Total suspended load transport as a natural stochastic process

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Abstract Stochastic analyses were prepared for processes of rainfall, runoff and suspended load transport for the gaging station Botun on the River Sateska in the Republic of Macedonia. From the analyses a stationary correlation was found among three processes. The best correlation occurred between runoff and total suspended load transport for rainy days and rising stage. Improvement of the correlation was achieved by substitution of total rainfall with effective rainfall and corresponding runoff and suspended load transport.

INTRODUCTION

Suspended load transport by streamflow is a natural process that can be described as a stochastic process. To define the expected variation and to search for the relation with other natural processes by which this process is caused, it is first necessary to do stochastic analyses of three processes. Study of suspended load transport as a natural process is of great importance for solving practical problems in river engineering, such as water diversion and reservoir construction. Stochastic study of the total suspended load transport, therefore, is important to direct the observation and data collection that are expected to be used for future prediction, control and management of suspended load transport as a natural process.

The transport of suspended load is directly caused by the rainfall/runoff process. Rainfall is the "initial impulse" on which the realization of the runoff process depends, as well as the surface erosion process and sediment entering the river net. The rainfall process is initially realized as independent, and the process from which the other two processes start. The development of runoff and suspended load transport processes act parallel in time and space and they develop under the same conditions. Much of space and its components (morphologic, hydrogeologic, geomechanical, vegetation cover, instantaneous humidity and others) have the same positive or negative effect on both processes of runoff and suspended load transport. It is known that as runoff decreases, the rate of sediment transport in the streamflow decreases as well (Fig. 1).

There is hardly any information in the literature (available to us) in which suspended load transport is treated as a separate natural process, the main reason for which is a shortage of information about the realization of the suspended load process. Bogardi (1974) and Graf (1971) present partial relations between total suspended load transport and stream stage (indirect with discharge). Kasprzak (1977) and Macura & Hlaveova (1993) suggest relations between rainfall and rate of surface erosion. Shamov (1959)

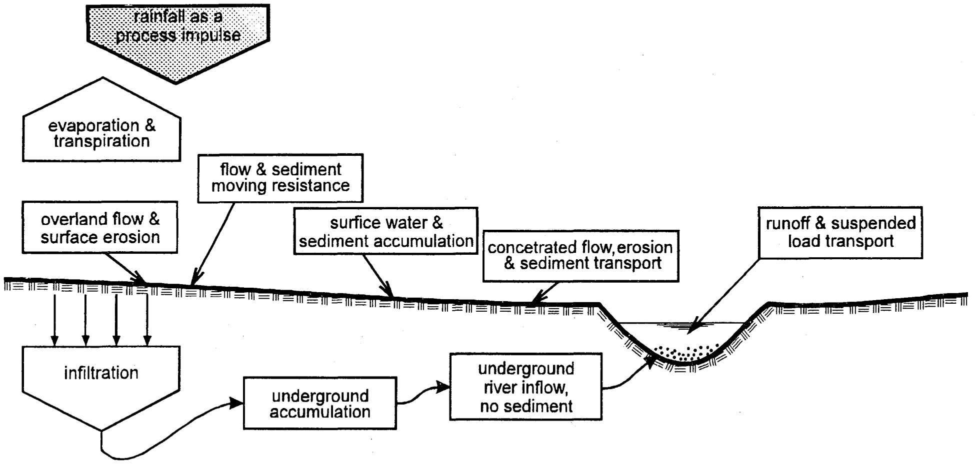

Fig. 1 Diagram showing structure of the runoff and suspended load transport process.

presents hydrographs with total sediment transport for many gage sites on rivers of the former Soviet Union.

The aim of the present paper is to identify a stochastic structure for suspended load transport as a separate natural process and to find a possible link with two other natural processes: rainfall and runoff. The intention is to point out the need for "parallel" study of the runoff and suspended load processes.

