Methodology of hydrologic model building[1]

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ABSTRACT: Two alternative representations of the infiltration component of a rainfall-runoff simulation model are investigated. Four criteria for choice between the two which are considered are accuracy of prediction, simplicity of the model, consistency of parameter estimates, and sensitivity of results to changes in parameter values. Sensitivity plots are presented for obtaining an understanding of interactions in the model by studying several error functions simultaneously. The tools presented are shown to aid in the subjective decisions which must be made in the construction and use of a rainfall-runoff model.

RÉSUMÉ : Deux réprésentations alternatives des constituants d'infiltration d'un modèle similateur d'écoulement sont étudiées. Les quatre critères retenus pour choisir entre les deux représentations qui sont considérées sont: l'exactitude de la prédiction, la simplicité du modèle, la consistance des estimations de paramètres et la sensibilité des résultats aux changements de valeur paramétrique. Les tracés de sensibilité sont présentés afin d'obtenir une compréhension des interactions dans le modèle à l'aide d'études simultanées de plusieurs fonctions d'erreurs. Il est montré que les instruments présentés aident à décider subjectivement les problèmes de construction et d'utilisation d'un modèle d'écoulement de pluie.

INTRODUCTION

Hydrologic modeling has expanded in response to the widespread availability of digital computers. Many complex models of the rainfall-runoff phase of the hydrologic system have been presented [1]. Those models which are based on physical laws controlling the various processes are semi-subjective, for in practically no case is there a generally accepted method for describing the pertinent physical laws. The empirical approximations used not only affect the accuracy of prediction of the model, but, because of the involved interactions in hydrology, they may affect the apparent response of other components of the hydrologic system. In addition, the criterion of accuracy or goodness of fit influences the set of "optimum" parameters which describes the physics of a particular basin [2].

The potential user of a model usually is interested in prediction for design purposes. Among the many models presented in the literature, one wishes to choose the "best" for a particular prediction problem. Just as the criteria for goodness of fit affect the fitted parameters in a given model, so the criteria for "best" will influence the model chosen by a particular user, for there is no one "best of all possible models."

Both the goodness of fit and the appropriateness of a model are related not only to the criteria of fit but to the sensitivity of prediction to changes in model parameters. The model developer should be as concerned with these two means for judging models as is the model user. No model is perfect, and therefore each is continually in a state of evolution. Each change made improves the model in the mind of the builder. The more understanding gained through study of criteria and sensitivity, the more easily can the model builder judge when an improvement has occurred.

The fitting of model parameters to a given set of data under a particular criterion can be accomplished by objective hill climbing methods [3, 4], random search methods [4], or more subjective methods [5]. The study of sensitivity of model response to parameter changes has been discussed [3, 5], but not in a detailed manner. The study presented here

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describes a set of tools for accomplishing sensitivity analyses, and describes the comparison of two alternative ways to describe a particular component in runoff simulation. The purpose is to present methods of comparison between models, rather than to describe a particular recommended model.

DESCRIPTION OF THE MODEL

The relative accuracy of fitting as measured by several alternative error criteria is a function of the particular model used. Similarly, the sensitivity of the various error criteria depends upon the parameters used and how those parameters are used. Therefore, a description of the model used in this study will aid in understanding the discussion of model comparisons.

Input to the model is rainfall (time increment variable) and pan evaporation (optionally average or recorded). Output is stream discharge. Optionally other components of the water budget that are computed may be printed out. The model operates in the following steps:

1. Rainfall is divided into:
   (*a*) Infiltration (two alternative descriptions are considered);
   (*b*) Direct runoff (spill in excess of detention storage);
   (*c*) Detention storage (spill level to surface runoff is a parameter);
   (*d*) Evaporation from detention storage (at potential rate, if storage is available).
2. Soil moisture accreted from infiltration, depleted by evapotranspiration (portion of potential varied exponentially with soil moisture [6], with exponential decay rate a parameter) and deep percolation to ground-water storage (spill level a parameter).
3. Potential evapotranspiration is computed from pan data (pan coefficient a parameter).
4. Surface storage is routed through a linear reservoir, and ground water through a non-steady linear reservoir [7]. (Reservoir storage coefficients a parameter).
5. Soil moisture is two-layered, with drainage rate from upper to lower layer proportional to differences in moisture between the two layers.
6. Precipitation may be adjusted to "effective" precipitation on the basin (coefficient a parameter).

