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Model of pyroclastic flow and its numerical simulation

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Abstract A gravity flow of granular bodies is considered as a model of main body of pyroclastic flow. Characteristics of the granular flow can be described by Kanatani's constitutive equations (Kanatani, 1984), which are obtained in consideration of the energy loss caused by the only inter-particle friction by the collision between the particles. Using this model, the 1991 pyroclastic flow at Mt Unzendake were reproduced by the numerical simulation. The result of calculations roughly agree with the actual phenomena. This study also shows inter-particle friction decreases as a result of the formation of a pressure gradient by gas ascent velocity.

INTRODUCTION

Pyroclastic flow is one of many types of damage-causing volcanic activities such as lava flow, volcanic mudflow and volcanic ash fall. Being a very rapid phenomenon, it has the potential to cause great damage. Then, in the past, several pyroclastic flows have occurred, and caused disaster around the volcanos.

Information about pyroclastic flows is indispensable when preparing hazard maps, establishing warning and evacuation systems, and planning facilities for use as countermeasures.

In this study, we examined pyroclastic flow fluid models and equations describing its composition, treating pyroclastic flow as a particle flow. We designed a pyroclastic flow simulation model to simulate the flows at Mt Unzendake in 1991, and compared the computed results to the actual statistics. We also examined the relationship between steam generated from contact between flow bed and hot pyroclastic flow, and fluidity of pyroclastic flow.

MODEL AND CONSTITUTIVE EQUATIONS OF PYROCLASTIC FLOW

There are relatively few instances worldwide of pyroclastic flow being directly observed and recorded (e.g. Aramaki, 1973; Mizuyama & Yamada, 1990; and Mizuyama *et al.*, 1990). Unfortunately, the data available covers only a relatively narrow range of the diverse spectrum of pyroclastic flows, and most of these are similar types of flows.

Pyroclastic flows move along slopes. The force maintaining the flow is believed to be gravitational force, as shown in Fig. 1. Pyroclastic flows consist of a gravitational flow layer of coarse particles in the lower part of the flow (the 'lower layer') and an upper mixed solid/gas layer of fine particles and gas (the 'upper layer'). In this study, we will examine the flow mechanism in the lower layer only, treating the flow as a particle flow.

Kanatani (1984) has proposed a model which takes particle flow to be the flow of completely elastic particles, and the mechanism of energy consumption to be the conceiving friction between particles. Applying the composition equation derived by Kanatani to a two-dimensional shear flow and approximating it slightly gives:

$$P = \frac{1}{200} \frac{c}{1-(c/c_*)^{1/3}} T_e \sigma D^2 \left[\frac{\partial u}{\partial z} \right]^2 \tag{1}$$

$$\tau = \frac{3}{200\sqrt{10}} \frac{c^{4/3}}{1-(c/c_*)^{1/3}} T_e \mu \sigma D^2 \left| \frac{\partial u}{\partial z} \right| \left[\frac{\partial u}{\partial z} \right] \tag{2}$$

Here, P and τ represent flow pressure and shearing stress respectively, $\partial u/\partial z$ is the velocity gradient (z-axis is perpendicular to the flow bed), T_e is a constant expressing the state of flow, σ is particle density, D is particle diameter, μ is coefficients of friction and of restitution between particles c_* is the particle concentration at the time of deposition and c is particle concentration. From equations (1) and (2), the concentration in a state of local equilibrium particle concentration can be obtained by the following equation:

$$c = \left[\frac{\sqrt{10}}{3} \frac{i_e}{\mu} \right]^3 \tag{3}$$

Here, i_e is the energy gradient expressed as τ/P.

We will use these constitutive equations of Kanatani's as the flow model of the pyroclastic flow.

