

Correlation analysis of rainfall: classification method of rainfall in view of sediment yield and transport

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Abstract It is important to understand the characteristics of rainfall in order to make sediment disaster prevention plans. We use the method of correlation analysis to know the temporal changes in the spatial distribution of precipitation, and find that the method is efficient to know the characteristics of rainfall.

INTRODUCTION

The processes of erosion and deposition in rivers and streams are determined by the hysteretic relationship between discharge and yield of sediments depending on rainfall and its runoff, and these processes influence sediment disaster patterns. Therefore, when making sediment disaster prevention plans, it is important to understand the characteristics of rainfall.

So far, when making the plans, the plan precipitation has been defined as the precipitation causing sediment yield, because the sediment, which is passing at a point in river stream and domains sediment disaster, is yielded at anywhere in upstream watershed and transports by rainfall. Recent improvements in numerical calculating method of river beds variation make it possible to take into account discharge processes of sediments. Therefore, it becomes possible now to regard the plan precipitation as the precipitation which cause not only yielding but transporting sediment. The sediment transport at any point in a river depends on flood discharge at the point determined by temporal changes in the spatial distribution of precipitation in the watershed. Therefore, rainfall properties must be treated as temporal changes in spatial distribution of precipitation.

This paper will present a method of approaching a precipitation time series from the perspectives mentioned above.

COMPUTATION OF REPRESENTATIVE SPATIAL AND TIME SCALES FOR PRECIPITATION

Individual time series precipitation data having specific spatial and time scales will be treated as 'noise signal' that fluctuates in terms of both time and space. When precipitation is treated in this manner, its spatial and time scales are characterized by the reciprocity in the correlation between temporally and spatially lagged data. In correlation analysis, which usually deals with time series data, the correlation between

data can be evaluated using the cross-correlation coefficient as shown in equation (1):

$$R_{xy}(\tau) = C_{xy}(\tau) / \sqrt{C_y(0) \cdot C_x(0)} \quad (1)$$

$$C_{xy}(\tau) = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} X(t) \cdot y(t + \tau) dt \quad (2)$$

In equation (2), $x(\tau)$ and $y(\tau)$ are time series data, while τ is the temporal lag. When the time series $x(\tau)$ and $y(\tau)$ are identical, the correlation coefficient is called auto-correlation coefficient, as shown in equation (3):

$$R_{xx}(\tau) = C_{xx}(\tau) / C_{xx}(0) \quad (3)$$

Auto-correlation coefficient under random movement decreases as τ increases, which usually can be expressed as one of the equations (4), (5):

$$Ra(\tau) = \exp\{-|\tau|/\tau a\} \quad (4)$$

$$Rb(\tau) = \exp\{-(\tau/\tau b)\} \quad (5)$$

Here, τa and τb express representative time scales of changes in the time series, with each satisfying its respective equation:

$$\tau a = \int_0^{\infty} Ra(\tau) d\tau \quad (6)$$

$$\tau b = \frac{2}{\pi} \int_0^{\infty} Rb(\tau) d\tau \quad (7)$$

What is referred to here as a representative time scale does not indicate the time between beginning and end of precipitation but rather the duration of rainfall of similar intensity. Figure 1 shows an example of $R_{xx}(\tau)$ derived from actual precipitation data. The change in $R_{xx}(\tau)$ is approximated in equation (4). Then, a suitable measure for indexing a representative hyetograph time scale is shown in equation (6).

The autocorrelation coefficient or the time scale τa exhibits temporal characteristics of hyetograph as 'noise signal' at certain observation points but it does not show spatial and time scales of precipitation. In order to know the spatial scale of rainfall and associated temporal changes, it is necessary to compare precipitation time series from

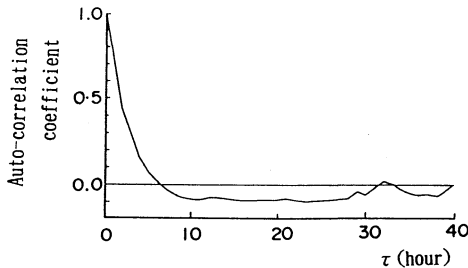


Fig. 1 An example of auto-correlogram.

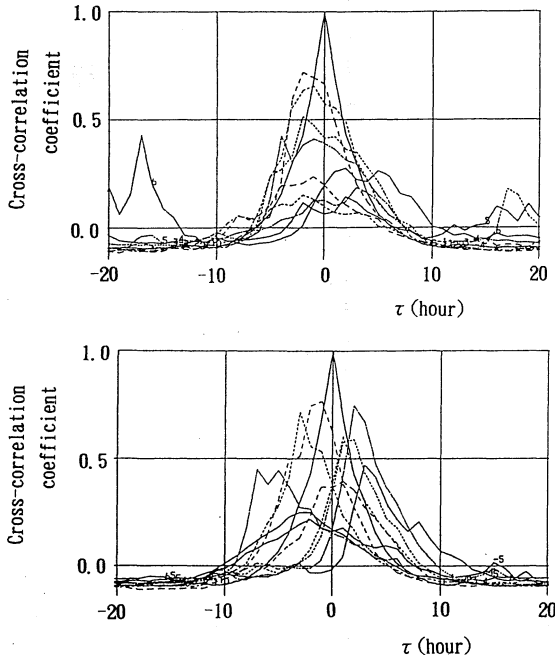


Fig. 2 An example of Cross-correlogram.

several observation points. Figure 2 shows the correlation coefficient calculated between precipitation time series obtained at the center and the other observation points which are selected at the nodes of north-south and east-west grid. For observation points along the north-south axis, the maximum correlation coefficient is at roughly $\tau = 0$, which means there is at most only a small time lag in the precipitation pattern. On the other hand, the correlation coefficient along the east-west axis peaks at the point τ is not equal to zero, showing a time lag in the precipitation pattern. Furthermore, if we examine the relationship between time lag τ and the correlation coefficient of the peak in east-west axis, we can see that the coefficient of cross-correlation becomes smaller as the time lag becomes larger. These two results shows that the rainfall moves in east-west direction.

