) is the net rainfall detachment rate. The assumption of quasi-steady state allows deletion of time terms from the equation (1). Further, R_{DT} is negligible since the transport capacity of rainsplash is very low. D_F in arid zones, where the initial potential sediment load is always in excess of the transport capacity (Foster *et al.*, 1980), has been estimated by a first order reaction model of the type:

$$D_F = G(T_c - Q_s) \tag{2}$$

where G (m^{-1}) is a first-order reaction coefficient and T_c (kg s^{-1} m^{-1}) is the flow transport capacity.

The hydrological input to the model is the flow depth which is estimated from the Manning equation as:

$$h = \left(\frac{q_w n}{\sqrt{s}}\right)^{0.6} \tag{3}$$

where h (m) is overland flow depth, q_w (m^3 s^{-1} m^{-1}) is the flow discharge, n is the Manning roughness coefficient (taken as 0.046 for a good vegetative cover and rough surface/depressions of 10 to 15 cm depth; a moderate value (Foster *et al.*, 1980)), and s is mean slope. Although the Darcy-Weisbach equation with a varying friction factor for laminar flow might be more accurate for calculation of depth in some cases, most users are better acquainted with estimating Manning's n. The shear stress acting on the soil is calculated as:

$$\tau_s = \gamma h s \tag{4}$$

where τ_s (kg m^{-1} s^{-2}) is the shear stress and γ (kg m^{-2} s^{-2}) is the specific weight of water.

Several generalized formulae have been developed for computing T_c. However, Alonso *et al.* (1981) concluded that the Yalin equation (Yalin, 1963) provided reliable estimates of T_c for shallow overland flow associated with upland erosion. The Yalin equation is defined as:

$$\frac{T_c}{(\mathrm{SG})\,\mathrm{d}\rho_w^{Y2}\tau_s^{Y2}} = 0.635\delta\left[1 - \frac{1}{\beta}\ln(1+\beta)\right] \tag{5}$$

$$\beta = 2.45(\mathrm{SG})^{-0.4}\,(Y_{cr})^{0.5}\,\delta \tag{6}$$

$$\delta = \frac{Y}{Y_{cr}} - 1 \qquad (\text{when } Y < Y_{cr},\ \delta = 0) \tag{7}$$

$$Y = \frac{\tau_s/\rho_w}{(\mathrm{SG}-1)gd} \tag{8}$$

where SG is particle specific gravity (2.65 for fine sand and silt), τ_w (kg m^{-3}) is the mass density of water, d (m) is particle diameter, Y is the dimensionless shear stress, Y_{cr} is the dimensionless critical shear stress from the Shields' diagram, g (m s^{-2}) is the acceleration of gravity, and β and δ are parameters as defined by equations (6) and (7). The modified Yalin equation (Foster, 1980) capable of dealing with mixtures of particles of varying diameter and density was used in the analysis.

Combining equations (1) and (2) the upland soil erosion model is derived as:

$$\frac{\partial Q_s}{\partial X} + GQ_s - GT_c = 0 \tag{9}$$

Equation (9) is a linear nonhomogeneous ordinary differential equation which can be solved analytically as:

$$\ln(T_c - Q_s) = -GX + \ln C \tag{10}$$

where C (kg m^{-1} s^{-1}) is the integration constant and is equal to $T_c - Q_s$ at $X = 0$; thereby explaining the discrepancy between the sediment transport capacity and the actual soil loss at a point of initiation of runoff within the basin.

CALIBRATION OF THE MODEL

Conceptually, the outlet of a drainage basin may be considered as a gate which controls the amount of sediment leaving the basin. Steeper slopes at the outlet may result in higher sediment discharge rates because of greater soil detachment rates within the basin. When the slope at the outlet is reduced, large amounts of sediment may be deposited rapidly. Therefore, the conditions at the outlet of the basin can be used to calibrate the upland sediment transport model, with the expectation that the model will provide the highest degree of accuracy at this critical location. The calibration options are as follows.