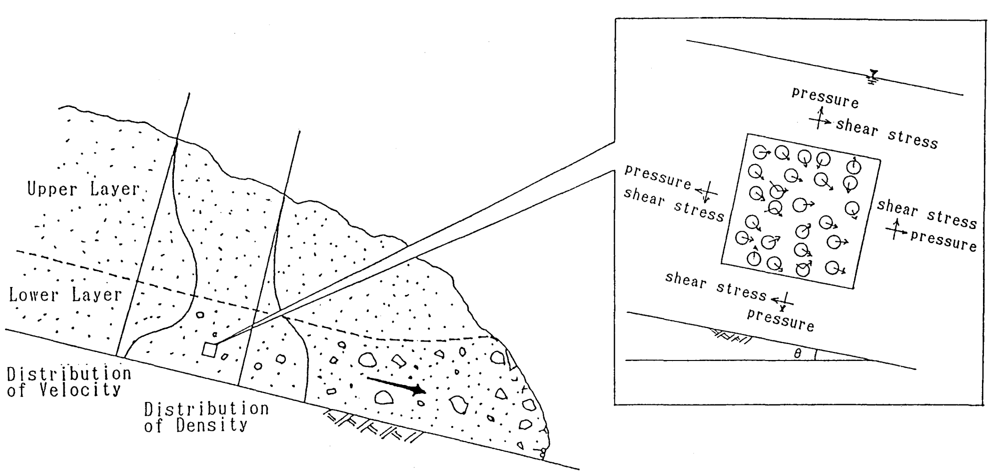

Fig. 1 The model of pyroclastic flow.

GAS ASCENT AND COEFFICIENT FRICTION BETWEEN PARTICLES

Under normal conditions, dry debris particles flowing from a slope steeper than the angle of repose will usually come to a halt when the gradient decreases. For example, data from Mt Komagatake in 1921 and Mt Fugendake, Unzen in 1991 (Ikeya & Ishikawa, 1991) indicate that pyroclastic flows were deposited at gradients much smaller than 30°, the normal angle of repose for debris flow.

Thus, it can be concluded that apparent μ will always be equal to the smaller than μ between particles. This suggests that pressure from particle collision and friction between particles has decreased, and that another pressure to support the weight of the particles has developed. The first pressure is presumed to be balanced with the weight of the particles. The new pressure gradient is created by gas filling the pores. Here we will consider the generation of steam via contact between hot pyroclastic products and pore water in the flow bed deposit layer, and gas ascent and the formation of the pressure gradient caused by the generation of steam.

As in Fig. 2, drag F working on a single particle can be expressed as:

$$F_i = \frac{1}{2}\rho C_D \left[\frac{\pi}{4}D^2\right] u_b{}^2 \tag{4}$$

gas density. Using N as the number of particles per unit volume, the pressure gradient is the with u_b the velocity of gas ascent from the flow bed. Here, C_D is a drag coefficient and ρ is sum total of drags at that location. If N is represented by the particle concentration c, the pressure gradient can be expressed as follows:

$$\frac{\partial \rho}{\partial z} = NF_i = \frac{2}{3D}\rho C_D c u_b^2 \tag{5}$$

Considering the balance of forces in the control volume shown in Fig. 2, the apparent gradient increases via the formation of the pressure gradient. Furthermore, taking into

