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Rheological modelling and peculiar properties of some debris flows

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Abstract In order to better understand the particular properties of debris flows, a fundamental study of the rheology of their components is proposed. A complete model, based on a thermodynamic approach and taking into account interparticle links and microstructure evolution, is developed to describe the behaviour of clay-water mixtures. This model predicts the transition from solid-like behaviour to flow by a continuous evolution with a final bifurcation. In simple shear a non-monotonous flow curve with a minimum is a possible solution of the general equations. In this case under given shear stress flow may be unstable. Experiments show that bentonite-water mixture exhibit such properties. Peculiar possible consequences for debris flow characteristics are then discussed.

INTRODUCTION

Catastrophic debris flows are rare in France, but small events often occur and these can cause much damage. It therefore appears necessary to improve our knowledge of such flows. In fluid mechanics the first step towards a better understanding of a flow is to proceed to a complete rheological study of the material flowing. Some rheological measurements of the fine parts of mudflows have already been done, for example by O'Brien & Julien (1988) or by Fei Xiangjun (1982). A Bingham model was generally found as fitting flow curves in a certain range of shear rates. However it has been stated by Wang Yuyi (1989) that these materials are shear thinning. Other authors studied the anti-thixotropic behaviour of some hyperconcentrated fluids and proposed a model (Hu Guangdou & Fang Zongdai, 1985). But many other authors, Wan Zhaohui (1982) and Moore (1959) for example, claim that one of the characteristics of some clay-water mixtures is the thixotropy. On the other hand Takahashi

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(1980) used a Bagnold dilatant model to describe rock debris flows. So there is a large diversity of constitutive relations proposed to describe debris flows. There are two main reasons: firstly, the possibilities to get fluids with distinct behaviours from some of the basic components of a debris flow are infinite; secondly, the various and complex mixtures obtained are often difficult to understand or to study with the usual rheometrical techniques.

To bring a partial answer to these problems, it was decided to make direct measurements of the rheological properties of natural fluids. For that purpose a special big rheometer (with coaxial cylinders, 1.2 m wide) was built. Measurements on fluids with maximum particle diameter of 2 cm are now beginning.

A study of the behaviour of the fine fraction of debris flows samples was also undertaken. This aims at improving rheometrical techniques, at understanding the variations of behaviour with clay type, size distribution and solid concentration, and at getting a better view of the characteristics of their behaviours. Additionally water and fine particles of debris flows constitute an interstitial fluid which could have a large influence on the behaviour of the whole flow.

Some clay-water systems have been studied. With a bentonite-water mixture, a very peculiar behaviour was found experimentally and described by a theoretical model. The results confirm the instability observed by Engelund and Wan Zhaohui (1983). In the following, attention is focused on this behaviour. It is hoped that such a complete description of a material, even if far from most natural debris flow materials, will bring an advance for rheometrical techniques in the field of debris flows, a better understanding of microstructural processes, and some explanations for peculiar natural flow characteristics. In the first section general equations will be introduced and special effects discovered in simple shear will be studied; in the second section results of experiments with bentonite-water mixtures will be described; and in the third section possible consequences on the debris flow characteristics will be discussed.

RHEOLOGICAL MODEL FOR CLAY-WATER MIXTURES

General equations

Each clay particle is made of a various number of unit-layers stacked parallel to each other. Some aluminium or silica atoms of these unit layers are replaced by atoms of lower positive valence, which results in an excess of negative charge. This charge is compensated by the adsorption on the layer of cations which are too large to be accommodated in the interior of the lattice. In the presence of water, compensating (or exchangeable) cations diffuse slightly in water, resulting in the formation of a double layer. Clay particles have also positive charge on their edges. In water at a certain pH the particles are mainly linked by highly attractive interactions (the nature of these interactions is still under discussion (Chen *et al.*, 1990). For montmorillonite, repulsion between negative double-layers is especially high.

