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Drainage of a glacial lake through an ice spillway

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Abstract Debris cover on glacier surfaces forms topography that can trap water in surface lakes. Such lakes may overflow and drain by melting a gorge through a low point in the ice barrier. If the spillway elevation melts downward faster than the lake level drops, a rapid (unstable) discharge results. We show that unstable discharge will occur when the area of the lake exceeds a critical area evaluated in terms of the slope and width of the channel through the ice dam and the temperature of the water in the lake. If the lake water is at the freezing point, the critical area is large (order 1 km^2 or more) for typical slopes (order 0.1 or less) and unstable drainage is unlikely. Modest warming of the lake water (order 1°C) can reduce the critical area significantly and promote unstable drainage. The filling of depressions and subsequent drainage, whether fast or slow, is one mode of water storage change on a glacier that affects the timing of runoff. These theoretical concepts are used to explain observations from Black Rapids Glacier, Alaska, where a marginal lake drained through a lateral moraine onto the surface of the glacier with an initial unstable phase lasting about one day followed by a stable multi-day gradual drainage.

INTRODUCTION

Lakes impounded behind ice barriers can drain in a variety ways. Most well known are glacier outbursts (jökulhlaups) that are associated with catastrophic drainage of lakes by rapid thermal enlargement of subsurface tunnels. This phenomenon has been examined extensively through observations and theory (Björnsson, 1974; Nye, 1976; Clarke, 1982; Spring & Hutter, 1981; Björnsson, 1992). Ice-dammed lakes sometimes drain by mechanical collapse of the ice dam. Discharge through gaps between the wall of the enclosing valley and the ice can accelerate by sideward back melting and calving of the ice to increase the width of the channel (Walder & Costa, 1996).

Lakes often form in depressions on the irregular surface of debris-covered glaciers. Commonly, such lakes do not drain through pressurized tunnels or marginal moats, but instead drain over an ice floored spillway either on the ice surface or in a partially filled, near-surface tunnel. Ice cored moraine may also dam lakes that drain over spillways. This paper explores the potential for rapid, unstable discharges from a lake, when the lake overflows a spillway and the discharge water melts the floor of the spillway downward faster than the level of the lake drops. We also examine stable drainage of such lakes, when the spillway floor and lake level drop more or less in unison. Both the unstable and stable modes of drainage are of interest with regard to the evolution of surface water storage.

We illustrate features associated with drainage over a spillway with data from a marginal lake on the Black Rapids Glacier, Alaska. This lake drained with both unstable and stable phases through a gorge that was deepened by the discharge water.

DRAINAGE OF A LAKE ON BLACK RAPIDS GLACIER

We observed the 1993 drainage of a lake (latitude 63°29.5'N, longitude 146°31.0'W) on the north margin of the Black Rapids Glacier. The lake forms just upstream from the tributary entering from the east side of Aurora Peak (Fig. 1). We refer to this lake as "Aurora Lake". The depression that contained the lake was formed by a combination of factors associated with dynamic thickening of the trunk/tributary junction and with differential melting caused by debris cover and proximity to the rock slopes of the valley. Outburst-like discharges from this lake onto the ice surface have been observed on several occasions (Sturm & Cosgrove, 1990).

Discharge started sometime on 13 June 1993. At this time the surface of the glacier was still covered with a snow thickness of about 1 m. The lake had ice and snow slush floating in it. The water released from the lake ran several hundred metres through an ice gorge breaching the ice dam into a number of snow-bounded channels. Some of these channels were ice floored and flowed about 1 km downslope from the lake before dissipating into the snow or descending into the glacier. Figure 1 gives an over view of the lake and gorge.

The exact timing of the onset of drainage and the peak discharge are not known directly from observation. The drainage started to affect the speed of the glacier during

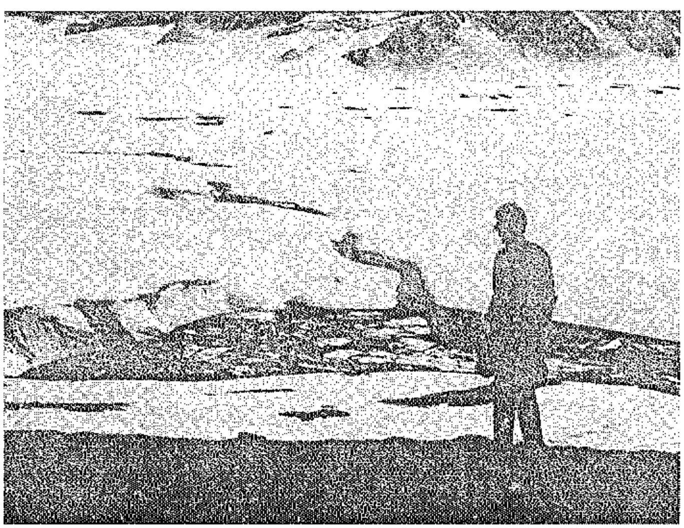

Fig. 1 View looking south across the eastern part of Aurora Lake and Black Rapids Glacier before the drainage started. Note the pre-existing gorge though which drainage occurred. "Pot holes" in the background are related to the drainage of other lakes (Sturm & Cosgrove, 1990).

