

## Effect of temporal scale on runoff and erosion modeling

**D. STEPHENSON**

*Department of Civil Engineering, University of the Witwatersrand, Johannesburg, South Africa*

**Abstract** A model based on simplified hydrodynamics equations was compiled to reproduce runoff based on the physical laws of nature. The kinematic equations were used to include the effects of bed shear and the concentration of water from overland, subsurface and stream components. Monthly rainfall figures are required as well as the number of rain days per annum, which enables the program to estimate storm duration. Periods are further sub-divided to suit numerical analysis.

### INTRODUCTION

There have been a number of models proposed for estimating monthly streamflow. The Stanford watershed model (Crawford & Linsley, 1966) is perhaps the most recognized. There are, however, shortcomings in the models that are primarily due to the fact that the models are empirical rather than based on hydrodynamic principles. There can be times, however, such as periods of intense rainfall as well as drought periods, when a hydrodynamic model can be inaccurate. High rainfall figures result in non-linear rainfall/runoff relations so that flood flows cannot be modeled well for extreme storms. Dry flow conditions are primarily from groundwater contributions, and unless aquifer characteristics are accounted for the results of simple empirical models can be misleading.

Rafler (rainfall-flow-erosion) model, developed by Paling *et al.* (1989), is a conceptual deterministic model. It is based on simplified hydrodynamic equations for overland flow and the Green & Ampt (1911) infiltration model for flow into the soil. Some factors that are difficult to observe, such as aquifer characteristics, can be obtained by calibration. For overland flow, factors such as roughness, slope and width and overland flow length of the catchment are required. Because runoff is not in the form of perfect sheet flow, a factor to account for flow in rills is required.

In so-called hydraulic models, many hydraulic parameters are not the same as are measured in a laboratory. This is particularly so in the case of surface roughness and infiltration rate. Lumped models generally require higher roughness factors than engineers are accustomed to assign under laboratory or channel flow conditions.

### BASIS OF MODEL

The program attempts to reproduce runoff and silt yield on a monthly basis, using basic catchment parameters and monthly rainfall records. To estimate runoff and soil erosion

rates, however, it is necessary to operate the model using a much shorter time interval, such as hourly instead of monthly. An estimate of the distribution of monthly rain is made, therefore, using the average number of rainy days in a year. The program accommodates various combinations of three elements or modules (Fig. 1): plains, channels and reservoirs. The continuity principle applied to one-dimensional flow can be written as:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_i \quad (1)$$

where  $Q$  is volumetric flow rate,  $x$  is distance in direction of flow,  $A$  is cross-sectional area,  $t$  is time and  $q_i$  is lateral inflow rate per unit length along the  $x$ -axis.

The conservation of momentum for one-dimensional, unsteady, non-uniform flow is given by:

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{v}{g} \frac{\partial v}{\partial x} - \frac{1}{g} \frac{\partial v}{\partial t} \quad (2)$$

where  $S_f$  is friction gradient,  $S_o$  is bed slope,  $y$  is water depth,  $v$  is mean water velocity,  $g$  is gravitational acceleration and  $t$  is time. In addition, if the cross-section of the conduit is constant, a uniform flow may be assumed. This will reduce the conservation of momentum equation to:

$$S_f = S_o \quad (3)$$

which means the equation of motion can be approximated by a uniform flow formula of the general form  $Q = ay^b$ , where  $a$  and  $b$  are constants. The combination of the continuity equation and a uniform flow formula describes kinematic flow.