THEORETICAL BACKGROUND

Analyses of the structure and relations among the three processes are made by using stochastic methods for analyses of random time series. The realization of the three processes is defined by three random discrete time functions:

$x(t_1), x(t_2), \dots x(t_i), \dots x(t_n)$ runoff (1)

$y(t_1), y(t_2), \dots y(t_i), \dots y(t_n)$ suspended load transport (2)

$h(t_1), h(t_2), \dots h(t_i), \dots h(t_n)$ rainfall (3)

Stochastic analyses of the random time discrete functions are made according to Yevjevich (1972) and Kazakievich (1971). The analyses (Fig. 2) include (a) definition of autocorrelation functions $\rho_x(\tau)$, $\rho_y(\tau)$ and $\rho_h(\tau)$; (b) definition of cross-correlation functions $\rho_{xy}(\tau)$, $\rho_{xh}(\tau)$ and $\rho_{yh}(\tau)$; (c) definition of continuous spectra ω_x, ω_y and ω_h; and (d) correlation analyses among the three processes. The autocorrelation functions, cross-correlation and three spectra are defined by Kaevski's mathematical model and correlation analyses are made by means of commercial software.

APPLICATION AND DISCUSSION

Basic information

The stochastic structure and the relations among the three processes are estimated by

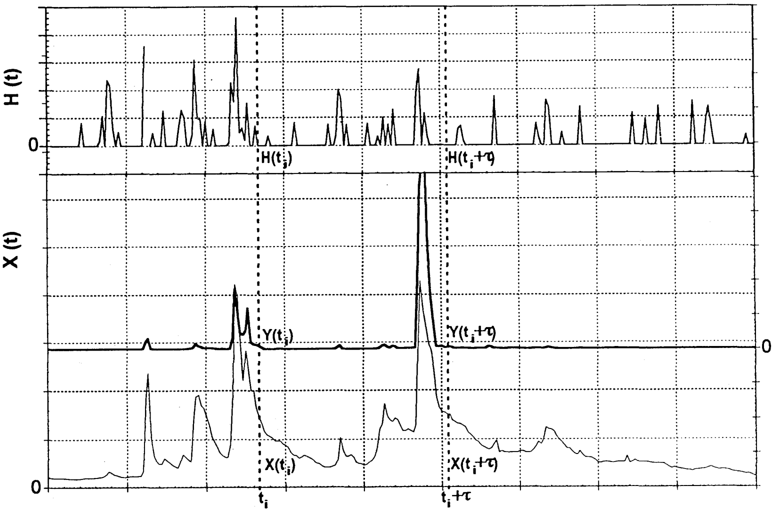

Fig. 2 Definition sketch of stochastic analyses.

analyses of the three random discrete time functions, which describe the relations of the studied processes. Each of the three functions have 13 years of daily observations (1975-1988). Runoff and total suspended load transport data are obtained from the Botun gage on the River Sateska (tributary to Ohrid Lake) in the Republic of Macedonia. Rainfall information is from the Belciste station. At the Botun gage the catchment area is 356.7 km^2. Average annual discharge is 6.34 m^3 s^{-1} and average annual suspended load is 1.12 kg s^{-1}. Mean annual rainfall at the Belciste station is 845 mm. The runoff function, $x(t_i)$, is in m^3 s^{-1}, the total suspended load transport function, $y(t_i)$, is in kg s^{-1} and the rainfall function, $h(t_i)$, in mm day^{-1}. Erosion categories for the catchment all show relatively high rates. Discharge is documented by stage recorder and hydrometric measurements. Suspended load is estimated daily by point sampling in the water profile and by sampling in combination with hydrometrical measurements. Rainfall is observed by raingage, providing daily readings of total rainfall.

The three basic time discrete functions are presented on Fig. 3. The corresponding variation and extremes of the functions are evident. The snow season usually extends from the second half of December until the beginning of March, and extensive snow melting occurs in March and April.

Stochastic analyses of random time functions

Stochastic analyses of the three random functions $x(t_i)$, $y(t_i)$ and $h(t_i)$ define the process structure and possible relation between them. Autocorrelation functions $\rho_x(\tau)$, $\rho_y(\tau)$ and $\rho_h(\tau)$, in which τ is time step (Fig. 2), are presented in Fig. 4, and the linear continuous spectra ω_x, ω_y and ω_h are presented in Fig. 5.