To describe the behaviour of some clay-water mixtures, use can be made of the more general model developed in Coussot *et al.* (1991) for the rheology of dispersed systems in a low molecular weight matrix, i.e. two-phase systems of colloidal solid particles in a low viscosity liquid. Let us now see the foundations of this model. A highly attractive interaction between the particles is assumed, so that they may create flocs, i.e. small agglomerates. Beyond a certain concentration of particles these flocs produce a continuous network structure. The particle interactions are supposed to be elastic. When a characteristic stress or strain is applied to a floc, it may be broken into smaller flocs. This phenomenon is called rupture. When external forces are released, some links may be restored. During reversible deformations of flocs, elastic energy is stored; the rupture is an irreversible process and dissipates energy. These qualitative considerations result in a viscoelastic behaviour of the dispersed systems with low molecular matrixes: elastic effects result from the attractive interparticle forces and the effect of flow arises because of the floc ruptures and the flow of floc debris, as in suspensions.

The complete set of general equations is written as follows:

$$\underline{\sigma} = \underline{\sigma}_m + \underline{\sigma}_p = -p\underline{\delta} + 2\eta\,\underline{e} + G\underline{C} \tag{1}$$

$$\frac{\mathrm{d}\underline{C}}{\mathrm{d}t} - (\underline{\nabla}\,\underline{v}).\underline{C} - \underline{C}.(\underline{\nabla}\,\underline{v})^T = -2\underline{C}.\underline{e}^{ir}(\underline{C}) \tag{2}$$

$$\underline{e}^{ir}(\underline{C}) = \frac{B}{4}\left[\underline{C} - \underline{C}^{-1} - (I_1 - I_2)\frac{\underline{\delta}}{3}\right] \tag{3}$$

$$B = \frac{\mathrm{f}(\xi)}{\theta} \quad = \mathrm{f}(\xi)\,\mathrm{f}(\hat{\xi}) = \left((1+\hat{\xi})^n - 1\right)/n \qquad \hat{\xi} = \frac{\xi}{(1-\xi)} \tag{4}$$

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} + \frac{\xi}{k\theta} = \frac{E}{kZ_c}(1-\xi) \qquad E = [2tr(\underline{e}.\underline{e})]^{½} \tag{5}$$

$$k = \exp\left[\frac{a}{1+bq\Gamma}\right] \quad \Gamma = \frac{E\theta}{Z_c} \quad q = \left|\frac{\Gamma}{1+\Gamma} - \xi\right| \tag{6}$$

The first equation of (1) expresses the fact that there are two separate contributions to energy dissipation processes: one from interstitial fluid deformations, and the other from particle interactions. These two types of phenomena contribute partial stresses, $\underline{\sigma}_p$ and $\underline{\sigma}_m$ respectively, to the total stress $\underline{\sigma}$ and seem to arise respectively from some specific matrix and particle submedia or "modes".

As the interstitial fluid has a low molecular weight matrix, the contribution of the matrix mode to stress is assumed to be purely viscous. η is a Newtonian viscosity which corresponds to the viscosity of the suspension of flocs considered as solid elements. At a fixed particle concentration, variations of η with the size and concentration of the flocs are slight, thus η is considered as a constant of the system.

The particle mode is a system able to accumulate recoverable strain under the effect of external actions. The mechanism of flow is partly reversible and partly irreversible. So the thermodynamic approach developed in Leonov (1987) has been used to describe the particle mode contribution and this leads in this special case to a Maxwell nonlinear representation. In this approach the state of the medium is described by only one parameter referring to reversible changes of the medium. For that purpose tensor $\underline{C}$ is used, defined as $\underline{F}.\underline{F}^T$ where $\underline{F}$ is the averaged elastic deformation gradient of flocs. So, G being the averaged elastic modulus of the flocs, the instantaneous shear stress contribution of particle mode is only due to elastic deformations, and we have the simple relation: $\underline{\sigma}_p = -p\underline{\delta} + G\underline{C}$, where p is the isotropic pressure and $\underline{\delta}$ is unit tensor.