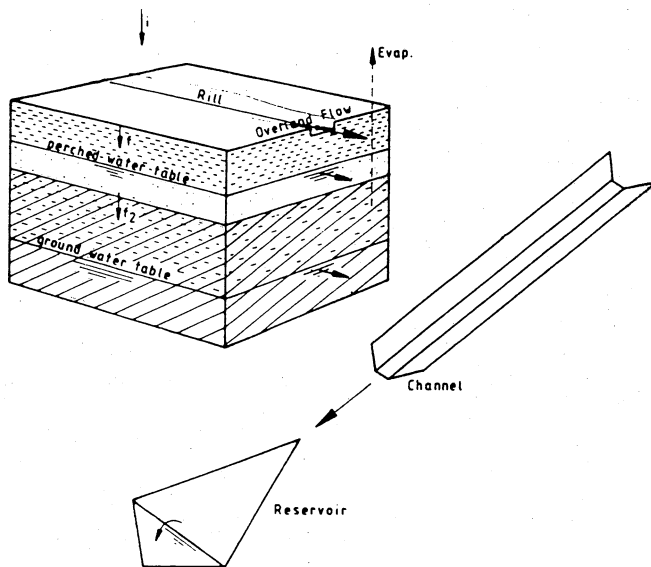


Fig. 1 Diagrammatic assembly of modules.

## RAINFALL-RUNOFF SIMULATION

The rainfall requirements for a continuous monthly simulation model can be less stringent than for event simulation. If rainfall records are available, daily data are generally obtainable, but they may pose economic and practical drawbacks. The data input effort may be prohibitively expensive. Therefore, the historical monthly rainfall records in this model form the basis for the simulated rainfall intensity and duration.

There is a tendency for rainy days to occur in clusters. Based on historical records, average number of rainy days for each month can be established. The number of hours of rain per month is assumed proportional to the number of rainy days per month multiplied by the ratio of the months rainfall to average rainfall, all to the empirical power of 0.75:

$$TR = AH * RD(M) * 12 * R(K, Y, M) / [MAP(K)]^{0.75} \quad (4)$$

where  $TR$  is number of hours of rain per month,  $AH$  is a constant,  $RD(M)$  is number of rainy days for month  $M$ ,  $R(K, Y, M)$  is monthly rainfall (mm) for region  $K$ , year  $Y$  and month  $M$  and  $MAP(K)$  is mean annual precipitation for region  $K$  (mm).

## SEDIMENT YIELD CALCULATION

Silt yield from catchments is calculated using Yalin's (1963) theory, which is based on Shield's critical shear slope criterion for erodibility. Total erosion is summed over each month and routed into the downstream element each time step.

The runoff model appears particularly sensitive to infiltration rates. The first factor to adjust in the model calibration for any particular catchment study is the infiltration rate, which can vary from one sub-catchment to another. The correct volume of runoff is an indication of correct infiltration rates. The soil suction is not found to be particularly influential on results. In order of sensitivity, other factors most readily obtained by calibration are number of rain hours per month (obtained from number of rainy days per year and a conversion factor), rill ratio, which has a direct bearing on infiltration as well, and cover factor, which affects erosion rate.

## OVERLAND FLOW

If the surface flow takes place along depressions, the cross-sectional area of flow is:

$$A = RR * W * Y \quad (\text{m}^2) \quad (5)$$

Due to the shallow nature of depressions and their spread over the width of the catchment, the wetted perimeter can be approximated by  $P = W$ . The surface flow equation can thus be written as:

$$QI = \frac{\sqrt{s}}{n} * W * (RR * y)^{5/3} \quad (\text{m}^3 \text{s}^{-1}) \quad (6)$$

The upper limit of flow is determined by the availability of water, or:

$$Q2 = Y * W * X/DT \quad (\text{m}^3 \text{s}^{-1}) \quad (7)$$

which may play a role if the time increment  $DT$  takes on a high value. The smaller value of  $Q1$  and  $Q2$  is selected to represent the actual surface runoff. Subsequently, the depth of the water is reduced to account for the runoff. Additional reduction will take place as a result of infiltration.

## APPLICATION

The use of the rainfall-runoff model is illustrated by application to the Mbashe River in Transkei (Fig. 2). A flow recorder collected data between July 1956 and June 1967. As a result there are ten years of information against which the simulation results can be calibrated. The total catchment area upstream of the gaging station is 4800 km<sup>2</sup>. The rainfall data are supplemented with data from the Weather Bureau. For calibration, an abbreviated version of the rainfall data files was used, starting with October 1955 (Fig. 3).

