Quantification of soil detachment by raindrop impact: performance of classical formulae of kinetic energy in Mediterranean storms

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ABSTRACT The eroding power of raindrop impact depends on raindrop properties, soil properties, and interaction properties. Raindrop properties govern rainfall erosivity through the kinetic energy of drops. This Kinetic Energy can be expressed in terms of the diameter and terminal velocity of raindrops. Direct measurements of these drop features are uncommon and kinetic energy of a given storm is generally computed from mean rain intensities using formulae proposed in the literature. This paper evaluates the performances of 9 common formulae when applied in Mediterranean climate. Two characteristic Mediterranean storms (about 3000 minutes of rainfall with intensities up to 90 mm/h) were measured with a Joss & Waldvogel disdrometer in the Cevennes region (south of France). Comparisons show that formula performance depends basically on the rainfall intensity range, leading to important discrepancies between them when rainfall intensity exceed 30 mm h⁻¹.

INTRODUCTION

In interrill erosion, the direct impact of raindrops on soil or on a thin layer of water over the soil surface constitutes the major force initiating soil detachment. Note for instance that kinetic energy of falling raindrops at terminal velocity is from one to two orders of magnitude greater than that of flowing water (Hudson, 1971).

The eroding power of raindrop impact depends on raindrop properties, soil properties, and interaction properties (see Park et al., 1982). Particularly, raindrop properties (mass, size, shape, and terminal velocity) govern rainfall erosivity through the kinetic energy of drops. Thus, kinetic energy is a widely used indicator of the potential ability of rain to cause erosion.

The Kinetic Energy of raindrops (KE) can be easily expressed in terms of the diameter of drops (D) and of their terminal velocity (V). Nevertheless this approach is hardly ever used since direct measurements of these drop features have been uncommon up to now. Thereby the KE of a given storm has been generally computed from mean rain intensities using the formulae proposed in the literature.

On the one hand, there are some empirical formulae providing estimates of the KE, using directly the rainfall intensity (normally the only rainfall data available). They are simple regressions between a transformation of the rainfall intensity and the KE obtained using data from rainfall simulation and, some times, from field data.

On the other hand there are expressions relying on average relationships between rainfall intensity (I) and drop-size distributions (either using a single representative median diameter or the complete distribution of diameters). The terminal velocities are then derived from experimental tables or from theoretical relations with the drop diameter.

Nevertheless, recent developments in disdrometers and optical rain spectrometers (devices able to measure D and V directly) provide the way to estimate rainfall erosivity
directly. This paper presents an application of these data in the soil detachment domain.

The performances of the most common formulae of both types (generally derived from USA or UK data) were evaluated when applied in Mediterranean climate. Two characteristic Mediterranean storms (about 3000 minutes of rainfall with intensities up to 90 mm/h) were measured with a Joss & Waldvogel disdrometer in the Cevennes region (south of France). So, $KE$ was computed through:

i) 9 different formulae taking into account the rainfall intensity

ii) a detailed formulation using the directly measured values of the drop diameter and their associated terminal velocities.

The aim of this work is to provide a reference framework in Mediterranean climate according to the requirements of the usual erosion studies presently carried out, where the rainfall intensity is the only rainfall measurement.

HOW TO CALCULATE KINETIC ENERGY: THE STATE OF THE ART

The kinetic energy of a raindrop is easily expressed in terms of its properties as:

$$KE = \frac{1}{2} m v^2 = \frac{1}{12} \pi \rho D^3 v^2$$

where $\rho$ is the water density, $D$ is the diameter of the equivalent spherical drop and $V$ is its terminal velocity. So, the $KE$ of a rainfall event per unit of surface can be directly computed knowing the $D$ and the $V$ of each raindrop falling on a surface unity.

However these drop features are not normally measured in erosion field studies (nor in others) and relations between $KE$ and rainfall intensity ($I$, generally the only rainfall measurement available) must be used. So, the question is how we can calculate $KE$ knowing $I$.