Incidentally, the usual way to quantify such correlations of random signals is coherence and phase as shown in equations (8), (9). In equation (10), $\tau\omega$ expresses phase as a function of time:

$$\text{coh}^2(\omega) = \frac{|\Sigma_{xy}(\omega)|^2}{\Sigma_{xx}(\omega) \cdot \Sigma_{yy}(\omega)} \quad (8)$$

$$\theta_{xy}(\omega) = \tan^{-1} \left[\frac{Q_{xy}(\omega)}{K_{xy}(\omega)} \right] \quad (9)$$

$$\tau\omega = \frac{\theta_{xy}(\omega)}{2\pi\omega} \quad (10)$$

$\Sigma_{xy}(\omega)$ is the cross spectrum of $x(\tau)$ and $y(\tau)$ and is defined by equation (11). Additionally, $Q_{xy}(\omega)$ and $K_{xy}(\omega)$, as shown in equation (12), define the complex composition of cross spectrum $\Sigma_{xy}(\omega)$:

$$\Sigma_{xy}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{C_{xy}(\tau) \cdot \exp(-i\omega\tau)\} d\tau \quad (11)$$

$$\Sigma_{xy}(\omega) = K_{xy}(\omega) - iQ_{xy}(\omega) \quad (12)$$

where i is the imaginary unit.

Coherence is to exhibit cross-correlation coefficient of each frequency component of random signal, assuming that the process varying in time is the set of frequency component. If the precipitation time series caused by the same rainfall are recorded at some points, coherency for them shows a strong correlation. Therefore, the similarity of rainfalls can be determined by the value of Coherency. Phase $\theta_{xy}(\omega)$ refers to the phase difference of a couple of time series for each frequency, the phase difference for the prevailing frequency derived from the coherence of the precipitation time series between two points for identical rainfall can be regarded as the time needed for rainfall activity to move between two observation points.

Coherence and Phase, derived from the precipitation which is used in Fig. 2, are depicted in Figs 3 and 4. The solid lines in the figures represent the calculated value for observation points between the center point and the point approximately 50 km away from the center, while the broken lines represent the same at a distance of about 120 km. This example shows that time-series of precipitation in short distance are more similar than one in long distance. This tendency is recognized obviously in the component of small frequency.

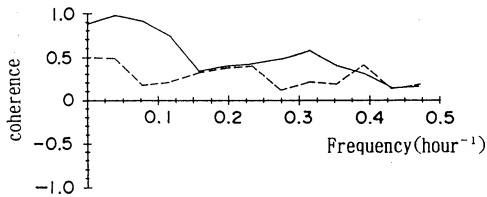


Fig. 3 An example of coherence.

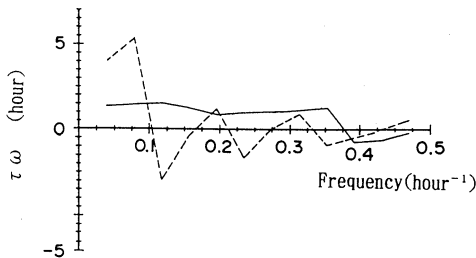


Fig. 4 An example of $\tau\omega$.

In order to depict the temporal and spatial characteristics of precipitation which is used in Fig. 2, Coherence and Phase for coherent frequencies between the center and the other observation points, are plotted against the distance from center observation point in Figs 5 and 6. Figure 5 shows that Coherence becomes small as distance increase. And Figure 6 shows that relation between distance of observation point and Phase is approximately liner, then the gradient indicates velocity of the rainfall moving.

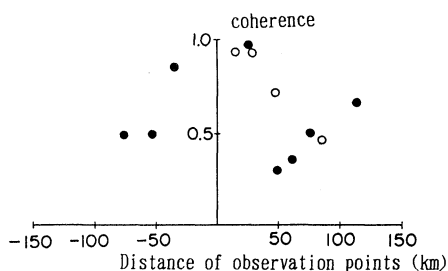


Fig. 5 The relation between coherence and distance.

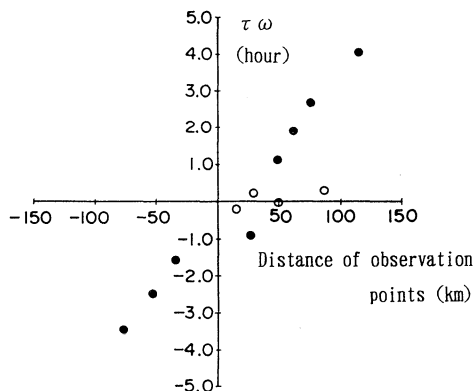


Fig. 6 The relation between $\tau \omega$ and distance.

CONCLUSIONS

The conclusions of this research are as follows:

- (1) A suitable measure for indexing a representative hyetograph time scale is τa in equation (6).
- (2) Cross-correlogram (Fig. 2) is efficient to know the characteristics of precipitation.
- (3) The degree of identity of a rainfall which may be observed at several observation points and the movement of it can be evaluated using Coherence and Phase of the precipitation time series at the observation points.

The methods, which are presented above and verified by actual data in this paper, are very useful in order to classify the rainfalls. After classify the rainfalls into some categories which are characterized by range and intensity of the rainfalls, and life time and movement of them, the long term precipitation pattern will be able to make as the plan precipitation which may cause not only yielding but transporting of sediment.