Thus, computed evapotranspiration may be obtained from the sum of its two components, that from detention storage and that from soil moisture. Surface streamflow may be computed by summing surface runoff and groundwater discharge to the stream. Changes in storage are computed for the various components in the basin, and net change for any period may be studied.

TWO ALTERNATIVE INFILTRATION SCHEMES

Within the framework of the model described above, two infiltration schemes were compared. Model I is a Horton type exponential decay [8], with the time of incipient ponding approximated. The Horton equation assumes

$$\frac{\delta w}{\delta x_{x=0,t=t}} = \left[\frac{\delta w}{\delta x_{x=0,t=t_0}}\right] e^{-K(t-t_0)}$$

where w is soil moisture content, x is depth below the surface, K is the Horton exponential decay coefficient, and t_0 is the time of incipient ponding. In order to estimate t_0, all infiltrated water is assumed to be contained in a saturated layer, and head is assumed to vary linearly in this layer.

At incipient ponding infiltration equals rainfall, so that

$$R = -P_{sat}\left[\frac{H_b}{x_{t_0}} - 1\right]$$

where R is rainfall rate, P_{sat} is soil saturated permeability, and H_b is the bubbling pressure [9, 10]. Therefore

$$x_{t_0} = \frac{-H_b}{\dfrac{R}{P_{sat}} - 1}$$

This modified Horton equation has three parameters: K, the Horton decay coefficient, P_{sat} or f_c, the minimum infiltration rate, and H_b, the bubbling pressure.

Model II is based on a Philip type approximation to infiltration [11]. Philip assumes a uniform constant hydraulic conductivity, k_h, over the depth of wetted profile. k_h replaces the saturated permeability of Model I, and should be proportional to P_{sat}. Thus, the Philip approximation models the total wetted layer rather than just the saturated layer, so that the layer modeled has greater thickness but less conductivity. With this assumption, the Philip equation for infiltration is approximated by

$$\frac{di}{dt} = \frac{k_h}{\mu}\left[1 + \frac{(m - m_0)P}{i}\right]$$

where i is total infiltration up to time t, μ is liquid viscosity, m_0 is initial liquid content of the soil, m is the average content at time t, and P is the capillary potential at the wetted front. Model II has two parameters rather than three. They are k_h/μ, which is analogous to P_{sat} in Model I, and P which is analogous to H_b. The operation of the second term in the parentheses replaces the Horton exponential decay.

Model I therefore has ten parameters, whereas Model II has but nine. Each has five initial condition variables. By choosing the starting time for simulation at the end of a period without rainfall, three of the initial condition variables may be assumed zero. These three are surface storage, detention storage, and thickness of the saturated soil layer. Two must be estimated. These two are average soil moisture content and groundwater reservoir storage.

INPUT DATA

The data used for testing the model were for the gaging station on Santa Anita Creek near Sierra Madre, California. The drainage area above the station is 9.7 square miles. Santa Anita Creek is a tributary to the Los Angeles River, and drains a portion of Mt. Wilson. The rainfall record used for input was hourly recorded precipitation for Mt. Wilson. The rainy season records for the water years of 1941, 1943, 1944, and 1959 were used for fitting. The fitted years represent two years of above average rainfall, 1941 and 1943, a year of below average rainfall, 1959, and a year of about average rainfall, 1944. The monthly recorded precipitation during the periods of fitting are shown in table 1. In addition, the totals for the fitted periods recorded at another gage within the basin, Hoegee's Camp Ivy, are shown for comparison. The records at Mt. Wilson for 1943 and 1959 measured considerably less precipitation than that for Hoegee's.

Average monthly pan evaporation data collected at Camp Baldy over the period 1931 to 1935 was reduced to 10-day averages. The computed 10-day average values were used for determining daily potential evapotranspiration. In this manner, general seasonal fluctuations are accounted for, but exceptionally hot, cold, or dry periods are not. The Camp Baldy pan is located at an elevation of 4,300 feet whereas the median elevation of the basin simulated is 3,700 feet. Therefore, it might be expected that Camp Baldy data should be adjusted upward to be representative of the basin.

