Scale behavior of hydraulic conductivity during a pumping test

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Abstract Studies show that hydraulic conductivity of an aquifer seems to increase as size (scale) of the portion of the aquifer tested increases. These studies relied on different methods to determine hydraulic conductivity at each scale of interest, raising the possibility that the cause of observed hydraulic conductivity increase is the measurement method, not scale. This study analyzes hydraulic conductivity with respect to scale during individual pumping tests in a carbonate aquifer in Wisconsin. It shows that hydraulic conductivity generally increases during a test as the volume of aquifer impacted increases. This scale dependency of hydraulic conductivity is thus independent of measurement method and occurs in units dominated by both porous flow and fracture flow within the aquifer.

INTRODUCTION

Hydraulic conductivity (K) is known to vary with scale when comparing measurements from different methods (Herzog & Morse, 1984). The question remains, however, whether the relation seen is due to a scale dependency of the hydraulic conductivity parameter or is an artifact of the inaccuracy of the measurement methods. In the carbonate aquifer of southeastern Wisconsin, which consists of porous, double-porosity, and fractured media, hydraulic conductivity varied during individual pumping tests, showing increase with discharge volume. Discharge volume can be seen as scale measure, because the larger is discharge volume the larger is the rock volume affected by the aquifer test. Hence, a scale dependency can be shown that is independent of measurement method.

SCALE INDEPENDENCE OF THEIS AND COOPER-JACOB METHODS

Pumping test responses of porous media were analyzed with the Theis matching curve procedure (Theis, 1935) and the Cooper-Jacob method (Cooper & Jacob, 1946). Hydraulic conductivities were calculated for successive five-point sets of observations using the Theis and Cooper-Jacob methods. The methods use the first five data points of the pumping test to calculate a hydraulic conductivity, then skip the first point and add the sixth data point, calculate a second hydraulic conductivity, and so on. The discharge volume at the time of each five-point data window is used to represent the scale of measurement.

As a first step, the Theis and Cooper-Jacob methods were checked for scale independence (Table 1). Using the successive five-point method with the Theis matching curve procedure, hydraulic conductivity was calculated for 18 successive sets of values. As testing time and scale increased, hydraulic conductivity remained constant (Table 1). Hydraulic conductivity was also constant with scale when using the Cooper-Jacob straight line approximation (Table 1). Deviations from the early pumping test responses using the Cooper-Jacob method are due to its derivation; it is valid only if u is small (usually <0.1).

Table 1 Scale independence of Theis and Cooper-Jacob method. Data points were taken from values of $W(u)$ for values of u from the data table provided by Wenzel (1942); $1/u$ values were equated to time values in min, $W(u)$ values to drawdowns in m. Pumping rate was arbitrarily set to 1 $m^3 s^{-1}$, aquifer thickness to 10 m.

First value used $1/u$*	Data points	Time interval (s)	Discharge volume (m^3)	Theis method K ($m s^{-1}$)	Cooper-Jacob K ($m s^{-1}$)
1/9	1-5	6.7-60	12	0.0077	no match
1/7	2-6	8.6-66.7	20	0.0081	no match
1/5	3-7	12-85.7	60	0.0081	no match
1/3	4-8	20-120	66.7	0.0080	no match
1/1	5-9	60-200	85.7	0.0079	0.014
1/0.9	6-10	66.7-600	120	0.0081	0.012
1/0.7	7-11	85.7-667	200	0.0080	0.011
1/0.5	8-12	120-857	600	0.0080	0.0098
1/0.3	9-13	200-1200	667	0.0079	0.0089
1/0.1	10-14	600-2000	857	0.0080	0.0084
1/0.09	11-15	667-6000	1200	0.0081	0.0082
1/0.07	12-16	857-6670	2000	0.0080	0.0081
1/0.05	13-17	1200-8570	6000	0.0081	0.0081
1/0.03	14-18	2000-12000	6670	0.0080	0.0081
1/0.01	15-19	6000-20000	8570	0.0080	0.0080
1/0.009	16-20	6670-60000	12000	0.0080	0.0080
1/0.007	17-21	8570-66700	20000	0.0080	0.0080
1/0.005	18-22	12000-85700	60000	0.0080	0.0080

