Analyses of a 238-year long monthly precipitation time sequence by use of filtering techniques

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Abstract This study aims at investigating the occurrence and characteristics of drought patterns by analyzing a 238-year long monthly precipitation time sequence observed at Lund in Sweden using an ordinary Kalman filter (OKF) and an adaptive Kalman filter (AKF). The OKF identifies the periods (in the time sequence) with anomalous precipitation by comparing the observed and average precipitation patterns. The AKF detects the changes of the long-term precipitation patterns in the time sequence. These changes are associated with the abrupt changes in the parameters of a periodic-stochastic model. The time sequence is divided into several precipitation epochs, where an epoch is uniquely represented by one set of parameter values. The precipitation pattern in each epoch reveals whether the risk of drought occurrence is high or not in an epoch.

INTRODUCTION

Recently, evaluation and estimation of climatic changes such as global warming by greenhouse gases have become an important issue. Global scale estimation of climatic change is usually done by use of general circulation models (GCMs) under a certain scenario. At present, however, due to many uncertainties, the results of different GCMs are not unique even under the same scenario, especially for small scale catchment areas where practical water resources problems occur. Under these circumstances, long term records of observed precipitation and temperature are indispensable to compare with the results from GCMs and to evaluate local climatic changes. At the city of Lund in the south of Sweden observation of monthly precipitation and temperature has been made since 1753 and based on these data we will discuss long term fluctuations of climatic changes.

Droughts are often caused by so-called adverse weather (Nemoto, 1974), and their impact on the economy, society, and political situation can be disastrous. The need to limit the undesirable consequences of droughts has increased society's interest in long term fluctuations of adverse weather. In this paper, long term fluctuations of precipitation and its relation to temperature are studied. Specifically, this paper aims at investigating the occurrence of droughts by analyzing the dynamic characteristics of a 238-year long monthly precipitation time sequence using an ordinary Kalman filter (OKF) and an adaptive Kalman filter (AKF). The OKF is used to detect periods in the sequence with anomalous precipitation and to quantitatively determine the period's magnitude of anomaly. We classify the periods into three different types and evaluate the possibility of drought occurrence in each type. The AKF is used to detect abrupt changes, so-called climatic jumps, in the precipitation record. The precipitation pattern in an epoch is assumed to be terminated by a rather instantaneous shift to another epoch with a different precipitation pattern. The shifts in the precipitation pattern divide the time sequence into several precipitation epochs, where an epoch represents one pattern. Each epoch is characterized, and the risk of drought occurrence is estimated as high or not high in the epoch.

GENERAL TREND

The general tendency of the annual precipitation and annual average temperature is shown in Fig. 1. Here, temperature data from 1821 to 1833 are missing, so that the monthly average temperature data during this period are interpolated as mean values of the entire period. From the linear trend lines in the figures, it is seen that both time sequences have obvious increasing tendency. The increase rate for annual average temperature is about 1.1 degrees per 100 years and 70 mm for annual precipitation. The cause for this increase may be explained by the increase of carbon dioxide after the industrialization in late 18th century as reported by Kitano (1990) but also the effect of urbanization in the city of Lund. It is, however, not easy to separate global changes from local urbanization effects. Since this is not the aim of this paper we do not discuss this further. We only note that the urbanization effect is probably one of the reasons for increasing temperature trend in Lund.