TABLE 1. Recorded precipitation in inches for Mt. Wilson and Hoegee's Camp Ivy gages during periods of simulation

		Dec	Jan	Feb	Mar	Apr	May	Total	Ratio
1941	HCI	11.08	5.09	20.66	15.52	12.11	.11	64.57	1.04
	MW	10.56	3.79	23.14	15.71	9.12	T	62.32	
1943	HCI	2.42	42.95	10.93	13.48	2.74	0	72.52	1.35
	MW	2.28	28.59	9.71	10.98	1.97	0	53.53	
1944	HCI	12.28	1.97	18.51	6.54	2.46	0	41.76	1.02
	MW	12.79	1.79	18.33	6.05	1.86	T	40.82	
1959	HCI	—	4.12	10.70	0	1.86	.01	16.69	1.33
	MW	—	3.10	8.07	0	1.36	T	12.53	

METHODS AND CRITERIA FOR GOODNESS OF FIT

The fitting of model parameters was accomplished through a hill-climbing method based on the Rosenbrock algorithm [3, 12]. All parameters plus the two necessary variables for initial storages were optimized for each model. The criterion used was the minimization of the sum of the squares of the difference between the logarithms of estimated daily discharges and the logarithms of the measured daily discharges. Figure 1 shows both the simulated and the measured hydrographs for Model II for 1943. This fitting is typical of the results achieved by simulation with these models. A comparison of the two hydrographs gives an indication of the accuracy of simulation. The parameters were fitted on the basis of all daily discharges, as stated above. However, similar sums of squared differences were computed separately for certain designated peak flows, for days with measured rainfall, and for days without measured rainfall. Thus, a comparison of these different components of simulation error could be made during the fitting process. The sets of optimum parameters which resulted for each of the four years for each model when fitted by optimizing to daily flows are listed in table 2.

GENERAL INADEQUACIES OF THE MODELS

Most of the detail in a rainfall-runoff model is involved with the distribution of "losses" in time. By losses is meant that part of rainfall which does not result in streamflow. The distribution of losses in time, however, influences later determinations of surface runoff and ground-water accretion, so that there are strong couplings or interactions between losses and streamflow. Thus, accretions to soil moisture plus depletions of soil moisture by drainage and evapotranspiration are used in these models to develop a complex antecedent precipitation index. The sum of all losses determines the water balance.If the water balance over any reasonable length of time, say a month, is to be reasonably correct, a constraint is placed on the loss function, and thus, indirectly, on the streamflow distribution in time.

TABLE 2. Final parameter values as fitted to each of the test years for the two models. Model I is Horton-type and Model II is Philip-type infiltration

Model	Year	KH	DK	f/k	H/P	KS	MM	MX	PR	PE	KG	M	GW	U
I	1941	0.42	0.22*	0.102	7.9	43*	12.6	0.39	0.58	1.38	1392	4.25	3.11	15.00
	1943	.11*	.14	.140	9.0	25	11.2	.40	.81	.99	1267	5.81	4.04	9.91
	1944	.40	.10	.080	9.0	20	13.6	.41	.71	1.03	1604	4.20	5.23	22.56
	1959	.42	.04	.093	10.0	29	10.0	.41	.75	.93	3555*	3.58	4.20	9.69
	Average	.41	.09	.104	9.0	25	11.8	.40	.71	1.08	1421	4.5	4.2	
II	1941		0.23*	0.108	5.4	39*	14.9	.415	0.6	1.42	1368	5.1	3.7	10.23
	1943		.12	.115	14.1	22	12.0	.418	.7	.84	1150	7.42	5.12	8.83
	1944		.05	.065	13	26	12.7	.49	.66	1.15	1125	4.35	6.4	21.73
	1959		.07	.042	20.5	24	9.2	.43	.72	1.46	3620*	4.02	3.94	10.73
	Average		.08	.082	13.3	24	12.2	.44	.67	1.22	1214	5.2	4.8	

* Parameter values not used to obtain average parameter values.

N.B. The parameter symbols have the following meaning: KH = Horton exponential decay, DK = maximum detention storage in inches, f = Horton minimum infiltration in inches/hour (Model I), k = saturated soil permeability in inches/hour (Model II), H = bubbling pressure in inches (Model I), P = suction at the wetted front in inches (Model II), KS = surface storage coefficient in hours, MM = maximum soil moisture retention in inches, MX = soil moisture exponential depletion coefficient, PR = precipitation coefficient, PE = pan coefficient, KG = ground-water storage coefficient in hours M = starting soil moisture storage in inches, GW = starting ground-water storage in inches, U = utility function, sum of squared log deviations for all days.