* Every other value from Wenzel's table (1942) was used. As an example, the first 7 values for u are 9,7,5,3,1,0.9,0.7. For the 1st row of the table the first 5 were used (starting at 1/u = 1/9). For the 2nd row, values 2 through 6 were used, and so on down the chart.

SCALE DEPENDENCY OF AQUIFER UNITS

The same procedure was used on actual pumping test data from a carbonate aquifer in southeastern Wisconsin to examine scale behavior. The pumping test responses of the carbonate aquifer were either porous, double-porosity or fractured. The Mayville Formation of the carbonate aquifer is a geological unit in which fluid flow is transmitted primarily by the porous vuggy medium, and secondarily by fractures. Figure 1 shows the scale dependency during a pumping test conducted in the Mayville Formation. Hydraulic conductivity increases with scale due to the inherent heterogeneities in the aquifer. Because the pumping test response was porous, the hydraulic conductivities were calculated by the Theis and Cooper-Jacob methods. The methods yielded the same hydraulic conductivities.

Another example is two single well pumping tests conducted in the Thiensville Formation of the carbonate aquifer. The Thiensville Formation is a porous medium with a unimodal lognormal distribution of hydraulic conductivity as measured by injection pressure tests (Rovey & Cherkauer, 1994). Single wells were tested at two sites less than 20 m apart as preparation for a major tunneling project. At one point (vent shaft, Fig. 2), scale dependency of hydraulic conductivity is apparent, whereas it is not apparent at the other site (drop shaft, Fig. 2). This difference suggests that scale dependency of hydraulic conductivity in porous media is not universal. The aquifer must be homogeneous at the drop shaft site. In contrast, the response at the vent shaft (Fig. 2) suggests heterogeneity. Hydraulic conductivity in the relation (Fig. 2) locally increases with scale, with a break in slope. At some point on a larger scale the scale dependency of hydraulic conductivity of this location diminishes; the reason for different response at nearby sites is not known.

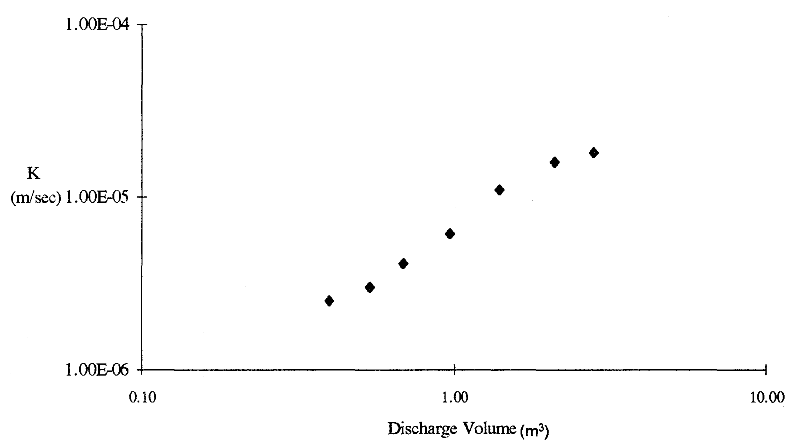

Fig. 1 Scale dependency of hydraulic conductivity during a pumping test at a well in a dominantly porous flow unit of the carbonate aquifer. Hydraulic conductivity increases with increasing discharge volume during the pumping test. Hydraulic conductivity and volume are calculated as described in the text for five-point windows of data from the full test set.