We consider 9 formulae solving this question and compared them. They can be classified in two main groups: those that relate empirically $KE$ and $I$ and those relying on average relationships between $I$ and the drop size distributions. They cover fairly well the state of the art of measuring $KE$.

Regression relationships between $I$ and $KE$

These formulae are obtained from rainfall simulations or field experiments during which $KE$ was derived from equation (1). $D$ is measured by one of the two classical methods: the flour pellet proposed by Bentley in 1904 (described by Laws & Parson, 1943 among others), or the filter-paper technique proposed by Wiener in 1895 and improved by Blanchard (1953) (see for instance Hall, 1970 or Mason & Andrews, 1960). $V$ is generally obtained by using the classical tables provided by Laws (1941), Gunn & Kinzer (1949), or Best (1950). Knowing the sizes and velocities of the drops for each examined rain intensity, $KE$ is computed and the regression relationship is established.

They are the most widely used in erosion studies due to their simplicity. However, their applicability is a priori restricted by the characteristics of the rainfall samples used for the regression.

Wischmeier & Smith, 1958 Perhaps the most known expression since it forms part of the USLE equation (Universal Soil Loss equation, Wischmeier, 1959; Wischmeier & Smith, 1978). Originally expressed in American unities, Foster et al., 1981 converted it into S.I. unities. So, the kinetic energy (in J m$^{-2}$) of a rainfall of intensity $I$ (in mm h$^{-1}$) falling over a surface unity during a time step $\Delta t$ (in h) is:

$$KE = (11.9 + 8.73 \log I) I \Delta t$$

(2)
Park et al., 1980 Park et al., known by their theoretical study of splash erosion Park et al., 1982 and by previous work (for instance Bubenzer & Jones, 1971), proposed the formula

\[ KE = 21.1069 \frac{I^{1.156}}{\Delta t} \]  
(3)

(As in everywhere, \( KE \) is expressed in \( \text{J m}^{-2} \), \( I \) in \( \text{mm h}^{-1} \) and \( \Delta t \) in h.)

**Formulae using raindrop size distribution modelling**

Instead of deriving \( KE \) from measured drop diameters like above, these formulae associate each rain intensity to a statistical featuring of the Drop Size Distribution (DSD), from which \( KE \) is computed.

There are two classical possibilities to characterize statistically the DSD. A first one is to fit relationships between the median-volume drop diameter \( (D_{50}) \) of rainfall and its intensity \( I \). This \( D_{50} \) is supposed to be an effective diameter able to reproduce the properties of the whole raindrop set. A second possibility is to consider a model of the complete DSD of rainfall rather than \( D_{50} \).

Both ways need the knowledge of drop terminal velocities, but while the use of median drop diameter only needs to know a unique velocity \( (V(D_{50}) \), easy to deduce from published tables), that of a DSD model needs to introduce the complete spectrum of velocities. For this reason the formulae based on the complete DSD have been more rarely used. However we introduced in this study both ways of calculating \( KE \), but using the Beard’s equations of terminal velocity (Beard, 1976; Beard, 1977a; Beard, 1977b; Beard, 1980). These equations take into account previous published data and introduce corrections in terms of pressure and temperature. They are presently considered the best theoretical relations between \( V \) and \( D \) (Epema & Riezebos, 1983).

**Formulae using \( D_{50} \)**

Using the \( D_{50}(I) \) relations from the literature the kinetic energy can be expressed as:

\[ KE = \frac{I \Delta t}{2} V^2(D_{50}) \]  
(4)

where \( V(D_{50}) \) is the terminal velocity (in \( \text{m s}^{-1} \)) of a raindrop of diameter \( D_{50} \) (in mm). Four known formulae of \( D_{50} \) were chosen.

