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STECCANELLA, P., Progetto della reimmissione nel Lago Trasimeno del Tresa e Rio Maggiore.
STECCANELLA, P., Progetto per la bonifica delle gronde del Lago Trasimeno.

EFFECTS OF LAKES ON OUTFLOW CHARACTERISTICS*

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SUMMARY

This manuscript presents the study of outflow characteristics affected by properties of natural lakes when inflows are log-normally distributed. The general differential equation for water storage, based on the continuity equation, is adapted to this case, with properties of inflows and lakes given. Lake properties are described by the storage capacity function and by the outflow rating curve function. Independent inflows are described by the log-normal distribution function with two parameters: the mean and the standard deviation of logarithms. Sequential mathematical models are derived for outflows by integrating the differential equation under the assumption that the average inflow of a natural lake is equal to the average outflow. Data of inflows generated on a CDC 3600 computer consisted of 10,000 independent standard normal numbers, with mean zero and variance unity. These numbers were then transformed to a log-normal distribution with various values of standard deviation of logarithms, I_v.

The seven parameters in the mathematical model were reduced to three parameters which were used in generating the outflow sequences. These three parameters were: (1) I_v, the index of variability of inflows; (2) n, the ratio between the powers for storage function and the outflow rating function; and (3) d, a lumped dimensionless parameter descriptive of inflow, lake properties and time interval, Δt, used for the finite difference integration of differential equation.

The digital computer produced the independent log-normal numbers, solutions of outflow generating equations, parameters of the outflow distribution, and the first ten serial correlation coefficients of outflow series. Two hundred and ten outflow sequences were generated. They represented the following combinations: (1) five values of I_v ($I_v = 0.15, 0.25, 0.40, 0.60, 0.90$); (2) seven values of n ($n = 1/3, 1/2, 3/4, 1, 3/2, 2, 3$); and (3) six values of d ($d = 0.3, 1.0, 3.0, 10.0, 30.0, 100.0$). The values obtained for the variance, skewness coefficient, excess coefficient of outflow distribution and for the first ten serial correlation coefficients are presented in graphs showing their relationship with the parameters I_v, n and d. Whenever feasible, the equations are given for the parameters of lake outflows as functions of I_v, n and d.

RÉSUMÉ

Les effets des lacs sur les caractéristiques des débits effluents

Le sujet de cet article est l'étude des caractéristiques des débits effluents en tant qu'ils sont influencés par les propriétés des lacs naturels eux-mêmes. Les apports dans un lac sont supposés être distribués par la loi de probabilité logarithmique normale. L'équation différentielle générale pour la régularisation des débits par les lacs naturels, basée sur l'équation de continuité, est adoptée pour le cas où les propriétés des apports et des lacs sont connues. Les propriétés des lacs sont décrites par les fonctions de la capacité et de la courbe des débits, qui dépendent du niveau de l'eau. Les débits affluents sont décrits par la loi de probabilité logarithmique normale avec deux paramètres : la moyenne et l'écart type du logarithme des débits, I_v. Les séquences des débits effluents ont été obtenues par l'intégration de l'équation différentielle en supposant que la moyenne des débits effluents d'un lac natural est égale à la moyenne des apports donnés. Les apports obtenus par la méthode Monte Carlo sur la calculatrice CDC 3600 consistaient en un échantillon de 10 000 nombres indépendants distribués d'après la loi de Gauss standardisée, dont la moyenne est zéro et l'écart type est égal à un. Ces nombres ont été ensuite transformés de telle manière que leur distribution suive la loi de probabilité logarithmique normale.

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Les sept paramètres qui entrent dans le modèle mathématique se réduisent à trois paramètres. Ils sont employés pour créer des séquences de débits effluents par la méthode de Monte Carlo. Ces trois paramètres sont : (1) I_v, l'index de variabilité des apports; (2) n, le rapport de la puissance de la fonction de capacité à celle de la fonction de la courbe des débits; et (3) d, un paramètre qui est la combinaison des propriétés des apports, du lac, et de l'intervalle du temps, Δt, utilisé pour l'intégration numérique de l'équation différentielle.

La calculatrice a été utilisée pour obtenir les nombres indépendants, les solutions de l'équation des débits effluents, calculer les paramètres de distribution et les dix premiers coefficients d'autocorrélation des débits effluents.

Deux cents dix séquences des débits effluents ont été obtenues par ce calcul pour les combinaisons suivantes des paramètres : (1) cinq valeurs de I_v (0,15; 0,25; 0,40; 0,60; 0,90); (2) sept valeurs de n (1/3, 1/2, 3/4, 1, 3/2, 2, 3); et (3) six valeurs de d (0,3; 1,0; 3,0; 10,0; 30,0; 100,0).