The 30-year moving average temperature curve indicates a sharp decrease until

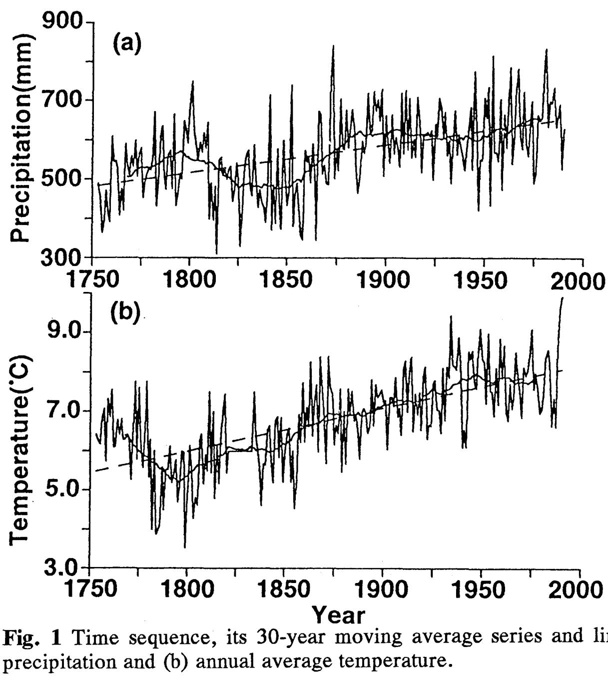

Fig. 1 Time sequence, its 30-year moving average series and linear tend of (a) annual precipitation and (b) annual average temperature.

around 1800. From then on, there has been a constant increase except for small fluctuations. For the precipitation, on the other hand, there are clear long-term fluctuations, i.e., a rapid decrease during the 1790s-1820s, a gradual decrease during the 1890s-1940s, and a sharp increase during 1850-1890, 1750-1790 and after 1950 up to now. It is probable that if the temperature rises, the precipitation amount may also increase. According to the above, however, the precipitation fluctuations indicate a decrease around 1800 when the temperature starts to increase, and then the precipitation turns into upward tendency with about a 30-year time lag against the temperature. It therefore seems as if the temperature effect the precipitation after a long time lag.

METHODOLOGY

Modelling of the precipitation sequence

The 238-year long (1753-1990) monthly precipitation sequence at Lund forms the basis for the analysis in this study. The use of monthly data provides a suitable approach for the investigation of long term variations in precipitation patterns. A cubic-root transformation of the 2856 (238 years) data points was done to normalize the data to treat both anomalously much and less precipitation on the same scale. Also the cubic-root transformed data were smoothed using a recursive low-pass filter shown in equation (1) (Bendat & Piersol, 1976).

$$y(k) = (1-\alpha)\sqrt[3]{z(k)} + \alpha y(k-1) \tag{1}$$

where k is the time step; y is the smoothed cubic root transformed monthly precipitation; α is a smoothing coefficient and z is the raw monthly precipitation. Here we choose $\alpha=0.6$. Histograms of the above tree sets of data are shown in Fig. 2. The smoothed cubic-root transformed monthly precipitation $y(k)$ at time step k is modeled by a periodic function as:

$$y(k) = M_y + \sum_{i=1}^{q} (A_i \cos 2\pi f_i k + B_i \sin 2\pi f_i k) + w(k) \tag{2}$$

where M_y is mean of the smoothed cubic-root transformed sequence; q is the number of significant frequency components; f_i is the frequency component; A_i and B_i are the amplitudes of the frequency component; and $w(k)$ is the stochastic component which is assumed to be white Gaussian noise with zero mean and variance $W(k)$. The reasons for smoothing, choosing $\alpha=0.6$ and modelling the sequence by a periodic function are mentioned in detail by Kawamura *et al.* (1985). Fig. 3 shows the MEM spectrum of $z(k)$ and $y(k)$ using $\alpha=0.6$. Comparison of the power spectra of $z(k)$ and $y(k)$ shows similar dominant peaks, indicating that the periodic properties of $z(k)$ are retained in $y(k)$. According to the results of spectrum analysis, We set $q=1$ in equation (2). The dominant frequency component used in this study is only $f_1=1/12$ (cycle/month), which corresponds to one-year period.