**Laws & Parson, 1943** Based on their own data and on those from Laws, 1941 they proposed the relation:

\[ D_{50} = 1.238 I^{0.182} \]  
(5)

**Atlas, 1953** Based on data from Marshall & Palmer, 1948 \( D_{50} \) is expressed as:

\[ D_{50} = 0.92 I^{0.21} \]  
(6)

**Brandt, 1989** Based on her own data, Brandt proposed the relation

\[ D_{50} = 1.416 I^{0.123} \]  
(7)

**Willis, 1984** Based on data from two tropical cyclones collected by an optical rain spectrometeter Willis proposed:

\[ D_{50} = 0.97 I^{0.158} \]  
(8)

At first glance the comparison of these formulae is amazing since four published studies leads to a similar relationship \( (D_{50} = a I^{b}) \) but with significantly different parameters. These
differences may result either from sampling reasons or, more probably, from the variety of the considered meteorological context.

**Formulae using the complete drop size distribution**

To use distribution functions of the raindrop size is the most attractive and intuitively complete way to solve our problem, even if in erosion studies they are not generally applied.

Various studies have related DSD to rainfall intensities. They can be used to deduce very general formulae of \( KE \) that will take into account the whole DSD and not only the median drop diameter.

These distributions provide \( N(D) \Delta D \), the number of drops per unit of volume (in \( \text{cm}^{-3} \)) with diameters between \( D \) and \( D + \Delta D \) (\( D \) in cm and \( N(D) \) in \( \text{cm}^{-4} \)), which generally depends univocally on \( I \). So, the flux of \( KE \) (\( J_{KE} \) in J m\(^{-2}\) s\(^{-1}\)) can be expressed as the summation of the raindrop elementary kinetic energy (1) for diameters between \( D_i \) (the minimum diameter considered) and \( D_f \) (the maximum diameter) reaching a surface unity during a unit time step

\[
KE = J_{KE} \Delta ts = \frac{10^3 \pi}{\sqrt{12}} \sum_{D_i}^{D_f} D^3 V^3 N(D) \Delta D \Delta ts
\]

with \( \Delta ts \) the considered time step in seconds. Obviously we need the \( V(D) \) relation. In our case the Beard's equations (1976) were used.

**Marshall & Palmer, 1948** Initially derived from moderate intensities observed in widespread rain situations, the Marshall-Palmer distribution has been largely tested on experimental data around the world and it is of common use in meteorological studies. It is the simplest one and is a very good approximation to the DSD when sufficient averaging in space and/or time is performed (Ulbrich, 1983 p. 1764). \( N(D) \) is considered to be a decreasing exponential function on \( D \):

\[
N(D) = N_0 e^{-AD}
\]

where the slope \( A \) depends on \( I \), \( A = 41.1 \cdot 0.21 \) (in \( \text{cm}^{-1} \)); and \( N_0 = 0.08 \) (in \( \text{cm}^{-4} \)). So, \( KE \) can be expressed in terms of \( I \) using (9) and (10) as

\[
KE = J_{KE} \Delta ts = \frac{80}{12} \pi \sum_{D_i}^{D_f} D^3 V^3 \exp \left\{ -41.1 \cdot 0.21 D \right\} \Delta D \Delta ts
\]

(\( \Delta ts \) is the time step in seconds.)

Although attractive in its simplicity, this exponential DSD shows some inadequacies in describing observed instantaneous spectra (sampling time of 1 minute for instance). It generally tends to overestimate the number of both the smallest drops (Waldvogel, 1974) and the largest drops (Joss & Gori, 1978), particularly for high-intensity convective storms. For this reason, even if it remains the most widely used, some authors have recently proposed improvements using multiparameter distributions as \( \Gamma \)-distribution (Ulbrich, 1983) or lognormal distribution (Feingold & Levin, 1986). We chose a simplified parametrized \( \Gamma \)-distribution to test what those refinements can bring.