## SOIL EROSION AND SEDIMENT TRANSPORT

The fact that the rainfall-runoff model is based on hydrodynamic principles enables water velocities overland and in streams to be calculated. This, in turn, facilitates calculation of erosion rates. The rainfall-runoff-erosion model was applied to the Mbashe

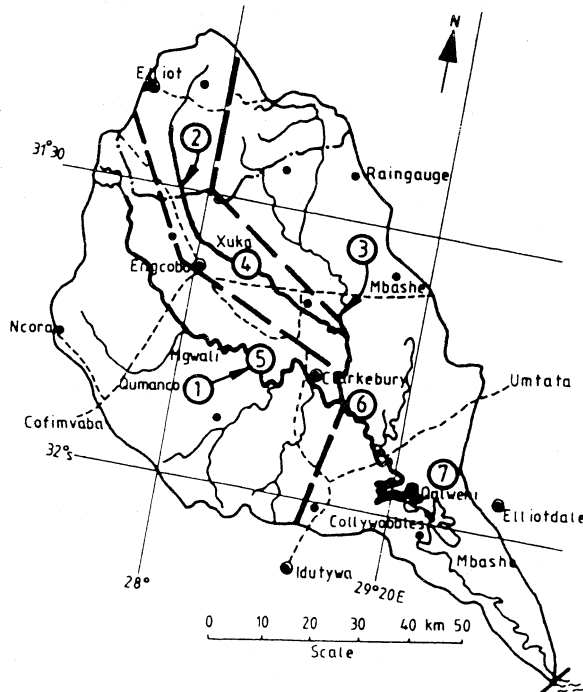
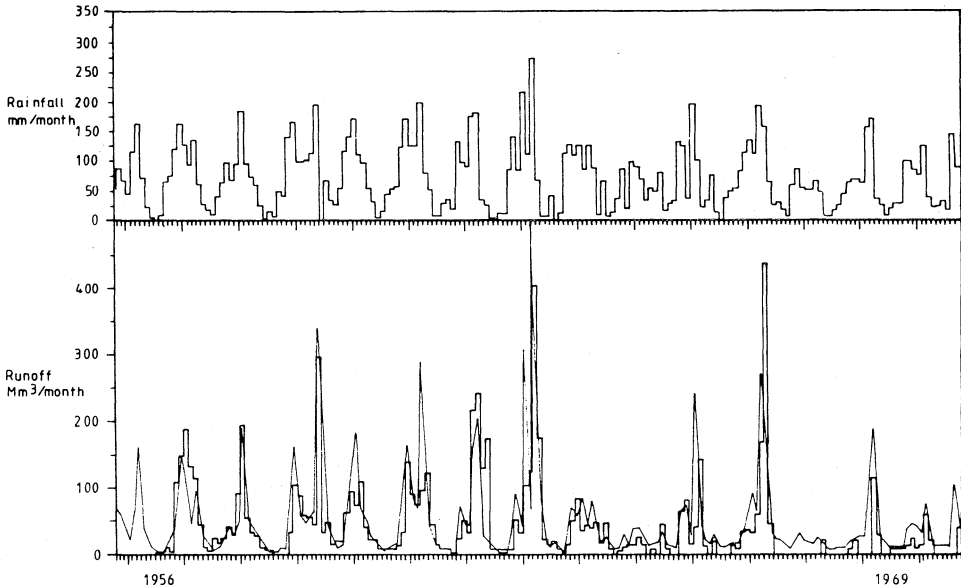


Fig. 2 Map of Mbashe catchment with sub-catchments and modules for Rafter.



**Fig. 3** Rainfall for the Mbashe region and recorded runoff (block diagram) and simulated runoff (graph) for Mbashe River.

River in Transkei. Siltation problems at the Collywobbles hydroelectric scheme have resulted in several studies related to sediment deposition in the reservoir (e.g. Watermeyer, 1987). Silt surveys indicate that in a period of approximately 4 years, the total reservoir capacity was reduced by 70%.