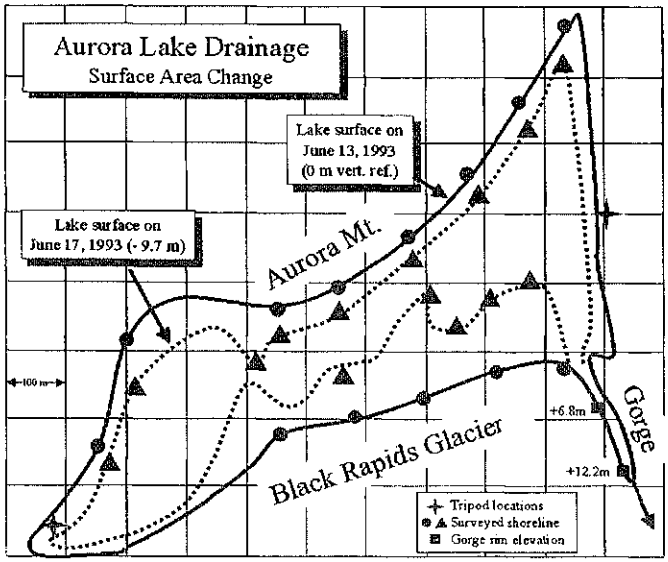

Fig. 2 Map of shorelines of Aurora Lake in local coordinates on 13 and 17 June 1993.

the night of 13–14 June (Nolan & Echelmeyer, 1999). Flooding of the surface suggested a peak discharge about midday on 14 June. At 17:00 h on 14 June the discharge in the main downslope drainage path below the gorge was roughly 12 $m^3 s^{-1}$, but this may not account for possible drainage in other smaller streams beneath the snow cover. The discharge from the lake was distinctly higher at sometime before 17:00 h as evidenced by high water lines in the channels cut in the snow.

On the afternoon of 17 June we surveyed the shoreline of the remaining lake and the initial high shoreline of 13 June marked in the snow (Fig. 2). The survey shows that the initial area of the lake was 0.15 km^2. By 17 June the lake level had dropped 9.6 m and the area was reduced to 0.07 km^2 or about half the initial area. The lake was long and narrow with relatively uniformly sloped side banks. Therefore, we assume a linear relationship between lake area A_l and lake surface elevation h_l (Fig. 3) given by $A_l(h_l) = 0.80 \times 10^4\ h_l$, where $h_l = 0$ m, 9.2 m and 18.8 m are respectively the bottom of the lake, the surface on 17 June and the surface on 13 June. The implied volume released from 13 to 17 June was $1.1 \times 10^6\ m^3$.

Figure 4 shows the gorge through which water was discharged from the lake. A sequence of water levels is clearly marked on the walls of the gorge. On 22 June at

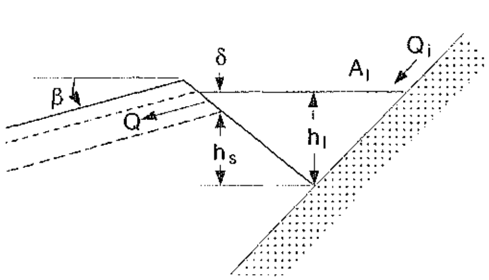

Fig. 3 Descriptive cross-section of lake and spillway.

Fig. 4 View looking into the gorge that formed the spillway for Aurora Lake. Note the series of water levels cut in the walls of the gorge.

18:00 h we measured the elevations of the water level marks. The number of levels corresponded to the number of days since the peak of discharge on 14 June. Subsequent visits to the gorge showed the creation of one new level mark each day. The measurements, therefore, enable the reconstruction of the history of the lake level and corresponding discharge on a daily basis (Fig. 5). Most of the water stored in the lake prior to 13 June was released rapidly in a 2-day interval. Subsequently water was released from the lake much more slowly at a relatively steady rate.

SPILLWAY MODEL

Figure 3 defines variables associated with an idealized geometry of a lake and spillway that forms the basis for the following analysis. The following relationships are introduced to predict the time evolution of drainage through the spillway.