**Willis & Tattelmann, 1989** Based on previous studies from tropical storms, Willis propose a simplified \( \Gamma \)-distribution, parametrized in terms of \( D_{50} \) (from Willis, 1984).

\[
N(D) = N_G D^\alpha e^{-AD}
\]
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with $\Lambda = \frac{5.588}{D_0} ; \alpha = 2.16$ ; and $N_G = \frac{512.85 M \times 10^{-6}}{D_0^4}$;

where $D_0$ is the median-volume diameter (in cm), and $M$ is the liquid water content (g m$^{-3}$) given by

$$D_0 = 0.157 M^{0.1681}$$

and

$$M = 0.062 I^{0.913}$$

Brandt, 1990 In order to simplify the application in erosion studies, Brandt derivates an expression of $KE$ in terms of $I$ but including the $V$ estimate by itself. It assumes i) a Marshall-Palmer DSD, and ii) that terminal velocities are those proposed by Clift (1978) and Beard (1976).

Instead of taking the general expression of $KE$ (like in equation (11)), Brandt generated rainfall from equation (10), calculated $KE$, and fitted a simplified expression of $KE(I)$. Even if it is a less accurate procedure than using directly $N(D)$, this formula has the advantage of being ready to be used. Thus, the expression is:

$$KE = \frac{8.95 + 8.44 \log I}{\Lambda} I \Delta t$$

It should be noted that Brandt, 1990 also provides formulae to take into account the forest cover in the $KE$ computation.

DISDROMETER DATA

In the last decades, the measure of raindrop features has been made easier by the development of disdrometers and optical rain spectrometers. However, problems related to data acquisition made these devices of exceptional use.

In this work, data from a disdrometer type Joss & Waldvogel, (1967) were used. This device is based on an electromechanical principle and determines the size ($D$) of the raindrops from measurement of the vertical force applied by the drops falling on a transducer. The signal delivered is processed to obtain $D$ in 25 equally spaced diameter classes of 0.2 mm. The sampling surface, $S$, is 50 cm$^2$ and the range of measurable drop diameters is from 0.2 mm up to 5.2 mm. The raindrop size spectrum is recorded each 30 s. $V$ should be calculated by theoretical relations or from published tables. In our case the Beard 1976 formulation was used. A complete description of the device and of the error analysis is provided by Salles, 1991.

Data from two storms collected in the framework of the Cevennes radar experiment (Andrieu et al., 1989) were used. The first event is a summer storm (150 minutes) recorded in July 1985 with a maximum of intensity of 89.4 mm h$^{-1}$. The second one, recorded in November 1986 with a maximum intensity of 66 mm h$^{-1}$ and 2847 minutes record length. A wider description of the data is given by Delrieu et al., (1991) and by Salles, (1991).

For each event the disdrometer provides the number of raindrops ($X(D)$) of each diameter class, $[D, D + \Delta D]$, falling over the disdrometer surface ($S$) during the time step $\Delta ts$. The relation between $X(D)$ and $N(D) \Delta D$ (in cm$^{-3}$) is given by:

$$N(D) \Delta D = \frac{10^{-6} X(D)}{V S \Delta ts}$$

where $S$ is the sampling surface (5 $10^{-2}$ m$^2$), $\Delta ts$ the sampling time step (in s) and the $\Delta D$ the diameter class length (0.02 cm). Thus $KE$ can be expressed (using (9) and (16)) as:
\[ KE = J_{KE} \Delta t_s = \frac{10^{-3} \pi}{12} \sum_{D_i} D_i^3 V^2 X (D) \]  

(17)

where \( D_i = 0.02 \text{ cm} \) and \( D_f = 0.52 \text{ cm} \).

Rainfall intensity can also be expressed as:

\[ I = \frac{\pi}{6} \rho \sum_{D_i} D_i^3 V N (D) \Delta D = \frac{0.6 \pi}{S \Delta t_s} \sum_{D_i} D_i^3 X (D) \]  

(18)

where \( I \) is in mm h\(^{-1}\) and \( \Delta t_s \) is the sampling time step in seconds.

