

## A discussion on the velocity of debris flow

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**Abstract** Since debris flow is the shearing motion of solid grains submerged in fluid or mud, it can be described by the equation of motion:  $\tau = B + G(du/dy) = B + K(du/dy)^2$ . The friction is composed of two parts, the static friction  $B$  and the kinetic friction  $G(du/dy)$  produced by the collision of grains. By integrating the equation, the mean velocity  $U$  along the vertical can be obtained and expressed by  $U = C(R_h S)^{1/2}$  in the form of Chezy's or Manning's formula.  $C$ , the coefficient of velocity, relates to many factors and is very complication. So the velocity formulae for debris flow vary a lot for different states of motion, different properties and quantities of solid grains delivered and the geological and morphological conditions of different basins. The existing velocity formulae of debris flow for different districts in China will be reviewed. The process of vertical velocity profile will be illustrated.

### DEBRIS FLOW – FLUIDIZED MOTION OF SOLID GRAINS

Since debris flow is the shearing motion of solid grains, it may be conceptionally described by the equation:

$$\tau = B + G \frac{du}{dy} \quad (1)$$

where  $B$  is the internal resistant stress between two adjacent layers of grains submerged in fluid or mud;  $G$  has the same dimension as viscosity which is produced by the mutual collision of solid grains.

According to the analytical and experimental results of Bagnold (1954), the normal dispersive stress  $P$  and the tangential shear stress  $T(\tau)$  produced by the collision of solid grains, in full turbulent inertia regime, can be expressed by:

$$P \text{ or } T(\tau) \sim \lambda^2 \rho_s D^2 \left(\frac{du}{dy}\right)^2 \quad (2)$$

in which,  $\lambda$  = the linear concentration,  $\rho_s$  = unit mass of solid grains,  $D$  = diameter of solid grains.

Comparing this equation with equation (1),

$$G \sim \lambda^2 \rho_s D^2 \frac{du}{dy} \quad (3)$$

Let

$$K \sim \lambda^2 \rho_s D^2 \quad (4)$$

then

$$\tau = B + K \left( \frac{du}{dy} \right)^2 \quad (5)$$

Integrating equation (1) or equation (5), the velocity distribution  $u = f(y)$ , and mean velocity  $U$  along the vertical can be obtained in the form of Chezy's or Manning's formula  $U = C(R_h/S)^{1/2}$  or  $U = (1/n)R_h^{2/3}S^{1/2}$  with  $(1/n) = (C/R_h^{1/6})$ . However, Chezy's  $C$  for debris flow is related to many factors: the coefficient of resistance of the bed  $f_b$ , the coefficient of resistance of wetted perimeter  $f$ , the content of solid grains  $C_v$ , the content of clay and silt  $C_s$ , the unit mass of solid grains  $\rho_s$ , the unit mass of fluid or mud  $\rho_f$  to convey the solid grains, and rheological properties Bingham yield stress  $\tau_B$  and rigidly coefficient  $\eta$ , the diameter of solid grains  $D$  and grading  $P_i D_i$ , flow depth  $d$  and thickness of core flow  $d_o$ , the resistant coefficient between solid grains submerged in fluid or mud  $\tan\Phi$ , the slope angle of the gully  $\tan\theta$ , etc.,  $C$  can be expressed by:

$$C = f(f_b, f, C_v, C_s, \rho_s, \rho_f, D, P_i D_i, \tau_B, \eta, \tan\Phi, \tan\theta, d_o, d, \text{ etc.}) \quad (6)$$

If, for one particular valley, the composition of solid grains, the geological and morphological conditions and state of motion of debris flow are nearly the same, then some of the variables can be omitted and one formula to present the average condition of debris flow for the valley may be possible.

## THE ACTIVE BED VELOCITY

Since there exists the tractive force  $\tau_b = \gamma_m R_h S$  on the bed, consequently there is friction velocity  $U_{*b} = (\tau_b / \rho_m)^{1/2} = (g R_b S)^{1/2}$  the active bed velocity. The debris flow has strong erosive action, because it occurs in a steep gully and has a large unit weight. The friction on the bed  $\tau_b$ , for steady and uniform flow can be expressed by Darcy's equation,  $\tau_b = f_b (\gamma_m / 4) (U_b^2 / 2g) = \gamma_m R_h S$ . So the bed velocity is:

$$U_b = \sqrt{\frac{8g}{f_b} \sqrt{R_h S}} = \sqrt{\frac{8}{f_b} U_{*b}} \quad (7)$$

While the friction on the bed is larger than the acting force, the debris flow decelerates and finally stops; deposition occurs. While the active bed velocity is larger than the critical velocity of motion of the bed material, erosion of the bed occurs; the larger the active bed velocity, the more intense the erosion of the bed.