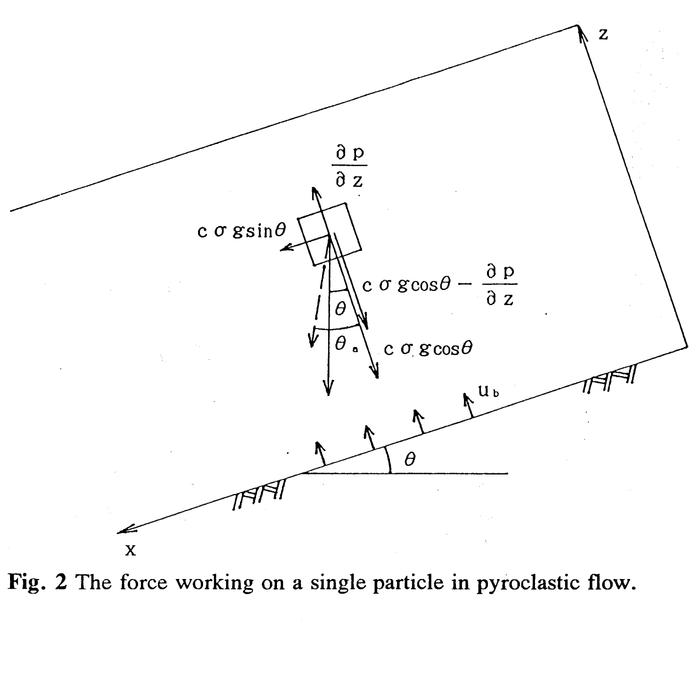

Fig. 2 The force working on a single particle in pyroclastic flow.

account the steady state, using μ_a as the apparent coefficient of friction between particles, from equation (3) the relationship between μ_a and μ can be expressed as:

$$\frac{\mu_a}{\mu} = \frac{\cos\theta}{\cos\theta - \dfrac{1}{c\sigma}\dfrac{\partial P}{\partial z}} c\sigma \tag{6}$$

Using the relationship $\mu_a/\mu = \tan\theta/\tan\theta_a$. Here, $\partial P/\partial z \geq 0$. Thus θ_a is proportionally small when the velocity of gas scent and pressure gradient are large, and that θ_a is proportionally small when the gradient is large. The decrease in θ can thus be quantitatively evaluated from u_b.

Figure 3 shows the relationship between dimensionless air ascent velocity and gradient determined from an experiment (Yamada *et al.*, 1990) where the experimental channel was set to a gradient lower than the angle of repose in order to examine the relationship between gradient and air ascent velocity from the flow bed. The values obtained in equation (6) (the continuous line in Fig. 3) are consistent with the experimental values. Here C_D is assumed to be governed by the shape and concentration of particles; if C_D is set to conform to the test results, then $C_D = 1.7$. We believe that the apparent decrease of θ can be explained by the above mentioned theory from these results.

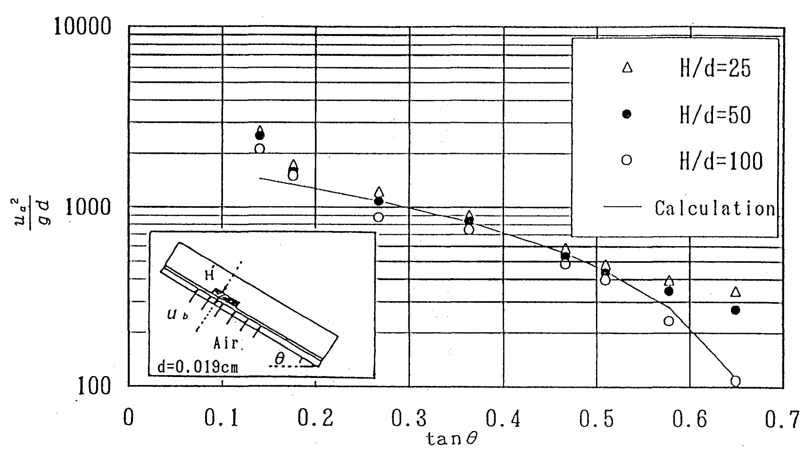

Fig. 3 The relationship between dimensionless air ascent velocity and gradient.

FUNDAMENTAL EQUATIONS FOR THE NUMERICAL SIMULATION

Pyroclastic flows are described in terms of governing equations for compressible fluids, including the laws of conservation of mass, momentum, and energy, and the state equation. We described these equations by dividing the law of conservation of mass into the law of conservation of flow volume (continuous equation) and the law of conservation of particle volume. Accordingly, we approximated the law of conservation of momentum as an incompressible fluid for our computations. We used a staggered scheme for basic equation differences and used windward differences for space differences in the inertia terms of momentum equations.