The autocorrelation functions indicate the yearly periodicity of events (Fig. 4) in which runoff has an autocorrelogram with an oscillation in an interval ±0.30. The autocorrelograms of rainfall and suspended load oscillate over a small interval (−0.02

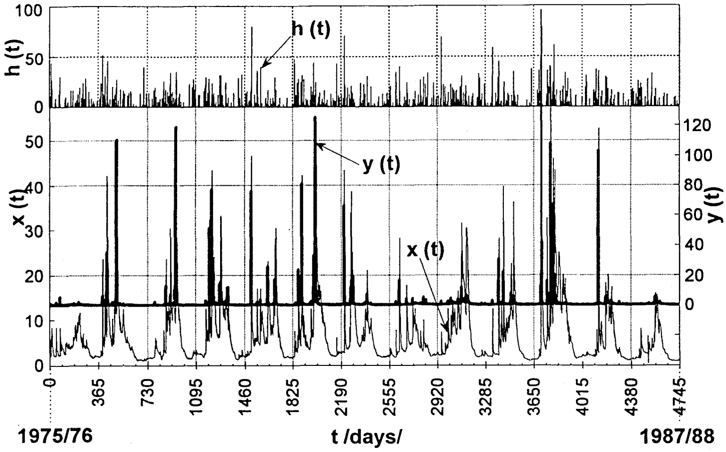

Fig. 3 Graph showing change with time of random discrete functions $x(t_i)$, $y(t_i)$ and $h(t_i)$.

to +0.15) and with a high oscillation frequency (Fig. 5). Results of the analyses suggest that rainfall and sediment transport processes have weaker yearly periodicities than does runoff, suggesting that the spatial effect more positively influences the smoothing of the runoff random component than it does the smoothing of the suspended load transport random component. This conclusion is supported by observations that underground flow has no effect on surface erosion. In addition, the higher random component of the sediment transport function may be caused by data of low quality.

The cross-correlation functions $\rho_{xy}(\tau)$, $\rho_{xh}(\tau)$ and $\rho_{yh}(\tau)$ are given in Fig. 6; the functions indicate (a) the relation between rainfall and runoff processes (rainfall and suspended load processes can be characterized as stationary); (b) that the relation between runoff and suspended load transport is also stationary, with an expressive

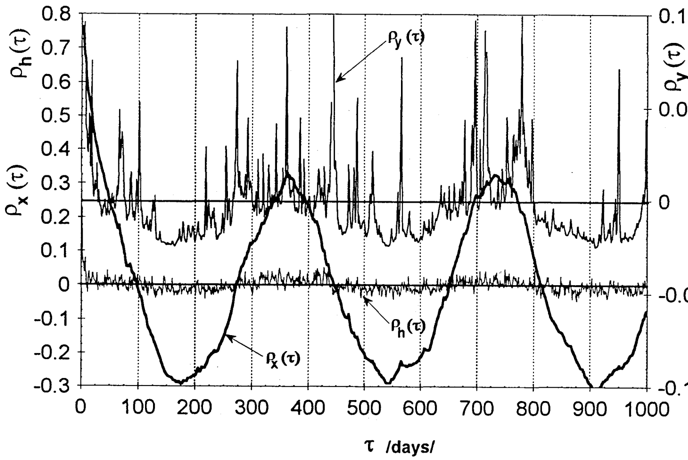

Fig. 4 Graph showing change with time of autocorrelation functions $\rho_x(\tau)$, $\rho_y(\tau)$ and $\rho_h(\tau)$.

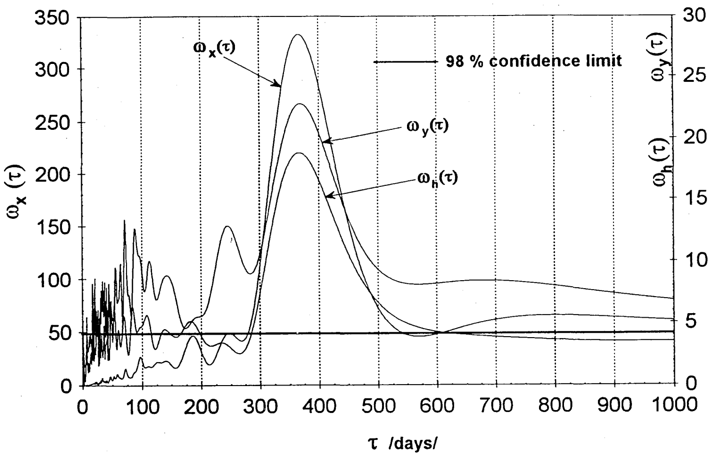

Fig. 5 Graph showing change with time of linear continuous spectra ω_x, ω_y and ω_h.

periodic component (further study of the relation between the two processes is necessary); and (c) the relation between runoff and suspended load transport processes can also be described as Markovian processes.