After a few simulations, a reasonable record was obtained with the following values for the erosion and sedimentation control parameters: density of loose soil for catchment,  $1500 \text{ kg m}^{-3}$ ; sediment size,  $0.2 \text{ mm}$ ; fall velocity,  $0.02 \text{ m s}^{-1}$ ; detachment coefficient,  $0.0138$ ; fragmentation rate,  $0.03 \text{ m month}^{-1}$ ; catchment cover density,  $0.8$ ; catchment erosivity factor,  $0.5$ . The results of the simulation are presented in Fig. 4. If the simulation results for the 1987-1988 season are excluded, the mean annual sediment runoff is  $5067 \text{ kt year}^{-1}$ . This exclusion is required as no rainfall data were available for the period from June through September 1988 and thus sediment data are incorrect.

## TEMPORAL SCALE

The hydraulic model needs much smaller time steps than conceptual models need. To reduce computational time, a zooming scale was used so that for periods of surface runoff the time increment was a few minutes. During low flow periods (groundwater contributions mainly), the time increment was much larger (a few days). This enables continuous modeling over many years and enables one to study statistics of monthly flows and sediment yield rates.

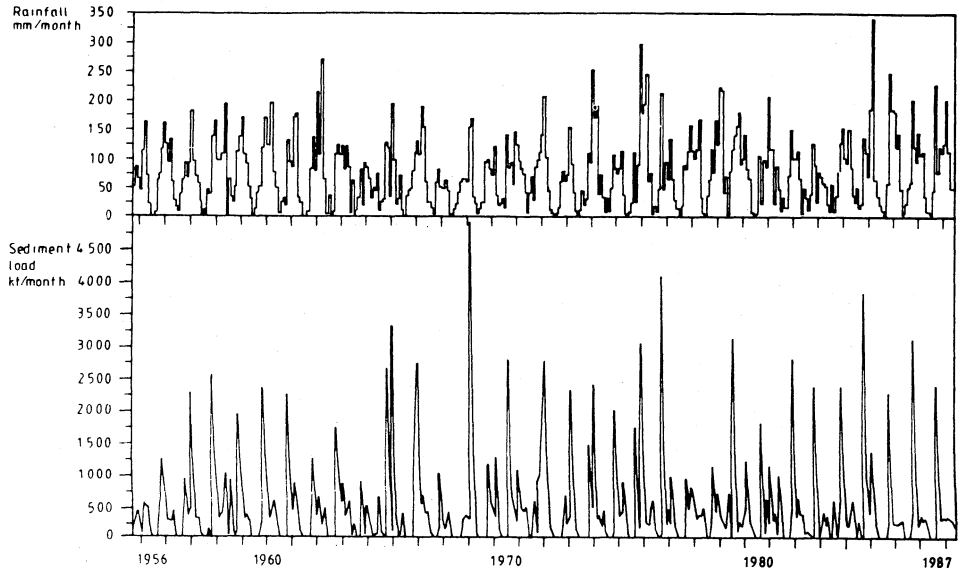


Fig. 4 Sediment load at Mbashe River.

## REFERENCES

- Crawford, N. H. & Linsley, R. D. (1966) Digital simulation in hydrology: Stanford watershed model IV. *Tech Report no. 39. Dept of Civil Engineering, Stanford Univ., California.*
- Green, W. H. & Ampt, G. A. (1911) Studies of soil physics. 1: The flow of air and water through soils. *J. Agric. Sci.* 4(1) 1-24.
- Paling, W. A. J., Stephenson, D. & James, C. S. (1989) *Modular rainfall-runoff and erosion modelling.* Water Systems Research Group, University of the Witwatersrand, Johannesburg.
- Stephenson, D. & Meadows, M. E. (1986) *Kinematic Hydrology and Modelling.* Elsevier, Amsterdam.
- Watermeyer, C. F. (1987) Discussion of "Reservoir sedimentation: the stable non-equilibrium state" by G. W. Annandale. *Civil Eng. S. Afr.* 2, 60-65.
- Yalin, Y. S. (1963) An expression for bed-load transportation. *J. Hydraul. Div. ASCE* 89(HY3), 221-250.