Kalman filter formulation

Now we consider the problem of identifying M_y and A_i and B_i by the OKF and the

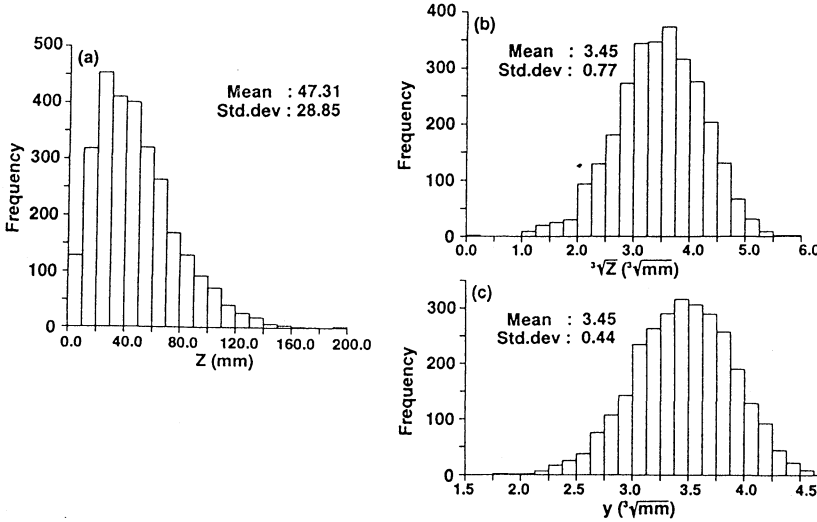

Fig. 2 Histogram of (a) monthly precipitation, (b) cubic root transformed monthly precipitation and (c) smoothed cubic root transformed monthly precipitation.

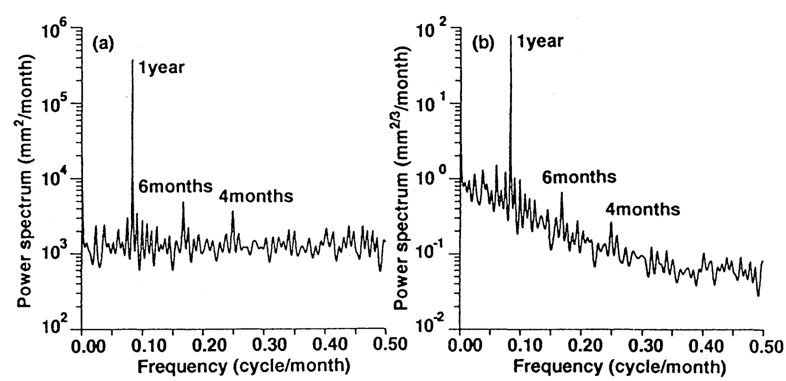

Fig. 3 MEM spectrum of (a) monthly precipitation and (b) smoothed cubic root transformed monthly precipitation.

AKF, assuming that f_i is known. The system equations for the OKF and the AKF are given in equations (3) and (4), respectively, while the observation equation for both the OKF and the AKF is given in equation (5).

$$x(k+1) = \Phi(k)x(k)+u(k) \tag{3}$$

$$x(k+1) = \Phi(k)x(k)+u(k)+\delta_{k\theta}G(k) \tag{4}$$

$$y(k) = H(k)x(k)+w(k) \tag{5}$$

where x is the $(n\times 1)$ system vector; Φ is the known $(n\times n)$ state transition matrix; u is the independent, zero mean, white Gaussian $(n\times 1)$ system noise vector with known covariance matrix U; G is the unknown $(n \times 1)$ vector expressing the magnitude of abrupt change; θ is the unknown time step when the abrupt change occurred; $\delta_{k\theta}$ is the Kronecker's delta ($\delta_{k\theta} = 1$ if $k = \theta$ and $\delta_{k\theta} = 0$ if $k \neq \theta$); y is the $(m \times 1)$ observation vector $(m \leq n)$; H is the known $(m \times n)$ observation matrix; and w is the independent, zero mean, white Gaussian $(m \times 1)$ observation noise vector with known covariance matrix W.