The following basic pyroclastic flow equations are obtained by taking the z-axis as a vertical axis and the x-y plane as a plane orthogonal to this axis, ignoring the bidirectional components of the flow and averaging through integration in z directions:

$$\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \tag{7}$$

$$\frac{\partial (ch)}{\partial t} + c_* \frac{\partial z}{\partial t} + \frac{\partial (cM)}{\partial x} + \frac{\partial (cN)}{\partial y} = 0 \tag{8}$$

$$\frac{\partial M}{\partial t} + \beta \frac{\partial (u_m M)}{\partial x} + \beta \frac{\partial (u_m N)}{\partial y} = -gh\frac{\partial H}{\partial x} + \frac{F}{\rho_t} u_m \sqrt{u_m^2 + v_m^2} \tag{9}$$

$$\frac{\partial N}{\partial t} + \beta \frac{\partial (u_m N)}{\partial x} + \beta \frac{\partial (v_m N)}{\partial y} = -gh\frac{\partial H}{\partial y} - \frac{F}{\rho_t} v_m \sqrt{u_m^2 + v_m^2} \tag{10}$$

$$F = \frac{3}{32\sqrt{10}} \frac{c^{4/3}}{1 - (c/c_*)^{1/3}} T_e \mu \sigma \left[\frac{D}{h} \right]^2 \tag{11}$$

Here, M ($=u_m h$) and N ($=v_m h$) are discharge fluxes in the x and y directions respectively; u_m and v_m are x and y-direction components of average velocity, h is flow depth z_b flow depth level, H the upper elevation of the flow layer ($=z_b + h$), and β ($=4.3$) is the coefficient of momentum correction. Also, ρ_t is the density of the flow as a whole, given by $P_t = c\sigma$.

COMPUTATIONS REPRODUCING THE PYROCLASTIC FLOWS AT MT UNZENDAKE

Reproduction calculations were carried out for the June 3 and June 8 flows at Mt Unzendake in 1991. The flow on June 3 inflicted heavy damage, accounting for 43

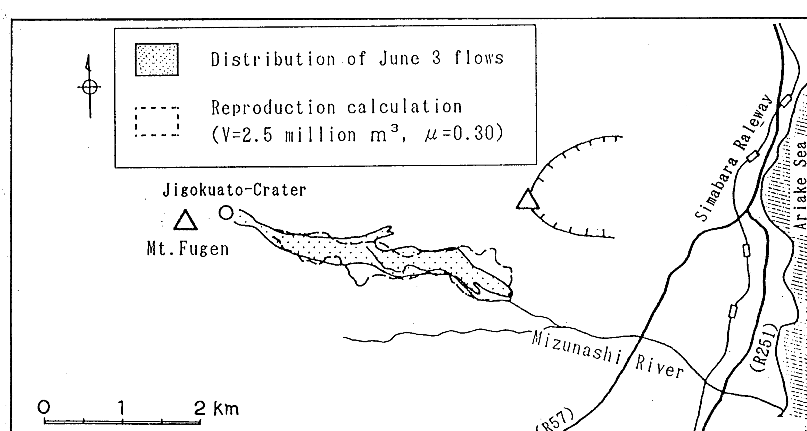

Fig. 4 The comparison of calculated extent of pyroclastic flows and actual outcome (June 3 flows).