Lake geometry relates height h_l of the lake surface to the lake volume V_l:

$$\frac{\mathrm{d}V_l}{\mathrm{d}t} = A_l(h_l)\frac{\mathrm{d}h_l}{\mathrm{d}t} \tag{1}$$

where A_l is the area of the lake surface.

Water volume conservation relates spillway discharge Q_s to change in storage in the lake and in flow rate Q_i:

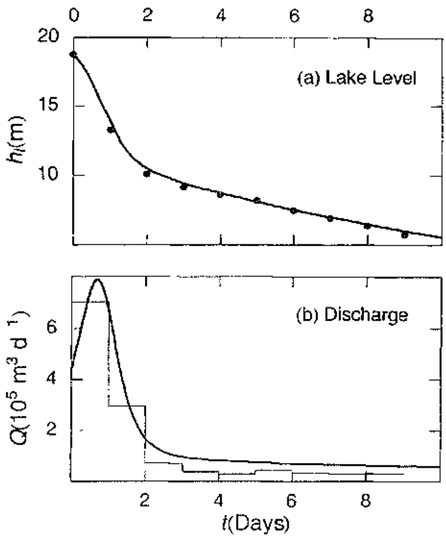

Fig. 5 History of water level (a) and rate of volume loss (b) for the lake. Data points in (a) and step plot in (b) show observations from Aurora Lake. Curves show the simulations described in the text.

$$Q_s = -\frac{dV_l}{dt} + Q_i \tag{2}$$

Hydraulics of the spillway relates Q_s, the lake level h_l relative to the spillway threshold h_s, width W_s and channel slope β:

$$Q_s = k\beta^{1/2} W_s (h_l - h_s)^{5/3}, \qquad h_l > h_s \tag{3}$$

where k is a channel discharge parameter depending on the roughness n and channel cross-section shape. Equation (3) corresponds to standard theory for steady open channel flow with $k = 1/n$ when the channel width is much larger than the water depth $\delta = h_l - h_s$. When $h_l < h_s$ there is no drainage from the lake ($Q_s = 0$). If the spillway were very steep or the width of the flow were not constrained down-flow from a breach, then the discharge would be limited by a transition to critical flow very close to the lake (e.g. Walder & Costa, 1996). Those conditions give a somewhat different flow relation with slightly different power (3/2 instead of 5/3) and substantially higher discharge unless β is quite large in equation (3). Because of the substantial length (~1 km) and modest slope (~0.1) of the gorge, it is appropriate to assume in this case that the flow is limited by the discharge through the channel.

Energy conservation relates melting of the spillway floor to the energy loss in the water flow:

$$\rho_i L W_s \frac{dh_s}{dt} = \left(\rho_w C_w \frac{d\theta}{ds} - \rho_w g \beta \right) Q_s = \rho_w g (\beta + \gamma) Q_s \tag{4}$$

ρ_i and ρ_w are densities of ice and water respectively; L is the latent heat of fusion per unit mass; C_w is the heat capacity per unit mass of water; $d\theta/ds$ is the change in water temperature per unit distance s along the spillway. To simplify the expression of equation (4), we introduce $\gamma \equiv (C_w \mathrm{d}\theta/\mathrm{d}s)/g \approx C_w\Delta\theta/gl$ and regard $\mathrm{d}\theta/\mathrm{d}s$ as an input approximated by the temperature of the lake water above freezing $\Delta\theta$ divided by the distance l along the spillway over which the temperature drops to freezing. To determine the actual temperature of the water in the spillway would require introduction of a heat transfer equation. We instead assume that the potential energy conversion to heat and thermal energy relative to the melting point are transferred to the ice floor of the spillway uniformly over its length according to the average slope and rate of water cooling.

Inputs to these four equations are $A_l(h)$, β, γ, W_s, k and Q_i. They can be solved for the four unknowns $h_l(t)$, $h_s(t)$, $Q_s(t)$ and $V_l(t)$.