The interactions of the loss parameters in both models are similar, and indicate some of the problems encountered. The precipitation adjustment factor (PR) is strongly negatively correlated, whereas the evaporation adjustment (PE) is positively correlated with the amount of precipitation. Thus, in wet years these models produce less effective precipitation input from a given amount of measured precipitation, and depletion from soil moisture is more rapid. In addition, the detention storage coefficient (DK) is higher for wet years, so that less runoff occurs for a given storm and more moisture is available for loss at the potential rate after the rainfall ends. The general conclusion might be that there is an inadequate loss function in a physical sense, and that much of the adjustment of the parameters for these models is curve fitting only, rather than a physical adjustment.

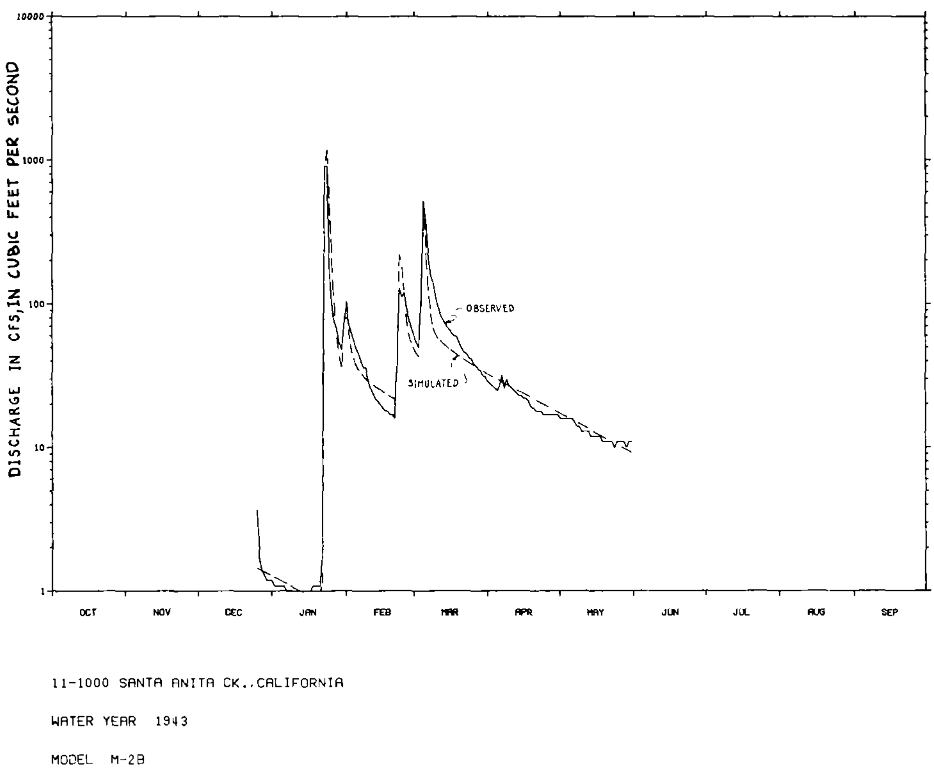

FIGURE 1. *Comparison of observed and simulated hydrographs for flood season of* **1943** *water year*

If these loss parameters were given a priori "physical" values, say 1.0 for precipitation and 0.7 for an evaporation "pan coefficient ", the water balance still must be met, and the curve fitting adjustments, of necessity, either would be magnified for those which now are affected, or would be transferred to other parameters, or both.

A rainfall-runoff model must abstract up to about 20 to 30 inches of rainfall and route the rest of the rainfall as runoff. The abstracted amount must be stored temporarily so that it is available for losses to the atmosphere throughout the year. The storage capability built into the model may be completely fictitious physically, but it must be capable of storing the 20 to 30 inches (not all at once) and holding on to it so as to distribute the losses through availability for evapotranspiration throughout the year. In addition, the manner in which the held water is used to generate an antecedent precipitation index is a determining factor in the generation of runoff. Therefore, results should be highly sensitive to changes in the storage parameters. If precipitation is not adjusted in the optimization process, some storage capability must be developed or some loss generated for the abstraction now included in the precipitation index.