But $\underline{C}$ does not refer in general to the global deformation of the system and $\underline{\sigma}_p$ must be expressed as a function of $\underline{e}$. A dependence of $\underline{C}$ as a function of $\underline{e}$ is found by writing the Jaumann derivative tensor of $\underline{C}$:

$$\overset{\nabla}{\underline{C}} = \underline{C}.\underline{e}^r + \underline{e}^r.\underline{C} \tag{7}$$

$\underline{e}$ can also be defined as the sum of some reversible and irreversible strain rate tensors:

$$\underline{e} = \underline{e}^r + \underline{e}^{ir} \tag{8}$$

From equations (7) and (8) one can get (2) where $\underline{v}$ is the velocity. An expression (3) for $\underline{e}^{ir}$ as a function of $\underline{C}$ is deduced from thermodynamic considerations. Parameter B depends on the two basic invariants of $\underline{C}$, i.e. on $I_1 = tr\ \underline{C}$, $I_2 = tr\ \underline{C}^{-1}$. Additional considerations developed in Coussot *et al.* (1991) are necessary to find an expression for the crucial parameter B. As it depends on the state of rupture at the present moment, a parameter ξ which has the sense of the concentration of ruptures per unit area, with $0 \leq \xi \leq 1$. By further physical considerations, the possible expression (4) for B and the equation of evolution of x (5) are proposed. n is a numerical parameter, θ a basic characteristic time, Z_c is the critical elastic strain before rupture, and a and b are additional parameters.

Simple shear

Let us consider now the case of simple shear. We will study only the evolution of shear stress, but the normal stress evolution might be found from the general equations. Elastic deformations of flocs are assumed to be small enough. This assumption can be checked easily in the simple shear experiment when imposing constant shear stress below yield stress. The set of rheological equations (1)-(6) takes the form:

$$\sigma = S + K\Gamma \tag{9}$$

$$\frac{dS}{dt} + f(\hat{\xi})S = \Gamma \tag{10}$$

$$\frac{d\xi}{d\tau} + \frac{\xi}{k} = |\Gamma| \frac{1-\xi}{k} \tag{11}$$

where we introduce the following dimensionless variables and parameters: $\tau = t/\theta$, $\Gamma = \gamma\theta/Z_c$, $S = C_{12}/Z_c$, $K = \eta/\theta G$, $\sigma = \sigma_{12}/GZ_c$. The first equation (9) gives the instantaneous value of shear stress as the two others ((10) and (11)) give the evolution of S as a function of τ and Γ.

Flow curve

When Γ is given ($\Gamma > 0$), the solution of the set (9)-(11) is:

$$\sigma = \frac{\Gamma}{f(\Gamma)} + K\Gamma \qquad f(\Gamma) = \frac{(1+\Gamma)^n - 1}{n} \tag{12}$$

Depending on the values of K and n we obtain different flow curves (Fig. 1):

(a) $0 < n < 1$, the flow curve corresponds to shear thinning; physically the microstructure is such as on average after a rupture less elastic energy is restored than dissipated. Then when the value of steady-state shear rate increases, the number of ruptures increases too, and so does the total stored elastic energy.

(b) $n = 1$, the flow curve corresponds to Bingham's behaviour; as soon as flow has been initiated there is an equilibrium between elastic energy dissipated by ruptures and energy stored due to restoration of flocs.

(c) $1 < n < 1 + 2K$, the flow curve corresponds to shear thickening; microstructure is such that following ruptures more elastic energy is dissipated than restored.

(d) $n > 1 + 2K$, the flow curve has a minimum and a decreasing, unstable part in the region ($0 < \Gamma < \Gamma_m$). As in case (c) much elastic energy is dissipated following ruptures but the matrix liquid viscosity is small. Then in the flow curve there is firstly a decrease of stress at low shear rates due to the decrease of stored elastic energy, and afterwards an increase of stress at high shear rates when viscous dissipation becomes significant. In the following attention is focused on this very interesting case.

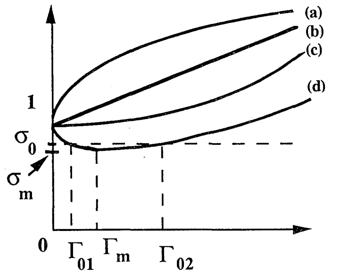

Fig. 1 Different types of flow curves predicted by the model depending on different values of parameters n and K.