Here n is 3 and m is one; $x = [M_y\ A_1\ B_1]^T$; $\Phi(k) = I$; the observation equation (5) corresponds to equation (2); and $H(k) = [1 \cos 2\pi f_1 k \sin 2\pi f_1 k]$. Here the state variables are the system parameters M_y, A_1 and B_1. Details of parameter estimations by the OKF and the AKF and the various properties of the two filters for the analysis of time series expressed by a periodic function and the definition of abnormality detection index $\phi_*(k,L)$ which expresses quantitatively the system abnormality at time instant k are given by Ueda *et al.* (1984) and Kawamura *et al.* (1986).

ANALYSIS BY FILTERING TECHNIQUES

Analysis of anomalous precipitation periods

The OKF identifies the anomalous precipitation periods in the sequence by comparing the observed and average precipitation patterns through $\phi_*(k,L)$. Since the plot of the one-step ahead predictions by the OKF are almost the same as the average precipitation pattern (Ueda et al. 1984), we can calculate recursively $\phi_*(k,L)$ using the L-month series of residuals between the predicted and observed values. Thus we can determine the magnitude of anomaly of an L-month observed precipitation period from the magnitude of ϕ_*. Here we set $L = 12$. From the information on how a 12-month observed precipitation sequence differs from the average precipitation pattern, we can decide whether the observed precipitation of the period is anomalous or not.

Using the above procedure, the numbers of peak ϕ_* above $\phi_* = 6.0$, 5.0 and 4.5 are determined to be 2, 12 and 23 respectively. The respective recurrence intervals of the periods with anomalous precipitation identified by these peak ϕ_* are 120, 24 and 10 years on the average. In this paper we analyze the 23 anomalous precipitation periods arranged in descending order of peak magnitude ϕ_* in Table 1. Fig. 4 illustrates the three anomalous precipitation periods ranked 1, 4 and 9 in Table 1.

We illustrate the characteristics of the 23 anomalous precipitation periods detected by the OKF. Each period whose length is 12 months, starts from the time of occurrence of peak ϕ_*. This results in an identification of three types of anomalous precipitation periods: Type A, Type B and Type C. Type A is characterized by months with below average precipitation amounts as shown in Fig. 4(c). Type B is typified by months with above average precipitation amounts as illustrated in Fig. 4(a). As shown in Fig. 4(b), Type C is characterized by both extremely high and low precipitation amounts, which occur alternately at an interval of several months. The 23 anomalous precipitation periods are classified according to these types as shown in Table 1. Of the 23 abnormal precipitation periods, three, twelve and eight are Type A, B and C respectively, suggesting that Type B is most likely to occur and drought Type A is less likely to occur whenever there is an increase of ϕ_* for the precipitation in Lund. As shown in the above discussion, the risk of drought occurrence in each period can be evaluated by using the OKF.

Table 1 The anomalous precipitation periods detected by the OKF.

Rank	Magnitude	Time	Type	Rank	Magnitude	Time	Type
	Peak ϕ_*						
1	6.66	Feb 1873	B	13	4.85	Dec 1907	B
2	6.12	Nov 1979	B	14	4.78	Apr 1963	C
3	5.33	Aug 1950	B	15	4.73	Mar 1954	B
4	5.29	Jan 1920	C	16	4.71	Sep 1893	B
5	5.18	Feb 1898	B	17	4.70	Jan 1814	A
6	5.12	Sep 1800	C	18	4.68	Dec 1965	B
7	5.10	Jan 1967	B	19	4.66	May 1760	B
8	5.06	Apr 1883	B	20	4.62	Oct 1946	A
9	5.04	Apr 1844	A	21	4.58	Dec 1871	B
10	5.01	Feb 1983	C	22	4.57	Feb 1841	C
11	4.97	Nov 1796	C	23	4.54	Sep 1847	C
12	4.93	Feb 1840	C				