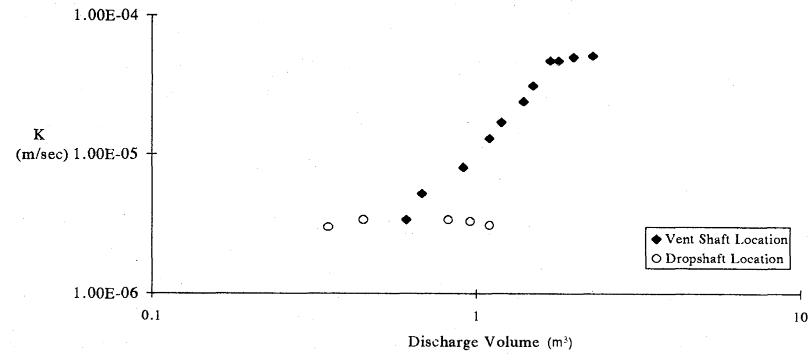

Fig. 2 Scale dependency of hydraulic conductivity during a pumping test in a porous flow unit of the carbonate aquifer. Although hydraulic conductivity is locally constant with scale at the drop shaft site, there is a significant increase of hydraulic conductivity at the vent shaft location, less than 20 m away. Hydraulic conductivity and volume are calculated as described in the text for five-point windows of data from the full test set.

FRACTURED ROCK PUMPING TEST RESPONSES

Fractured rock responses were observed in the Racine Formation of the carbonate aquifer. The Racine Formation, dominated by fracture flow, is mainly mudstone. The hydraulic conductivity distribution of the Racine Formation is bimodal, as indicated by injection pressure tests (Rovey, 1990). An increase of hydraulic conductivity with discharge volume representing scale was found in some of the fracture flow responses of the mudstone facies. Fracture flow responses typically exhibit a linear time-drawdown behavior on a log-log plot. Figure 3 shows a fractured rock response in the mudstone facies, for which the slope of the time-drawdown line decreases during the pumping test. Such decrease generally means that hydraulic conductivity of the aquifer response is increasing. In this case it likely means that continued pumping expanded the aquifer's

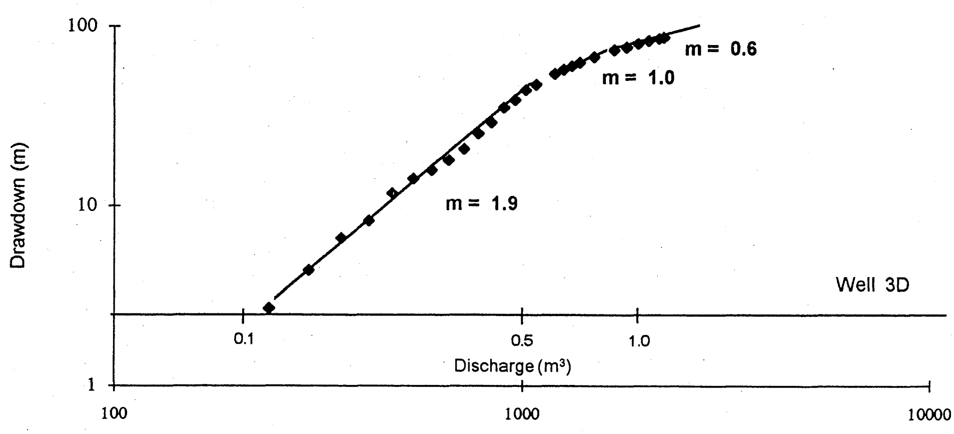

Fig. 3 Scale dependence of hydraulic conductivity during a pumping test at a fracture-flow dominated unit of the carbonate aquifer. Decreases of the linear slopes (*m*) represent increases of hydraulic conductivity during the pumping test.

response into another fracture network not active during early stages of the test. This expansion raises the mean hydraulic conductivity and causes the slope to decrease. Three different slopes can be distinguished in Fig. 3, although the second and third slopes might be summarized as one with a mean value. The procedure also shows that fracture response may mimic a typical porous Theis curve response, if slope change is gradual.