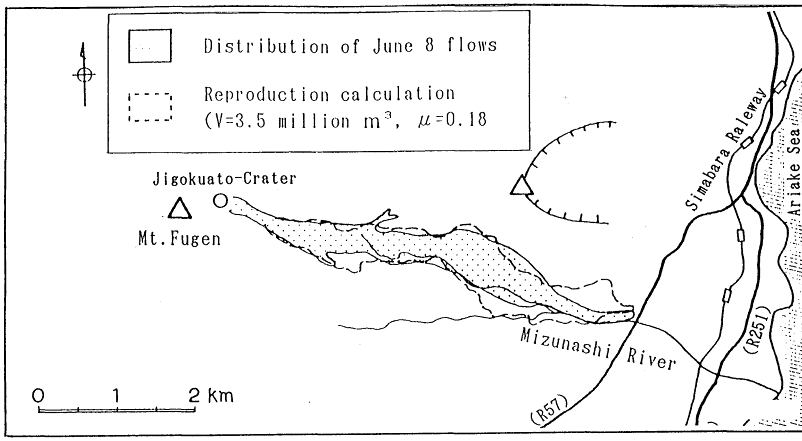

Fig. 5 The comparison of calculated extent of pyroclastic flows and actual outcome (June 8 flows).

dead and missing. Deposited sediments were estimated at 2.5 million m^3. And the larger flow on June 8, estimated at 3.5 million m^3, reached approximately 5.5 km from the crater, flowing along the Mizunashi River and blocking the channel. There were no casualties, although 73 houses were burned or destroyed (Ikeya & Ishikawa, 1991).

Normally chronological order for the volume of pyroclastic products is required as a boundary condition for calculation, but since this was difficult to estimate, an assumption of a steady supply of products lasting five minutes was made instead. The constants used in the basic equation were $D = 0.3$ m, $\sigma = 2\,500$ kg m^{-3} and $T_e = 1.0$. After some exploratory calculation, μ values of 0.28 for June 3 and 0.18 for June 18 were decided upon, in order that the computed reaching distance would agree with the actual reaching distance. Gas ascent from the flow bed is not taken into consideration in reproduction calculations.

Results of reproduction calculations

Figure 4 and Fig. 5 show the calculated extent of pyroclastic flows compared to the actual outcome. In both cases the calculated reach distance and extent of flow were basically consistent with the actual outcome. However, looking at the distribution of sediment deposits in the longitudinal direction if Fig. 6, the actual volume of deposits on the downstream side is relatively small compared to the calculated figure.

The value of μ is smaller for the June 8 flow, which had a larger volume of pyroclastic products and reached a point with a gentler gradient. In terms of the characteristics of materials, the value of μ should be roughly the same. The difference in the two μ values adopted seems to be attributable to the fact that a pressure gradient developed, for reasons including the generation of steam on the flow bed by flowing hot pyroclastic products. μ apparently changed because the pressure gradient differed in terms of space form one flow to another. Presumably, the reason why the distribution of deposited sediments was not consistent with the actual distribution is that the formation of the pressure gradient and its spatial distribution, were not taken into account in the reproduction computation.