Three kinds of bed friction occur: (a) friction between the debris flow with a sub-layer of mud; (b) friction of the original bed, after the sub-layer of mud is flushed; (c) internal friction between solid grains of debris flow and the predeposited solid grains.

## A REVIEW OF VELOCITY FORMULAE FOR DEBRIS FLOW

As mentioned above, the velocity formula of debris flow can be expressed by Manning's or Chezy's formula,  $U_D = (1/n_D)R_h^{2/3}S^{1/2}$  or  $U_D = C_D R_h^{1/2} S^{1/2}$ , in which  $n_D$  and  $C_D$  should be the coefficient of roughness and coefficient of velocity of debris flow which differ from that of the clear water. However, the Manning and Chezy formulae are applied only to one-dimensional, straight fixed boundary of clear water or low sediment-laden flow. They can be modified by the equation:

$$U_D = m_c R_h^x S^y \quad (8)$$

For a movable bed, the bed configuration varies with the flow condition, and the coefficient  $m_c$ , the exponents  $x$  and  $y$  vary correspondingly.

Since the content of solid grains is very high in debris flows, the content and the unit weight of solid grains should be considered firstly to modify the Manning formula.

(1) Srinivi (1940) established the formula:

$$U_D(\text{debris flow}) = \frac{1}{(\gamma_s \Phi + 1)^{1/2}} U_c(\text{clear water}) \quad (9)$$

In the formula,  $U_c = (1/n)R_h^{2/3}S^{1/2}$ ,  $\Phi = \gamma_D/(\gamma_s - \gamma_D)$ ,  $\gamma_s$  = unit weight of solid grains,  $\gamma_D$  = unit weight of debris flow.

Fleishman (1970) expressed the formula in another form:

$$U_D = \sqrt{\frac{1 - C_v}{1 + C_v(\gamma_s - 1)}} U_c \quad (10)$$

where  $C_v$  = the content of solid grains in the debris flow (percentage by volume),  $C_v = (\gamma_D - 1)/(\gamma_s - 1)$ .

Since:

$$\left[ \frac{1 - C_v}{1 + C_v(\gamma_s - 1)} \right]^{1/2} = \left[ \frac{\gamma_s - \gamma_D}{\gamma_D(\gamma_s - 1)} \right]^{1/2} \quad \text{and} \quad \left[ \frac{1}{\gamma_s \Phi + 1} \right]^{1/2} = \left[ \frac{\gamma_s - \gamma_D}{\gamma_D(\gamma_s - 1)} \right]^{1/2}$$

then formulae (9) and (10) are the same but with different symbols. Both the formulae are derived on the assumption that the liquid phase offers the driving energy to the motion of debris flow; obviously, they can only be applied to debris flows with low concentrations of solid grains. Since  $\{(1 - C_v)/[1 + C_v(\gamma_s - 1)]\}^{1/2}$  is smaller than 1, so with the same hydraulic factors ( $R_h$ ,  $S$ ,  $n$ ), the velocity of the debris flow is smaller than that of clear water. In fact, the solid grains are driven by their own energy, so the Srinivi formula should be further modified.

(2) **Modified Srinivi formula:**

$$U_D = \frac{M_c}{(\gamma_s \Phi + 1)^{1/2}} R_h^x S^y \quad (11)$$

In the above formula, the exponent  $x$  as in many formulae takes the value  $2/3$ , but the exponent  $y$  varies from  $1/2$  to  $1/10$  for different basins;  $M_c$  takes account of the bed material size and configuration. The value of  $M_c$  generally varies from 4 to 17.7 and can be found in tables for different formulae for different conditions. Taking the formula given by the Beijing Municipal Bureau of Planning, e.g.  $U_D = M_c/(\gamma_s \Phi + 1)^{1/2} \cdot R_h^{2/3} S^{1/10}$ , it is applied to a bed with boulders as large as 1.2-2.0 m in diameter and gravels 0.01-0.08 m in diameter, with the bed configuration varying from flat to irregular, so the range of  $M_c$  is as large as 2-40 to accommodate with  $S^{1/10}$  in the formula. Obviously,  $M_c$ ,  $R_h$  and  $C_v$  are dominant factors rather than  $S$ .

(3) **Formulae taking into account the viscosity of mud.** For the Hunshui Ravine, River Daxing, Yunnan,

$$U_D = \left[ \frac{\gamma_c}{\gamma_D} \right]^{0.4} \left[ \frac{\eta_{ec}}{\eta_{eD}} \right]^{0.1} U_c$$

in which,  $\gamma_c$  = unit weight of clear water,  $\gamma_D$  = unit weight of debris flow,  $\eta_{ec}$  = effective viscosity coefficient of clear water,  $\eta_{eD}$  = effective viscosity coefficient of debris flow.