Analysis of long-term precipitation patterns

The shifts in the precipitation pattern are associated with the abrupt changes in the system parameters M_y, A_1 and B_1. In this case, the AKF detects whether abrupt changes in the system parameters or state variables occur or not by evaluating the L-month long series of one-step ahead precipitation residuals using a generalized likelihood ratio test (GLRT). The GLRT compares the value of $\phi_*(\theta,L)$ with a threshold value η. If $\phi_*(\theta,L)$ is greater than η, the hypothesis (H_1) that an abrupt change occurred at time $k=\theta$ is accepted; otherwise, the hypothesis (H_0) that no abrupt change has occurred is accepted. When an abrupt change is detected, its time of occurrence and magnitude are estimated

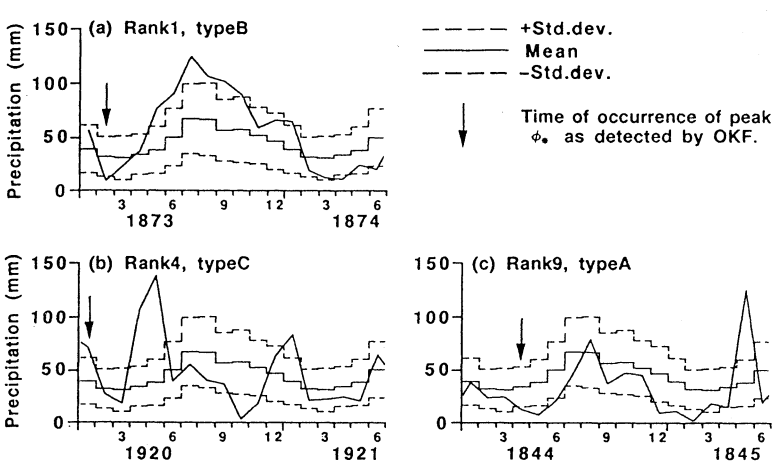

Fig. 4 Anomalous precipitation periods detected by the OKF.

quantitatively, and the state variables are appropriately corrected, according to the magnitude of this abrupt change, to allow the filter to adjust to the new precipitation pattern.

According to the statistical divisions made by the World Meteorological Organization (WMO), the interpretation of climatic change differs by time scale. A phenomenon regarded as anomalous in one time scale may be a normal event in a longer time scale. Since the 238-year precipitation sequence is analyzed using a monthly step, the changes in the precipitation pattern correspond to time scales from several years to several decades, which conform to the modern time scale defined by WMO (Nemoto, 1974). In this study, we try to detect the change in precipitation pattern, which recurs at an interval of about 30 years on the average which is defined as normal value by WMO. Thus we set η=5.0. Ten peak ϕ_* corresponding to ten anomalous precipitation periods (ranked 1-10 in Table 1) are above this level of η. Implementing GLRT, the AKF yields finally eight changes in the precipitation pattern in the 238-year sequence. This divides the sequence into nine precipitation epochs. Table 2 lists the precipitation epochs and the system parameters identified at the last time step k by the AKF for each epoch, and by the OKF for the whole sequence. Fig. 5 shows the nine precipitation epochs identified by the AKF and 23 anomalous precipitation periods identified by the OKF.

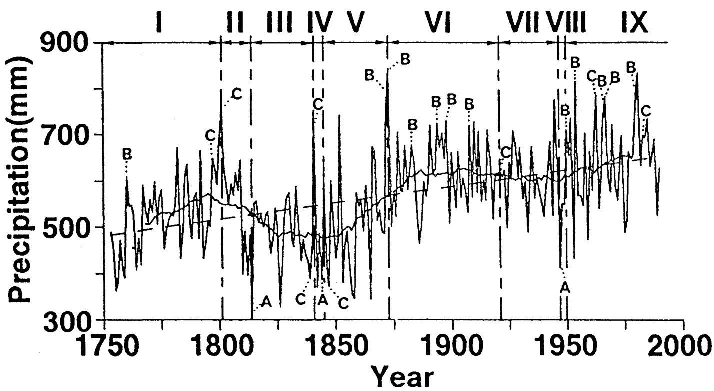

Fig. 5 The annual precipitation sequence divided by the AKF into nine different epochs. A, B and C are the types of the anomalous precipitations detected by the OKF.