Transition from solid-like behaviour to flow instability

Let us first consider the case when a shear stress σ_0 is suddenly applied to the material. The initial conditions are arbitrary: $\Gamma(0) = \Gamma_0$; $\xi(0) = \xi_0$. Thus the material may have been sheared or/and may have been flowing before the beginning of the test. We assume that σ_0 is higher than the minimum value of shear stress on flow curve but lower than yield stress, i.e. $\sigma_m < \sigma_0 < 1$. From the equations (9)-(11) we can deduce the dependency of $\hat{\xi}$ and Γ on time, and draw the phase diagram of the trajectories (Fig. 2). For $\sigma = \sigma_0$ there are three corresponding values of steady-state homogeneous shear rates: 0, Γ_{01} and Γ_{02} (Fig. 1). The corresponding points on the diagram are 0, P, and Q. Point 0 corresponds to static behaviour and is a stable point, and point P corresponds to stable flow. Point Q is an unstable saddle point. Then, the behaviour of trajectories depends on the position of the initial point ($\hat{\xi}$, Γ_0). If it is situated under (resp. above) stable moustaches of separatrix AQB going through the saddle point Q, this trajectory eventually will go to the static point 0 (resp. P).

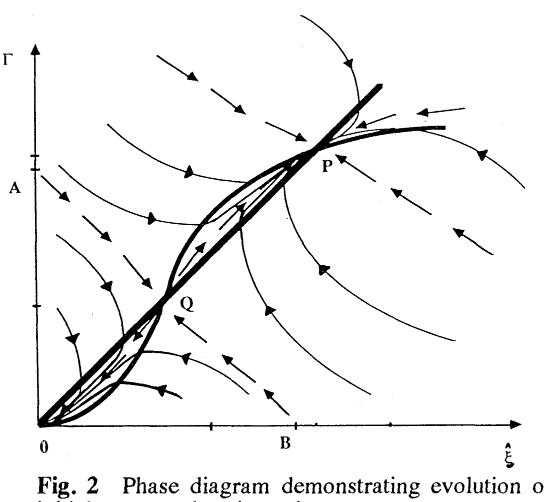

Fig. 2 Phase diagram demonstrating evolution of material depending on its initial state, under given shear stress.

The non-triviality of these results is that ***the change of steady-state regimes from a static one to flow is predicted theoretically as bifurcation, but not assumed by imposing an additional yield criterion*** as in all the visco-elasto-plastic-thixotropic theories.

This allows us to see some interesting consequences on natural flows. During some fully developed flows, shear stress may decrease because of material deposits on the riversides or because of a slight change of slope for example. Then depending on the initial state and velocity of the bulk, flow may either continue at a slightly smaller velocity or be suddenly stopped. The reverse phenomenon is also possible under inverted conditions and results in a rapid transition from rest to flow. So the flow may be quite unstable. This result has been already found by Wan Zhaohui from experiments with bentonite water-mixture, and could explain the clogging phenomenon reported by Qian *et al.* (1979) and mentioned by Engelund & Wan Zhaohui (1983).

Sandwich-like structure

Another original property of the material in case (d) one may (at least theoretically) observe is the appearance of ***inhomogeneous multiple "sandwich-like" flow patterns***, even in rheometers with highly homogeneous distribution of shear stress (like the cone-plate rheometer).

For example when $\sigma = \sigma_0$ is given, where $\sigma_m < \sigma_0 < 1$, there are two values of steady-state stable homogeneous shear rates corresponding to the given values of shear stress: 0 and Γ_{02}. They are shown in Fig. 3 by lines *OM* and *ON*; $x = 0$ is unmovable. From an arbitrary impulse-wise (equal to 0) or Γ_{02}) distribution $\Gamma(x)$, we are able to produce a "staircase" distribution of velocity $V(x)$ (Fig. 3) which is non-decreasing, continuous and satisfies the conditions $V(0) = 0$ and σ_0 constant throughout the gap.

A sandwich-like structure may also be obtained under conditions of a given shear rate. Additionally using the results of the previous paragraph we can see that from an arbitrary initial state of rupture there may be an evolution towards a sandwich-like structure when $\sigma_m < \sigma_0 < 1$.

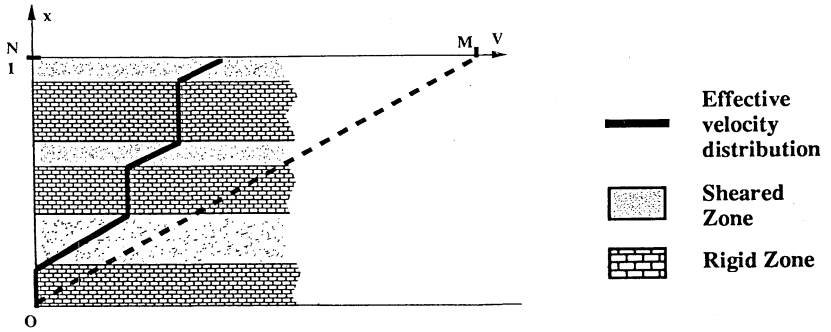

Fig. 3 Sandwich-like structure when shear stress is given.