Les résultats sont donnés sous forme d'abaques du carré de l'écart type, du coefficient d'asymétrie, du coefficient d'aplatissement, des coefficients d'autocorrélation des débits effluents tous en fonction des paramètres I_v, n et d. En quelques cas, les équations des propriétés des débits effluents sont données en fonction des paramètres I_v, n et d.

Introduction

Natural storage, storage by reservoirs or other artificial controls, and the characteristics of the outflow from natural lakes are important assets in the development of water resources. Unlike the outflow from a reservoir which is regulated by a hydraulic structure, the outflow from a natural lake is a result of the given inflow characteristics, outflow conditions (rating curve) and storage properties of the lake.

Outflow characteristics are described in terms of the most important parameters of inflow, outflow rating curve and lake storage. The effects of natural lakes with log-normally distributed and independent inflows on the outflow characteristics are particularly studied. The CDC 3600 digital computer was used: (a) to generate the independent inflow data; (b) to solve the equations; (c) to compute the parameters of the outflow probability distribution; and (d) to compute the first ten serial correlation coefficients of the generated outflow sequences. Generated data consisted of 10,000 independent random numbers, normally distributed, with the mean zero and the variance unity, which were transformed to log-normal probability distributions with various values of standard deviation of logarithms.

The approach used was:

1. The mathematical model for the generation of outflow sequences was derived from the general storage differential equation, which was obtained by using the outflow rating curve and the storage capacity function of a lake. The latter two relationships were assumed to be power functions of the water depth above the level of zero outflow. The average outflow was assumed equal to the average inflow for a sufficient time period. The difference between precipitation and evaporation, and seepage from the lake were neglected.

2. Only three parameters, which described the inflow, the outflow rating curve and the storage function of lakes were introduced in the mathematical model. The ranges of these three parameters were selected for practical cases.

3. The expressions for the variance, the skewness and the excess coefficients of the outflow probability distribution and for the serial correlation coefficients when the ratio of exponents of the lake storage and rating curve power functions is unity, were theoretically derived from the basic equation for given values of the other two parameters. The inflows were mutually independent and independent of previous outflows. The same statistical parameters of outflows for the ratio of exponents different from unity were computed from outflow sequences obtained by the data generation method.

The log-normal density function of X, defined as the density function of a variable X whose logarithms obey the normal law of probability, is

$$f(X) = \frac{1}{X\sigma_n\sqrt{2\pi}} \exp[-(\ln X-\mu_n)^2/2\sigma_n^2] \tag{1}$$

with μ_n the mean and σ_n the standard deviation of $\ln X$.

The mean μ and the variance σ^2 of X are given by

$$\mu = \exp(\mu_n+\sigma_n^2/2) \tag{2}$$

and

$$\sigma^2 = \exp(2\mu_n+\sigma_n^2)\,[\exp(\sigma_n^2)-1] \tag{3}$$

For $\eta = \sigma/\mu$, the coefficient of variation of X variable, the skewness coefficient γ_1 is

$$\gamma_1 = \eta^3 + 3\eta \tag{4}$$

and the excess coefficient γ_2 is

$$\gamma_2 = \eta^2(\eta^6+6\eta^4+15\eta^2+16). \tag{5}$$

For μ and σ given, then

$$\mu_n = \ln\frac{\mu}{\sqrt{1+\eta^2}} \tag{6}$$

and

$$\sigma_n = [\ln(1+\eta^2)]^{1/2}. \tag{7}$$

Equation for generating outflow sequences

1. *Basic storage equation*

The characteristics of outflow from a natural lake depend on: (a) the shape of the outlet cross-section, the slope and the roughness of the outflowing river reach which effects are integrated by the outflow rating curve; (b) the storage function which relates the storage capacities of natural lakes to the lake level or depths; and (c) the characteristics of inflows. The equation for the outflow sequence is derived from the above characteristics by using the continuity equation. It expresses the basic relation between inflow, outflow and storage as

$$P-Q = \frac{\mathrm{d}S}{\mathrm{d}t} \tag{8}$$

with P = the inflow discharge, Q = the outflow discharge, and S = the storage volume, t = the time, and $\mathrm{d}S/\mathrm{d}t$ = the rate of storage change with time.