Correlation analyses

The separate linear correlations among the three processes are examined by defining the linear correlation coefficients for (a) the three complete random functions → $x(t_i)$, $y(t_i)$ and $y(t_i)$; (b) periods with stable runoff → $|x(t_i) - x(t_{i-1})| \leq 0.25$ m^3 s^{-1}; (c) periods of rising runoff → $x(t_i) - x(t_{i-1}) > 0.25$ m^3 s^{-1}; (d) periods of falling runoff → $x(t_i) - x(t_{i-1})$

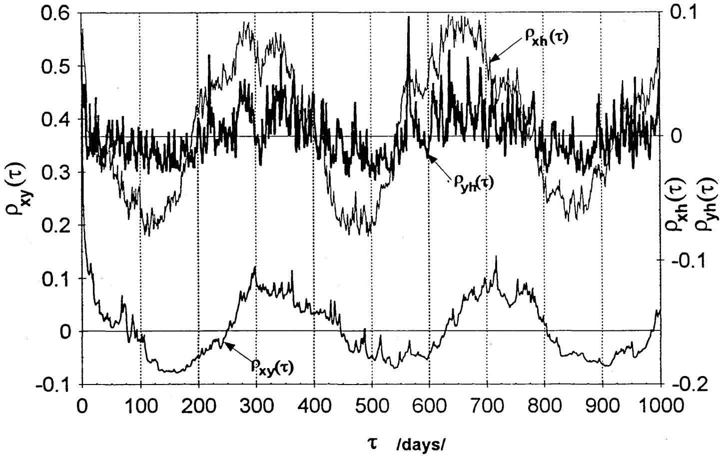

Fig. 6 Graph showing change with time of cross-correlation functions $\rho_{xy}(\tau)$, $\rho_{xh}(\tau)$ and $\rho_{yh}(\tau)$.

< 0.25 m^3 s^{-1}; (e) all rainy days; (f) rainy days only with rising runoff; (g) rainy days only with falling runoff; (h) rainy days only with stable runoff; and (i) total days with no rainfall. The estimated correlation coefficients, with standard errors, are presented in Table 1.

The linear correlations between rainfall and the other two processes is considerably weaker than is the correlation between runoff and suspended load process. The different effects of the catchment area on runoff and suspended load processes may be a possible explanation. If effective rainfall is an input to the catchment area instead of total rainfall, a much better correlation, especially in rainy periods of rising runoff, is expected. This conclusion is supported by extensive experience from parametric hydrology, in which runoff is estimated as a function of effective rainfall. In this research the correlation coefficients, r_{xh}, were estimated for all cases within the interval of 0.20-0.30 and indicated very weak correlation between rainfall and runoff. Correlation coefficients for rainy days and stable runoff were estimated using very low values ($r_{xh} = 0.04$ and $r_{yh} = -0.03$), which showed no correlation between rainfall and runoff and suspended load transport. Furthermore, the analyses showed that there was no effective rainfall that could cause runoff, erosion and sediment transport. Better correlation is expected if rainfall is taken into consideration in the analyses with two parameters – duration and intensity. The intensity of rain has a great influence on surface erosion and sediment discharge (Kasprzak, 1977).

For rainy days and rising runoff (with the best correlation coefficient), a polynomial function was obtained:

$$y(t_i) = -5.404 - 0.166\,x(t_i) + 0.268\,x^2(t_i) + 0.657\,h(t_i) \qquad (4)$$

that provides a possible stochastic prognostic model for suspended load transport that is limited to rainfall and rising runoff. The constants are statistically defined for the data and space. The function is tested by comparing predicted values against observed values for the suspended load process (Fig. 7). Although the correlation coefficient has a

Table 1 Correlation coefficients for linear correlation analyses, River Sateska, Republic of Macedonia.