The results obtained using these direct measurements were taken as the reference \( KE \).

**COMPARISON OF FORMULAE PERFORMANCES**

In a first step, we can compare for the range of rainfall intensity of the data set the \( KE \) estimates provided by the 9 presented formulae (see Figure 1). The comparison points out that there is a substantial dispersion of the estimates and an almost linear behaviour of the different relationships when \( I > 30 \text{ mm h}^{-1} \). For lower intensities, the behaviour is less linear and more convergent. A special case is the Park formula (3) which systematically provides much greater estimates than the others.

**TABLE 1** Nash efficiencies for the different estimates of \( KE \) for both events.

<table>
<thead>
<tr>
<th>ESTIMATE</th>
<th>NASH EFFICIENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wischmeier (2)</td>
<td>0.95</td>
</tr>
<tr>
<td>Park (3)</td>
<td>0.90</td>
</tr>
<tr>
<td>D-Laws (5)</td>
<td>0.96</td>
</tr>
<tr>
<td>D-Atlas (6)</td>
<td>0.79</td>
</tr>
<tr>
<td>D-Brandt (7)</td>
<td>0.93</td>
</tr>
<tr>
<td>D-Willis (8)</td>
<td>0.52</td>
</tr>
<tr>
<td>Marshall (11)</td>
<td>0.94</td>
</tr>
<tr>
<td>G-Willis (12)</td>
<td>0.85</td>
</tr>
<tr>
<td>Brandt (15)</td>
<td>0.87</td>
</tr>
</tbody>
</table>

In a second step, we can assess the global level of performances of each method by comparing their estimates, \( KE_i^* \), to the kinetic energy, \( KE_i \), computed from the disdrometer data over the \( N \) available observations (equations 17 and 18). Table 1 shows their performances, in terms of the Nash Efficiency criterion (\( E \), Nash & Sutcliffe, 1970)

\[ E = 1 - \frac{\sum_{i=1}^{N} \left( KE_i - KE_i^* \right)^2}{\sum_{i=1}^{N} \left( KE_i - \overline{KE} \right)^2} \]  

(19)

where \( \overline{KE} \) is the mean value of the disdrometer derived kinetic energy.

Considering \( E = 0.90 \) as the acceptability threshold, only 5 formulae provide satisfactory results, and the analysis can be confined to these formulae: Wischmeier (2), Park (3), D-Laws (5), D-Brandt (7) and Marshall (11).

Finally, \( KE_i^* \) from these retained formulae are compared to \( KE_i \) to analyze not only the global quality over the whole data set (given by \( E \)), but also its distribution. However, due to the great number of points (\( N > 2500 \)), this comparison is reduced to a percentile comparison. Thus only the percentile values of \( KE \) (from its distribution function) are compared to the estimates (see Figure 2). Note that the perfect estimate would follow the first diagonal.

As in Table 1, this plot shows that D-Laws estimate turns out to be the best one, especially for higher \( I \), where the formula differences are more significant. That confirms the well-known good quality of the work carried out by Laws (1941) and by Laws & Parsons (1943) that have been for long time one of the references for \( D \) and \( V \) empirical measurements. Wischmeier (2) and Marshall (11) give similar results up to intensities about 40 mm h\(^{-1}\) (\( KE \) about 20 J m\(^{-2}\)). For higher \( I \), only Wischmeier formula provides comparable performances to those obtained by the D-Laws one. On the one hand, it shows the robustness of this expression.
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that is the basis of the USLE (even for $I > 76 \text{ mm h}^{-1}$, the limit of its theoretical range of validity). On the other hand these results point out this expression as the best solution: It is the simplest one (it does not need $V$) and give fairly good results.

As for the Marshall formula (11), results accord to the evidences of that DSD's tend to be exponential only when sampling time is sufficient long. Thus, it presents some estimate problems at 1-minute $\Delta t$, and does not provide results as good as D-Laws. Analyses taking into

FIG. 1 Comparison of analysed estimates of $KE$ in terms of $I$ at 1 minute. The linear regression from equation (20) has also been added.