The interesting point of this structure is that it might allow a natural material to flow under low shear stress. Indeed one would see in this case a non-fully developed flow, at least at the beginning, with small parts of material sheared at high shear rates and other parts kept rigid. But when initiated like that the flow is quite unstable. It may reach steeper slopes, then its velocity increases and flow eventually becomes homogeneous. On the contrary it may reach slighter slopes and eventually stop.

EXPERIMENTS WITH BENTONITE-WATER MIXTURE

The bentonite which was used by Coussot *et al.* (1991) contains essentially montmorillonite (90%) and was mixed with water at a concentration of 4%. The pH during tests was 8.4. Stress controlled and velocity controlled shear rheometers were used for the experiments. The cone and plate geometry was found to be the most desirable geometry as shear stress is homogeneous through the gap. The maximum particle diameter was small compared to the gap between cone and plate. Because of high yield stress and thixotropy of this system, different problems were encountered during experiments: slip at the wall, fracture within the material, partial crack propagation, or crack separation. To avoid these problems one has to use rough surfaces, to survey free surface of the sample along with odd evolutions of shear stress or shear rate, and to get an insight of the part of sample effectively sheared.

Starting from an arbitrary initial-state (obtained after a certain time of rest), shear stresses were applied and released. Below a certain critical value of shear stress, the response was essentially elastic. Above this critical value a high shear rate was suddenly recorded. Around this value, the flow was quite unstable since it sometimes began by a slight strain and became a rapid flow. At the end of tests external observations could show that the shearing had been homogeneous. Under given shear rates values of shear stress corresponding to low values of shear rates could be obtained but below 10 s^{-1} the flow was proved to be non homogeneous. The flow curve exhibits a minimum, as the case (d) in the model (Fig. 4).

By using results of some particular tests, values of parameters in the model were estimated. All additional experiments such as transient or dynamic tests show a good agreement with theoretical predictions of the model.

COMMENTS

Bentonite-water mixtures with the same solid concentration but with different pH can behave very differently, for example resembling the behaviour of a suspension with non-attractive particles. This instability in the behaviour of bentonite-water mixtures might be a general property of some clay-water mixtures at certain pH.

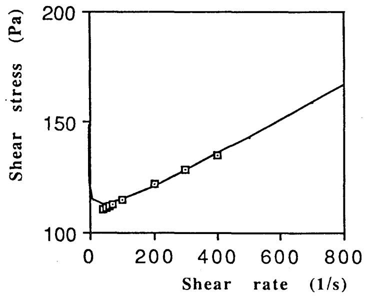

Fig. 4 Theoretical curve (thick line) and experimental points for the bentonite-water mixture.

Obviously the results obtained with bentonite-water mixtures can not yet be extended to all natural debris flows. Indeed an accurate rheological study of them is needed. But we suggest that a non homogeneous solid concentration in a debris flow may have similar effects (sandwich-like structure and instability) on flow even if the behaviour at an homogeneous concentration does not exhibit a minimum at all. Additionally the question is not only on the existence of an instability in the behaviour of some natural debris flow samples, but essentially on the importance of this effect on flow.

By now, it seems that this model and these experiments could explain some natural phenomena. It supports Engelund & Wan Zhaohui's (1983) experimental observations and could explain the instability of flows in China. It should be pointed out that Davies (1988) proposed a minimum in the flow curve as an explanation for debris flow surges.

CONCLUSION

A model based on microstructural and thermodynamic arguments has been proposed to describe the behaviour of clay-water mixtures. This model predicts the very peculiar characteristics of bentonite-water mixtures, such as a minimum in the flow curve. These characteristics may explain some interesting properties of natural flows in channels. But further study on the rheology of debris flows is needed to understand the various and complex behaviours of debris flows, and to know whether such characteristics have a crucial influence on natural flows or not.

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