For the Jiangjia Ravine, Dongchuan, Yunnan,

$$U_D = 25.38 \left[ \frac{d_{cp}}{R_h} \right]^{0.127} \left[ \frac{\eta_{eD}}{\gamma_D \sqrt{g R_h^3}} \right]^{0.0576} \sqrt{g R_h S}$$

in which  $d_{cp}$  = the mean diameter of solid grains in the debris flow. According to these two formulae, the exponent of the terms related to viscosity is very

small, and  $\eta_{eD}/[\gamma_D(gR_h^3)^{1/2}]$  is in the nature of Reynolds number; it is seen that in debris flows with high turbulence, the effect of viscosity of the mud matrix can be neglected.

However, there are creeping flows in small ravines in which the effect of viscosity is apparent, such as in the Majing Ravine, River Heisha, Sichuan,

$$U_D = 2.77 \left[ \frac{R_h}{d_{85}} \right]^{0.737} \left[ \frac{\eta_{ec}}{\eta_{eD}} \right]^{0.42} \sqrt{R_h S}$$

$$U_D = 740 \left[ \frac{\gamma_D}{\eta_{eD}} \right]^{1.4} R_h^{2.6} S^{0.5}$$

The formulae for velocity of debris flows are quite different for different basins. Furthermore, debris flows develop and decay with different states of motion, so that the formulae stated above are induced from the average condition of each intermittent flow.

### THE PROCESS OF VELOCITY PROFILE

A debris flow is intermittent in character. In the beginning, it slides against the bed friction integrally as core flow. As the tractive force increases to become larger than the internal friction between layers of solid grains, the lower part firstly turns into fluidized grain flow and the upper part remains as core flow. As the tractive force increases further, the whole depth of debris is fluid. The stronger the tractive force, the intenser the erosion of the bed and the intenser the dissipation of energy, the debris flow decelerates and finally stops. The

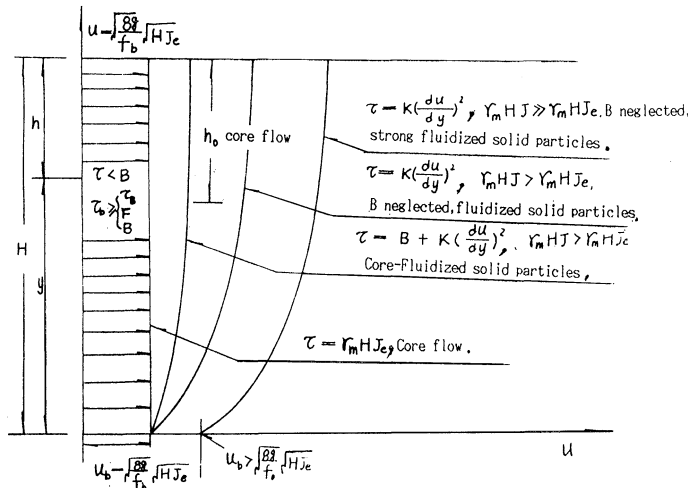


Fig. 1 The change in the velocity profile with increasing energy gradient.

upstream stage of stationary flow rises due to the inflow, becomes sufficient by deep to destroy the structure of the stationary flow, and the flow moves forward again. The debris flow finally disappears after several tens of such cycles.

The process described above is really the mutual alternation of the action of  $B$  and  $G$  ( $du/dy$ ) in equation (1) as the predominant factor in the motion.

The debris flow can be divided into three reaches: the "dragon head" is the reach of intense fluidized grain motion with strong power to erode the bed. The following reach is the "dragon body", the fluidized grain flow gradually turns to core flow in this reach. The rear reach is the "dragon tail", the velocity is weak, the core flow becomes stationary, and deposition occurs in this reach. In the reach of dragon head, the segregation action is predominant, the coarse gravels are displaced upward to the upper layer, while the small grains are displaced downward to the lower layer. The inverse grading of grains then results.

The transformation of velocity profile of debris flow is shown in Fig. 1.

## REFERENCES

- Bagnold, R. A. (1954) Experiments on a gravity free dispersion on large solid spheres on a Newtonian fluid under shear. *Proc. Roy. Soc., Lond.* **225A**, 49-63.
- Fleishman, S. M. (1970) *Seli L. Gidrometeoizdat.*
- Srinyi, M. F. (1940) Analiticheskii metod rascheta maksimal'nykh selevykh raskhodov. *Streitel'stvo Dorog no. 2-3.*
- Kang Zhicheng (1987) A velocity research of debris flow and its calculation method in china. *Mountain Res.* **5(4)**, 247-259.