FIG. 2 Percentile comparison of $KE$ estimates for $KE > 5 \text{ J m}^{-2}$.
account longer $\Delta t$ should be carried out to determine if this result is only related to this $\Delta t$, or if it could be generalized. The failure of the $\Gamma$-distribution from Willis (12), deduced for very intense storms (tropical cyclones), is not easy to understand. The only explanation we can suggest may be given by the high degree of parametrization that may decrease its robustness.

The difference between the estimates provided by the D-Brandt formula (7), and by the Brandt (1990) formula (15) should be also noted. This last expression, derived from the complete Marshall & Palmer DSD, was presented as an improvement of the first one relying on a single $D_{50}$. This is not the case with the used data set. Moreover the difference between Brandt (15) and Marshall (11) is also surprising since they are theoretically derived from the same DSD model.

Finally it is worth noting that all formulae, except that from Park (3), systematically underestimate $KE$.

A linear regression between $KE$ and $I$

A simple arithmetic plot of the $KE$ versus $I$ both computed from the disdrometer data for the 1-minute observations is presented in Figure 3. If a linear regression is established over the whole set, this regression fails to fit the points of higher intensity, and the relation underestimates again $KE$ (Figure 3, line a).

However, if only the high intensities are considered, a linear regression appears to fit the data satisfactorily ($R = 0.97$). For intensities $> 20$ mm h$^{-1}$, the $KE$ (in J m$^{-2}$) of a 1-minute rainfall of intensity $I$ (in mm h$^{-1}$) can be given by:

$$KE = 0.56I - 3.1 \quad (20)$$

Over the available data, this linear equation turns out to be a better and simpler estimate of $KE$ in terms of $I$ than the analyzed formulae (see figure 2), and the residual variance is apparently not related to the intensity. However, for low intensities ($I < 30$ mm h$^{-1}$), the residuals for a given intensity are not symmetrically distributed around their mean value. This is related to sampling considerations: On the one hand the density of large drops in light rain is so

![Figure 3](image-url)
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low that the probability of having any of it during one minute on 50 cm^2 is less than 1. On the other hand, $KE$ computation is very sensitive to larger drops.

It can be suggest that for low $I$ the raindrop size has a critical influence. However, when a certain range of $I$ is exceeded, the DSD is not a relevant factor any more and $KE$ can be expressed in terms of $I$ by a linear relation. The dispersion of the different estimates when $I$ increases is thus explained since most of them are taking into account the lower range of $I$ (the most usual measurements) and diverge when $I$ increases.

CONCLUDING REMARKS

A critical comparison of different formulae relating rainfall intensity ($I$) and kinetic energy ($KE$) was performed using direct measurements of drop sizes under Mediterranean climate.

A simple comparison of the 9 formulae taken from the literature shows that they are weakly non-linear and strongly diverging with increasing intensities. This comparison is valid for a 0-90 mm h$^{-1}$ intensity range and for any time step.

For a 1-minute time step and for the data considered, results point out that D-Laws (5) and Wischmeier (3) formulae exhibit the best skill, both in terms of the Nash Efficiency criterion and of the percentile comparison. When $KE$ is related to $I$, a simple linear regression is shown to be acceptable for $I > 20$ mm h$^{-1}$. The residual dispersion is weak and independent of the intensity, and the regression performs as good as the best formulae when they are compared on the calibration sample.

Thus very simple relations are apparently able to predict correctly the mean kinetic energy of rain knowing its intensity over very short time steps. This result needs of course to be confirmed by a larger data set in order to reflect a wide range of meteorological conditions. If it proves to be true, the erosion studies will not need sophisticated rain measurements devices provide that the mean kinetic energy is the appropriate indicator of the eroding power of raindrops.

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REFERENCES