Only two components of flow are simulated, and only mean daily flows are optimized. The simulated surface runoff component includes both direct storm runoff and any flow components intermediate between direct storm runoff and base flow depletion of ground water. The routing coefficient developed is more representative of an intermediate flow time coefficient, for it has a reservoir routing delay time of about 25 hours. The true surface routing coefficient would be on the order of an hour or two, for this basin, on the basis of storm hydrograph shapes. A planned adaptation of this model so that it can be used for simulation of storm hydrographs will require a separation of the surface component generated in these models into two sub-components.

COMPARISON OF THE MODELS

As shown by table 2, fitted values for most parameters are consistent among the years fitted, and for all parameters the average values of fitted parameters are consistent between the two models. Model II fits the test data considerably better, on the average, than Model I. The value of the objective function (U) for Model I is 47 percent greater than that for Model II for 1941, 12 percent greater for 1943, 4 percent greater for 1944, and 10 percent less for 1959. Also, Model II has one less parameter because it uses an approximation to the Philip two-parameter rather than the Horton three-parameter infiltration scheme. The Philip equation has a somewhat better physical interpretation as an approximation to the usual differential equation used to describe the infiltration process. Therefore, on the basis of fitting during the test periods, simplicity, and physical interpretation, Model II probably would be chosen.

The purpose of simulation is prediction, however. The only test for prediction ability is through simulation of a control not used for derivation of the parameters. The results of predictions based on the two models for several test years were quite similar. For much of the periods of prediction, accuracy of prediction was on the order of the accuracy of fitting during the control periods. However, there were gross errors for some storm periods which had an exaggerated effect on the computed error of prediction. A good measure of accuracy of prediction which can be used for choosing between models is yet to be developed for a daily flow model.

SENSITIVITY ANALYSES

The variability of the "goodness of fit" of the models as the values of the parameters are varied can give insight into the utility of the models.

Sensitivity plots for four parameters for the Philip-type model are shown in figure 2. *P*, pressure at the wetted front, and *K*, saturated permeability or minimum infiltration, are parameters with some physical meaning. The response surfaces for these parameters are smooth, and indicate that, although the parameter values chosen are important, a reasonable error of parameter value does not overly degrade the ability of the model to fit the input data. *EK*, the evaporation coefficient, and *PK*, the precipitation coefficient, are parameters without true physical meaning, although some meaning may be imputed to them in terms of adjustment of measured values to effective basin values of potential evaporation and precipitation. In effect, however, they are curve-fitting factors which help determine losses over the basin. The plots show that their fitted values are critical, and that the loss mechanism of the model is the weakest link, and must be developed further in order to obtain a more consistent response from the model.

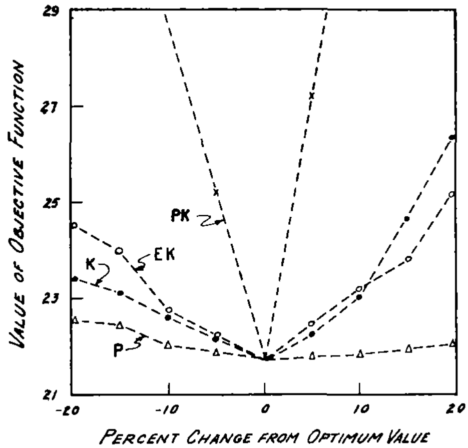

FIGURE 2. *Plot of sensitivity of Model II response to changes in values of parameters, P, pressure at the wetted front, K, saturated soil permeability, EK, evaporation coefficient, and PK, precipitation coefficient, for water year* **1944**.

CONCLUSIONS

On the basis of simplicity of the model, the Philip-type model is preferable because it contains one less parameter. Fitting results were in general somewhat better, and prediction results comparable, for the Philip, as compared to the Horton-type equation. Sensitivity plots were similar for both plots, and indicated that the major points still needing improvement are the soil moisture redistribution and depletion sub-systems. The purpose of this paper is not to recommend a particular model, however, but rather to stress that measures of performance may be used both to aid in model development and to indicate to the ultimate user both the strong and weak points of a given model. The measurements of performance used were objective criteria to measure accuracy of fitting and of prediction, and objective measures of sensitivity of results of simulation to changes in parameter values.

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