Periods	*N*	Correlation coefficients			
		$r_{xy} \pm \Delta r$	$r_{xh} \pm \Delta r$	$r_{yh} \pm \Delta r$	$r_{xyh} \pm \Delta r$
complete functions	4749	0.86 ± 0.007	0.37 ± 0.009	0.34 ± 0.008	0.58 ± 0.007
stable regime	2919	0.56 ± 0.009	0.08 ± 0.009	0.06 ± 0.002	0.56 ± 0.01
rising	592	0.69 ± 0.015	0.38 ± 0.024	0.37 ± 0.024	0.70 ± 0.014
falling	1237	0.51 ± 0.014	0.21 ± 0.018	0.30 ± 0.017	0.52 ± 0.014
no rainy days	3567	0.45 ± 0.009	/	/	/
all rainy days	1182	0.64 ± 0.011	0.33 ± 0.017	0.39 ± 0.017	0.65 ± 0.011
rising	392	0.71 ± 0.018	0.34 ± 0.033	0.37 ± 0.032	0.72 ± 0.018
falling	484	0.53 ± 0.022	0.20 ± 0.029	0.33 ± 0.027	0.53 ± 0.022
stable regime	368	0.66 ± 0.020	0.04 ± 0.035	−0.03 ± 0.035	0.67 ± 0.020

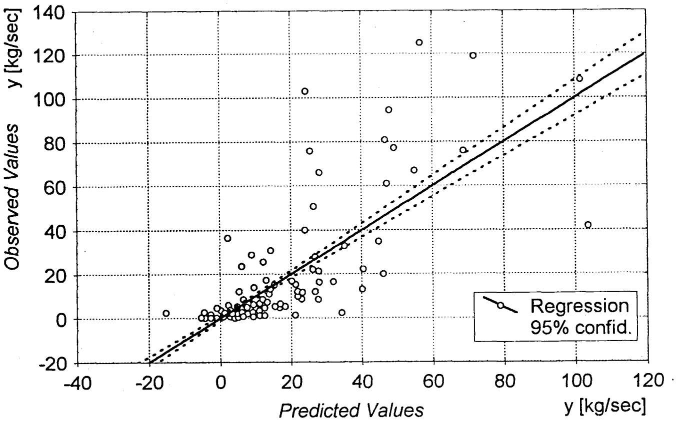

Fig. 7 Graph showing predicted values versus observed values.

relatively high value, the fitting points show a substandard dispersion out of the confidence intervals. These analyses confirm the previous statement of the necessity to use the effective rainfall data versus runoff. Further study in this direction may result in elaboration of mathematical simulation models regarding the process of total suspended load transport.

CONCLUSIONS

The following conclusions arise from the analyses:

(a) A weak stationary correlation, estimated among the three studied processes, requires further investigation.

(b) The studied case suggests that the best correlations are among the three processes for rainy periods of rising runoff.

(c) The stochastic model does not satisfy the testing criterion and it provides an estimate only.

(d) Improvement of the correlations among the three processes is obtained if effective rainfall is used as an input versus runoff and sediment transport.

(e) Improvement of correlations among the three processes is achieved by considering rainfall duration and intensity. Thus, a realistic condition for elaboration of mathematical models for suspended load transport in the river system is possible.

REFERENCES

Bogardi, J. (1974) *Sediment Transport in Alluvial Streams.* Academia Kiado, Budapest.

Graf, W. H. (1971) *Hydraulics of Sediment Transport.* McGraw-Hill, New York.

Kazakievich, D. I. (1971) *Osnovi Slucajnih Funkcii i re Primenenie v gidrometeorologii* (in Russian). Gidrometeoizdat, Leningrad.

Kasprzak, K. (1977) Empiricke Vztahy urcujici hodnotu mechanickeho pusobeni deste na pudu. *Vodohospodarsky casopis* **35**, CSAV Bratislava.

Macura, V. & Hlavcova, K. (1993) Modeling of overland flow and erosion processes. *Slovak J. Civ. Eng.* **1**.

Shamov, G. I. (1959) *Recnie Nanosy.* Gidrometeoroizdat, Leningrad.

Yevjevich, Vujica (1972) *Stochastic Processes in Hydrology.* Water Resources Publications, Fort Collins, Colorado.