It is useful to focus first on $h_l(t)$ and $h_s(t)$. Substitution of equation (3) into equation (4) gives:

$$\frac{\mathrm{d}h_s}{\mathrm{d}t} = -\frac{\rho_w g k \beta^{1/2}}{\rho_i L}(\beta+\gamma)(h_l - h_s)^{5/3} \tag{5}$$

Substitution of equations (2) and (3) into equation (1) to eliminate $Q_s(t)$ and $V_l(t)$ gives:

$$\frac{\mathrm{d}h_l}{\mathrm{d}t} = -\frac{k\beta^{1/2}W_s}{A_l(h_l)}(h_l - h_s)^{5/3} + \frac{Q_i}{A_l(h_l)} \tag{6}$$

The condition for unstable drainage is that the spillway elevation drops faster than the lake elevation ($\mathrm{d}h_s/\mathrm{d}t < \mathrm{d}h_l/\mathrm{d}t$ accounting for the signs of $\mathrm{d}h/\mathrm{d}t$). When inflow Q_i is neglected, equations (5) and (6) show that this condition holds when:

$$\frac{A_l(\beta+\gamma)}{W_s} > \frac{\rho_i L}{\rho_w g} \equiv H^* \tag{7}$$

The quantity $H^* = 3.0 \times 10^4$ m is the distance that a piece of ice at the melting point must fall under Earth's gravity g such that the potential energy loss is sufficient to melt the ice. It follows from equation (7) that unstable drainage requires that the lake area at the height of the spillway exceed a critical area A_0 given by:

$$A_0 \equiv \frac{H^* W_s}{\beta+\gamma} = \frac{\rho_i L W_s}{\rho_w g(\beta+\gamma)} \tag{8}$$

To further illustrate the stable *vs* unstable behaviour of this system, it is useful to consider explicitly the height difference $\delta = h_l - h_s$ that controls the discharge.
Subtraction of equation (5) from equation (6) gives:

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} \equiv k\beta^{1/2}W_s\left(\frac{1}{A_0} - \frac{1}{A_l(h_l)}\right)\delta^m + \frac{Q_i}{A_l(h_l)} \tag{9}$$

This equation shows that δ grows in a semi-exponential fashion when $A_l(h_l) > A_0$. When $A_l(h_l) < A_0$, then δ declines in a semi-exponential fashion. A time independent δ and corresponding discharge can be achieved when $\mathrm{d}\delta/\mathrm{d}t = 0$, which occurs when $\delta = [Q_i A_0/k\beta^{1/2}W_s(A_0 - A_l)]^{3/5}$ and $\mathrm{d}h_l/\mathrm{d}t = \mathrm{d}h_s/\mathrm{d}t = Q_i/(A_0 - A_l)$.

To examine the theoretical behaviour more generally it is useful to introduce the additional scales: height H, such that $A_l(H) = A_0$; time $T = A_0/(k\beta^{1/2}W_sH^{5/3})$; discharge

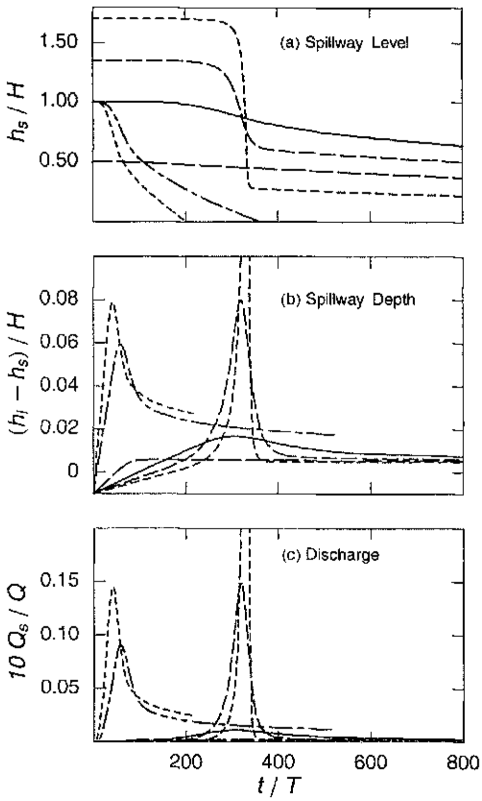

Fig. 6 Solutions for spillway level h_s (a), spillway depth $h_l - h_s$ (b) and discharge Q_s (c) displayed in dimensionless form using scales described in the text. A linear relation between lake area and lake surface height is assumed. Four initial spillway heights $h_s = 0.5H$ (long dashes), $h_s = 1.0H$ (solid), $h_s = 1.35H$ (intermediate dashes) and $h_s = 1.7H$ (short dashes) are considered, all starting with lake level 0.01 below the spillway and with dimensionless input flux 10^{-4}. Dimensionless input fluxes of 0.0014 (long-short dashes) and 0.0025 (short dashes) are also calculated for the case of initial $h_s = 1.0H$.

rate $Q = HA_0/T$. For example, with a channel slope of $\beta = 0.1$, freezing lake water ($\gamma = 0$) and a spillway width $W_s = 3$ m, the critical lake area A_0 is 9×10^5 m^2 (about 1 km^2). If the lake water is warm ($\gamma > 0$), then A_0 is smaller. Typical values for the other scales are H order 10 m, T order a few minutes and Q order 10^4 m^3 s^{-1}. These scales are used to display solutions to equations (5) and (6) for h_l, h_s and Q_s.