Reference Slope Method

The first calibration option determined τ_s based on the sediment discharge at the drainage basin outlet and the reference slope S_0. A reference slope was defined as a constant slope which passes through the end points of a complex profile lying within the basin (Fig. 1). This method was referred to as the Reference Slope Method.

Dual Slope Method

The second method of calibrating the sediment transport model was referred to as the Dual Slope Method. This method required computation of two hydraulic shear values at the outlet. The first hydraulic shear value was based on the reference slope S_0, while the second value was based on the actual slope at the outlet S_e. The value of τ_s was then defined as the average of these two shear stress values.

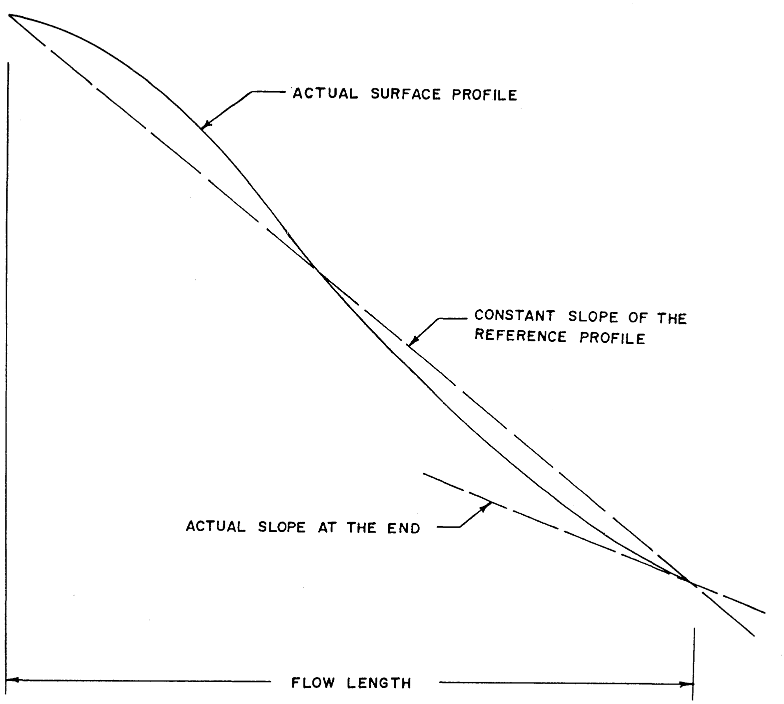


Fig. 1 Representation of a typical upland profile.