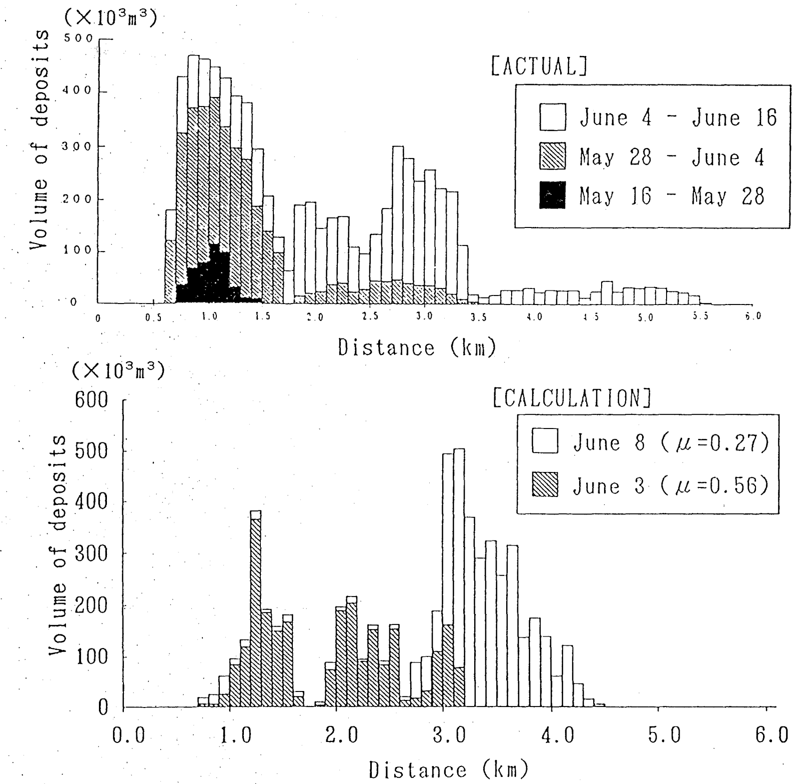

Fig. 6 The comparison of actual deposits volume and calculated figures.

GAS ASCENT VELOCITY

Gas ascent is cause dy steam generated through contact between hot pyroclastic products and pore water in the flow bed. Gas ascent velocity u_b can be given by the following equation as the quantity of generated steam per unit area and unit time:

$$u_b = \kappa \rho_w \alpha (1 - c_*) \frac{RT}{m \rho_0} \tag{12}$$

Here, m (=18.0153 x 10^{-3} kg mol^{-1}) is the molecular weight of water, ρ_w (kg m^{-3}) is the density of water, R (=8.31451 J mol^{-1} K^{-1}) is the gas constant, P_0 (Pa) is the pressure of generated steam, T (K) is the temperature of generated steam and κ (s^{-1}) is the proportion evaporated pore water per unit time. Also, ∂ (m) is the layer thickness by which heat exchange with hot pyroclastic products takes place in the flow bed deposit layer containing pore water. Here, assuming that RT/mP_0) is constant, u_b of gas depends on the conditions of the flow bed.

Here, let us infer gas ascent velocity and a, taking the example of pyroclastic flows at Mt Fugendake, Unzen for which reproduction calculations were carried out. It is assumed that μ is 0.6, the general value for debris, and therefore from the results of reproduction calculations that the apparent coefficient of friction between particles is 0.2. Also assumed is the value tan$\varnothing$ = Θ.1. ρ_w = 0.598 x 10^{-3} kg m^{-3} is taken as the gas density, assuming 100°C, 1 atm steam. When the gas ascent velocity is determined from equations (3), (6) and (12) using these values, the value obtained for u_b is 80 m s^{-1}. Assuming that $c_* = 0.6$ and $\rho_w = 1000$ kg m^{-3}, $\beta a = 0.12$ m s^{-1}.

Thus, assuming that pore water exists on the flow bed, it can be concluded that the deposit layer is rapidly eroded and mixed and that a high gas ascent velocity develops from the evaporation of pore water, significantly reducing friction between particles.

CONCLUSION

The calculations reproducing the pyroclastic flows at Mt Unzendake, using a fluid model of particle flow, were able to express fairly well the extent of deposition of the lower layer. This seems to indicate that, in general, the movement of the lower layer of a pyroclastic flow can be expressed by a particle flow composition equation, taking into account energy dissipation by μ only, and the coefficient of friction between particles.

This work has explained, and examined ways of evaluating the phenomenon whereby μ apparently decreases as a result of the formation of a pressure gradient by gas ascent velocity, with corresponding increase in the fluidity of the pyroclastic flow.

However, the technique of assigning drag coefficients for evaluating pressure gradients and the thickness of the heat exchange layer poses a continuing problem, where much research is needed.

FUTURE WORK

We are planning to prepare numerical simulation models which take into consideration the formation of pressure gradients. It is hoped that the accuracy of pyroclastic flow simulation can be improved by comparing results obtained with actual statistics from Mt Unzendake and other volcanoes.

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