Figure 6 shows situations where the initial height of the spillway corresponds to a lake area that is more or less than A_0 (equation (8)). These solutions are started at $t = 0$ with a lake level $0.01H$ below the spillway. The drainage is started by a constant inflow Q_i that gradually raises the lake level above the spillway height. When the lake area is above critical, there is an initial acceleration of discharge associated with down melting of the spillway faster than the lake surface drops. When the lake area is smaller than critical, a gradual lowering of the spillway releases water from the lake, which augments the input discharge. When the lake area is close to critical, the behaviour is sensitive to the input; high input rate tends to cause initial instability.

INTERPRETATION OF OBSERVATIONS

We now examine the characteristics of the lake drainage observed on the Black Rapids Glacier using the theoretical model for spillway evolution. We use $A_l(h_l) = 0.80 \times 10^4$ h_l m, $W_s = 3$ m and an initial lake level $h_l = 18.8$ m as inputs that are relatively well known from observations. Since β, γ, Q_i and initial h_s were not well known from observations, we adjusted them to produce a fit with the observations of lake level *vs* time (Fig. 5). The curves shown in Fig. 5 show the model prediction with $\beta = 0.06$, $\gamma = 0.63$, $W_s = 3$ m, $k = 1/n = 10$, $Q_i = 0.38 \times 10^5$ m^3 day^{-1} and initial $h_s = 18.0$ m. The gradual steady drop in lake level during the later part of the observation period is controlled primarily by the combination of $(\beta + \gamma)Q_i$. The amount of lake level drop in the initial phases is controlled primarily by $(\beta + \gamma)$ in combination with the known lake area. The combination of $k\beta^{1/2}$ controls the time scale for the initial phase. Through these rather distinct controls the measurements appear to constrain the input variables quite closely. The values for these parameters imposed by the fit are quite reasonable in terms of independent information on the slope of the spillway and expected roughness of an ice channel. Starting the simulation with an initial spillway level equal to the initial lake level about 2 days prior to the peak of the discharge also can fit the known lake level data. However, it is also possible and probably more likely that the pre-existing gorge was locally blocked with snow that failed and was quickly blown out of the gorge thus suddenly exposing its ice base to a fairly thick water flow.

The large value of γ (0.63) in comparison to β (0.06) indicates that the thermal energy stored in the lake played a dominant role in the lowering of the spillway height by melting. This conclusion is forced by the continued lowering of the spillway with a rather low discharge of water and small rate of potential energy release. The value of $\gamma = 0.63$ is produced with a lake temperature $\Delta\theta$ about 0.7°C above freezing and a drop to freezing over about 500 m. The combined value of $\beta + \gamma = 0.7$ gives a critical lake level of $A_0 = 1.3 \times 10^5$ m^2 (equation (8)), which is slightly less than the initial lake area at the onset of discharge. If the lake water had not been warmed above freezing, then A_0 would have been much larger and the unstable discharge would not have happened.

DISCUSSION

Although the model is simplified, its success in simulating the Black Rapids Glacier lake drainage suggests its basic physical foundations are applicable. One may, therefore, expect a range of potential behaviour as illustrated in Fig. 6. A question that arises more generally is what determines the input variables. The lake area in comparison to A_0 is a crucial factor. The area of basins that can trap water on a glacier will be related in some way to characteristics of the debris cover among other things. A_0 itself depends on the channel width W_s, the spillway slope β, and the lake temperature $\Delta\theta$ (equation (8)). All of these variables could be quite different from one circumstance to another depending on the shape of the ice barrier, the surface slope of the glacier and heat balance history of the lake. A steep spillway slope lowers A_0, but it is less likely to have large lakes on steep slopes. Thus, we expect that warming a lake and resultant lowering of A_0 is crucial for unstable drainage in most circumstances.

Lakes can become very warm (several degrees Celsius) when they are floored by debris (Sakai *et al.*, 2000).

Most lakes and ponds formed on glacier surfaces will be too small and cold to drain unstably. Nevertheless they comprise one mode of liquid water storage. They will reduce runoff in comparison to melt as they are filling during the early melt season. Later they will enhance runoff as they overflow and spillways are melting downward.

Seasonal snow may play an important role in the early melt season drainage of these lakes and can result in rapid unstable discharges, and should be an important consideration in the analysis of outburst hazard.

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