Average Shear Stress Method

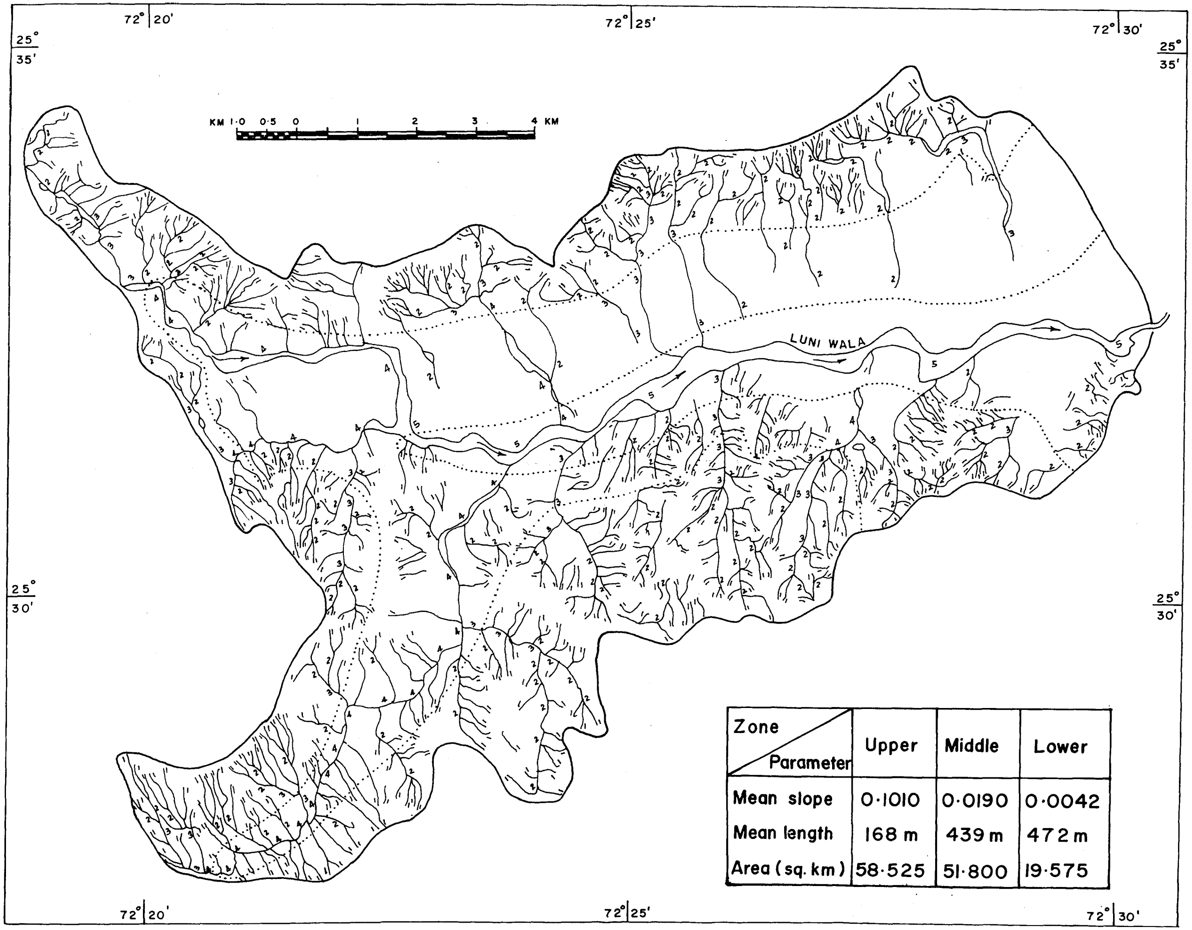
A third option for calibrating the model was termed as the Average Shear Stress Method, and is based upon the average shear stress along the entire flow path L (m). The average shear stress would be representative not only of the slope of the entire basin, but would take into account the combination of slope and sediment discharge along the flow path. The average shear stress was calculated as:

$$\tau_s = \frac{\int \tau_s(X)\,\mathrm{d}X}{L} \tag{11}$$

The upland sediment transport model was tested for 10 arid upland basins with areas ranging between 104 and 1520 km^2 located within the Luni river basin in the Indian arid zone. Basin complexity was accounted for by dividing the basin into three zones, namely, upper, middle and lower according to the degree of steepness and the stream order (Sharma & Murthy, 1990). One such subdivided basin is shown in Fig. 2 as an example. The model calibration options were computed from the characteristics of the individual zones. Values of G and C were determined by the least squares technique at the stage of the flow hydrograph viz. rising, peak and recession for 90, 68 and 76 events, respectively (Table 1).

EVALUATION OF THE MODEL

The reference slope, dual slope and average shear stress methods of calibrating G and C were evaluated using independent events for each stage of the hydrograph (Table 2). For the rising stage the root mean squared difference was consistently the lowest with the reference slope method. This is because the desert streams convey the highest sediment concentration during the rising stage (Sharma *et al.*, 1984) which may be attributed to the abundance of loose soil within the basin due to weathering and drying and the near-constant soil surface condition produced by preceding dry and wet phases; and thus, the average conditions within the basin i.e. the mean reference slope affects the sediment transport rates at the outlet. At the time of peak flow the flow conditions within the basin are at equilibrium i.e. $\mathrm{d}Q/\mathrm{d}t \rightarrow 0$; where Q (m^3 s^{-1}) is discharge and t (s) is time; and the reduced slopes at the basin outlets result in the deposition of a significant proportion of the sediment loads eroded from the upstream area before it leaves the basin. Therefore, the dual slope method of calibration resulted in the least root mean squared difference. Finkner *et al.* (1989) have also found the best agreement using the dual slope method of calibrating the sediment transport models. However, during the falling stage of the hydrograph the receding flow deposits sediment throughout the basin, since



Zone / Parameter	Upper	Middle	Lower
Mean slope	0·1010	0·0190	0·0042
Mean length	168 m	439 m	472 m
Area (sq. km)	58·525	51·800	19·575

Fig. 2 Subdivision of the Ramnia basin into small uniform zones.