We have confirmed that the parameters identified by the AKF for each epoch were almost the same as those identified by a least-square method for the same epoch, and the parameters identified by the OKF and by the least-square method for the whole sequence are equal. As a result, the mean and standard deviation of residuals from the fitted model for each epoch whose parameters are estimated by the AKF were much closer to zero for the mean and much smaller for the standard deviation than those by the OKF.

The change of precipitation pattern from one epoch to another can be investigated from the mean and one-year frequency amplitudes A_1 and B_1 shown in Table 2. We can also study in the same table whether an epoch has much or less precipitation from the estimate of mean M_y, and whether one-year frequency component is dominant in each epoch from the estimates of the amplitudes A_1 and B_1. The average precipitation pattern has the parameters estimated by the OKF shown in Table 2.

The first and the second lowest years occurred in epoch III and this epoch is typified by less precipitation as shown in Fig. 5. Also as shown in Table 2, the estimate of My of epoch III is the smallest in the nine epochs, and the amplitudes of the one-year period are small, suggesting that small precipitation continues during the entire epoch with small fluctuations. Hence the risk of drought occurrence is the highest among the nine epochs. Epoch IV is well characterized by very big amplitudes A_1, B_1 and low mean M_y, which means large fluctuations of small precipitation amounts. Hence, the probability of drought is regarded high during this epoch. In epoch VIII, the parameter values are similar to those in epoch III, which indicate a similar precipitation pattern during these epochs, so that the drought occurrence is also high. Droughts of Type A detected by the OKF also took place in these epochs.

In epochs VI, VII and IX (present epoch), the values of M_y in Table 2 are very large especially for epoch IX compared to other epochs, and the estimated amplitudes tend to be small. These precipitation patterns are stable with high precipitation amounts, so that many Type B but no Type A anomalous precipitation periods occurred as shown in epoch VI, IX (Fig. 5). Hence, we can safely say that drought is least expected in this epoch. According to Table 2, the change from epoch VI to VII is mainly caused by a phase shift of one-year period component. Here, it is interesting to note that drought-oriented epoch VIII occurred abruptly for a short period among consistent high precipitation pattern epochs, which suggests the possibility of drought epochs to occur even in the midst of periods with large precipitation amounts.

Table 2 Precipitation epochs and system parameters identified by the AKF and the OKF.

	Epoch		System parameter		
	Number	Inclusive dates	M_y	A_1	B_1
	I	Jan 1753 - Sep 1800	3.33	0.106	-0.285
	II	Oct 1800 - Jan 1814	3.42	0.176	-0.362
	III	Feb 1814 - Jan 1841	3.26	0.076	-0.271
	IV	Feb 1841 - Dec 1845	3.27	0.239	-0.653
AKF	V	Jan 1846 - Dec 1872	3.32	0.051	-0.319
	VI	Jan 1873 - Feb 1920	3.58	0.120	-0.243
	VII	Mar 1920 - Oct 1946	3.56	0.070	-0.299
	VIII	Nov 1946 - Jun 1950	3.27	0.073	-0.260
	IX	Jul 1950 - Dec 1990	3.64	0.168	-0.197
OKF	Total	Jan 1753 - Dec 1990	3.45	0.110	-0.295

CONCLUSIONS

In this paper, we have used two filtering techniques (OKF and AKF) to analyze a 238-

year long monthly precipitation time sequence at Lund in Sweden. The OKF has been used successfully to quantitatively detect periods with anomalous precipitation. The AKF was applied to divide the precipitation sequence into several epochs, where each epoch is characterized by one precipitation pattern. These epochs and the parameter shifts are usually indiscernible by visual inspection of the precipitation time series. The characteristics of the precipitation pattern in each epoch can be used as a basis for predicting the future general behaviour of the precipitation regimes and evaluating the risks for droughts.

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