GLOBAL SCALE BEHAVIOR

Empirical studies showing an increase of hydraulic conductivity with scale of measurement have been susceptible to the argument that observed hydraulic conductivity changes result from different methods being used at different scales. This study shows that observed hydraulic conductivity may increase with the volume of aquifer affected (scale) during a single test, a result independent of measurement method. It suggests observed global increases of hydraulic conductivity with scale, such as those for the Racine Formation of the carbonate aquifer are real (Fig. 4). Local variations of hydraulic conductivity are extremely difficult to predict due to the spatial variability of a geological unit or facies. On a global scale, however, the increase of hydraulic conductivity with scale of measurement in a geological facies seems to follow a predictable pattern. The Racine Formation of the carbonate aquifer shows a linear increase of hydraulic

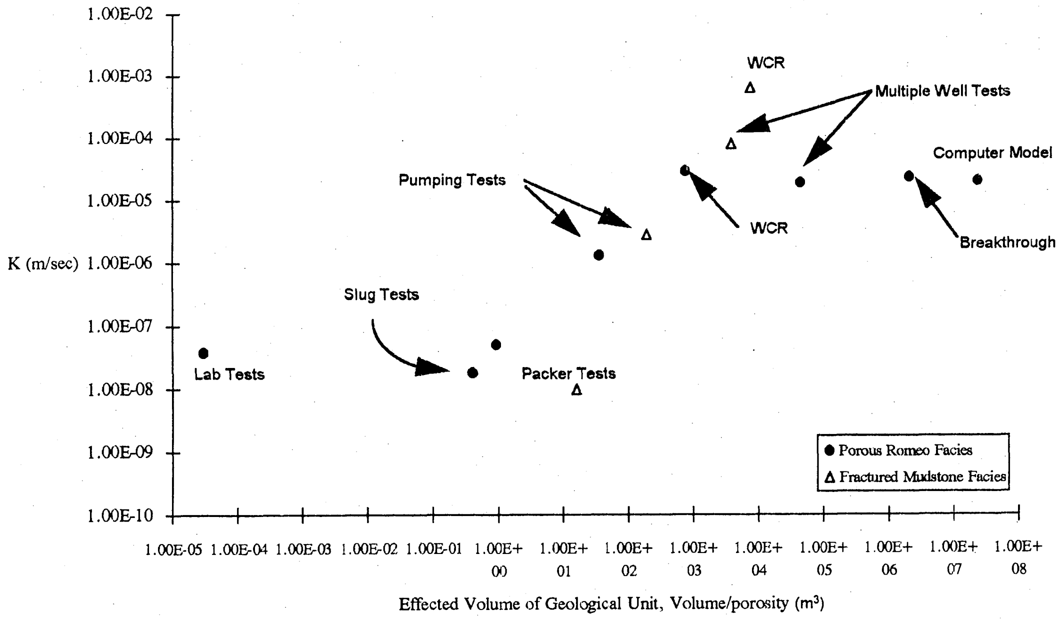

Fig. 4 Scale dependence for facies of the Racine Formation, part of the carbonate aquifer. The vuggy, porous Romeo facies shows linear increase of hydraulic conductivity with upper and lower bounds, whereas the fractured mudstone facies shows an unbounded linear increase with a steeper slope. Plotted points are geometric means of multiple tests. Numbers of observations on which geometric means are based: from left to right; Romeo facies N = (3, 4, 18, 2, 51, 2, 1, 1); fractured mudstone facies N = (10, 7, 1, 64). WCR = well construction reports (specific capacity data).

conductivity with scale for the fracture dominated mudstone facies on a log-log plot (Fig. 4). The porous, vuggy Romeo facies of the Racine Formation also shows linear increase on a log-log plot. The slope is different, however, and upper and lower bounds are found within which hydraulic conductivity is on average constant with scale.

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