Table 1 Estimated value of the upland sediment transport model parameters.

Hydrograph stage	Calibration method	Reaction coefficient (m^{-1})	Constant of integration (kg s^{-1} m^{-1})	No. of observations
Rising	Reference slope	0.0069	61.3	90
	Dual slope	0.0022	0.7	
	Average shear	0.0057	14.8	
Peak	Reference slope	0.0048	89.2	68
	Dual slope	0.0034	16.4	
	Average shear	0.0043	56.9	
Recession	Reference slope	0.0072	43.7	76
	Dual slope	0.0072	12.3	
	Average shear	0.0043	2.4	

the actual flow velocity is reduced below the critical value. This results in a rapid decrease in the sediment concentration towards the end of the flow. Consequently the average shear stress has the least root mean squared difference, since it represents not only the basin slope but also the combination of slope and discharge and its cumulative effect at the outlet.

A comparison of observed and predicted sediment transport rates (Fig. 3) shows good agreement. Furthermore, when using the optimum calibration method, the maximum deviation between the observed and predicted sediment transport rates was always less than 10% (Table 2).

CONCLUSION

The closed form solution of the continuity equation of sediment transport in arid upland basins, where initial potential sediment load is greater than the sediment transport capacity of overland flow, is preferred since it reduces the number of computations and reduces instabilities associated with the numerical solution. A model of this kind based on Manning's turbulent flow and Yalin's sediment transport capacity equations predicts the sediment transport rates in the arid upland basins with an accuracy of $\pm 10\%$ during the rising, peak and

Table 2 Summary of statistical analysis of the three calibration methods for the upland sediment transport model.

Hydrograph stage	Calibration method	Sum of squares	Root mean squared difference	Maximum deviation (%)	No. of observations
Rising	Reference slope	3.46	0.20	6.1	84
	Dual slope	4.73	0.24	6.4	
	Average shear	5.75	0.26	15.0	
Peak	Reference slope	114.51	1.33	25.5	65
	Dual slope	41.68	0.80	6.4	
	Average shear	43.95	0.82	6.67	
Recession	Reference slope	3.73	0.23	31.2	70
	Dual slope	1.21	0.13	4.5	
	Average shear	1.03	0.12	3.9	

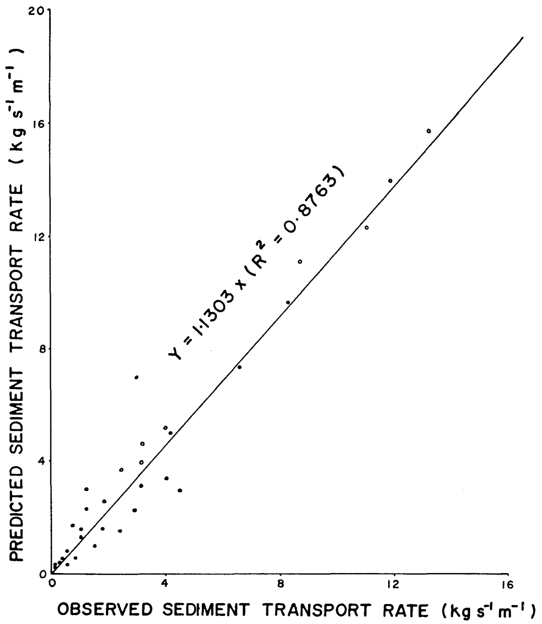


Fig. 3 Comparison of observed and predicted sediment transport rates.

recession stages of the flow hydrograph.

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