2. *Properties of lakes*

The relation between the lake volume and water depth can often be approximated by the power function

$$S = aH^m \tag{9}$$

with S the storage volume, and H the depth of water above the datum of zero outflow, and a and m the parameters, depending on the size and shape of the lake.

The outflow rating curve can also be approximated by a power function of the type

$$Q = bH^r \tag{10}$$

with Q the outflow discharge, and H the depth of water above the datum of zero outflow, with the parameters b and r which depend on the characteristics of the river reach at the lake outflow.

3. *Distribution of inflows*

It has been noted that discharge data of many rivers follow the normal law if the logarithms of the river flow are used instead of actual values. In this study, the log-normal distribution has been assumed to be a good distribution function to describe the inflow characteristics. For simplification, the inflows are assumed to be independent, though the same method can be used for dependent inflows.

4. *Derivation of equations for outflow sequences*

From equation (10), $H = (Q/b)^{1/r}$. Substituting in equation (9)

$$S = a(Q/b)^{m/r} = \frac{1}{c} Q^n \qquad (11)$$

where $n = m/r$ and $c = b^n/a$.

By differentiating equation (11) and by using equation (8), the differential equation becomes

$$\frac{dQ}{dt} - \frac{c}{n} Q^{-n+1} P + \frac{c}{n} Q^{-n+2} = 0. \qquad (12)$$

This equation can be expressed as a difference equation by letting

$$dQ = \Delta Q = Q_{i+1} - Q_i,$$

and $dt = \Delta t$, with $Q = \frac{1}{2}(Q_{i+1} + Q_i)$, where Q_i is the actual outflow at the i-th time Q_{i+1} at the $(i+1)$-th time; and $P = \frac{1}{2}(P_{i+1} + P_i)$, with P_i the actual inflow at the i-th time, and P_{i+1} at the $(i+1)$-th time. If the modular coefficients $X_i = P_i/P_0$ and $Y_i = Q_i/Q_0$ are used, then $P_i = X_i P_0$ and $Q_i = Y_i Q_0$, where P_0 is the average inflow, and Q_0 is the average outflow. The assumption of average inflow being equal to the average outflow of a lake gives $P_0 = Q_0$. Equation (12) becomes

$$Y_{i+1} - Y_i - \frac{c}{n} \Delta t P_0^{-n+1} \left(\frac{1}{2}\right)^{-n+2} (Y_{i+1} + Y_i)^{-n+1} (X_{i+1} + X_i - Y_{i+1} - Y_i) = 0.$$

Let $(\frac{1}{2})^{-n+2} c \Delta t P_0^{-n+1}$ be denoted by d, which is a dimensionless parameter, then

$$Y_{i+1} = Y_i + \frac{d}{n} (Y_{i+1} + Y_i)^{-n+1} (X_{i+1} + X_i - Y_{i+1} - Y_i). \qquad (13)$$

This is the equation for generating outflow sequences, Y_{i+1}. The difference between equation (12) and equation (13) is in their dimensions: the latter is dimensionless while the former has dimensions of L^3/t^2.

5. *Description of parameters in the equation for generating outflow sequences*

Originally there were seven parameters involved in the derivation of the equation: (a) Storage function parameters, a and m; (b) Outflow rating curve parameters, b and r; (c) Parameter Δt, to change the differential equation to the difference equation; (d) The parameter P_0, which is the average inflow; and (e) The parameter I_v, which is the index of variability of inflows to specify the characteristics of inflow. Only the following three parameters: n, I_v and d are used in the mathematical model and they are defined as follows:

(a) *Parameter n* with $n = m/r$, or the ratio of powers for storage function and outflow rating curve. This parameter describes the properties of the lake and the outflow with range from 1/4 to 4. The coefficient m lies within the limits 1 to 5, and depends on the range of levels and on the shape of lakes. For a high reference level and a small range of levels, m is usually 1.0 to 1.5 and rarely exceeds 2. For the highest range of levels it is from 2 to 5. The range of the coefficient r of equation (10) is 1.5 to 3.0. The parameters a and b are indicators of flatness of the storage function and rating curve function, respectively, while the parameter n indicates the ratio of rate-of-change of lake (storage function) and rate-of-change of outlet rating curve. The latter can be used as an index whether a natural lake has a good ratio of storage function to the outflow rating curve characteristics.

(b) *Parameter d*. This parameter, $d = (\frac{1}{2})^{-n+2}(b^n/a) \Delta t P_0^{-n+1}$, represents the relationship of average inflow, time unit selected, and the properties of the storage function and the outflow rating curve through $c = b^n/a$. The selection of a longer time unit creates a larger value for d. The effect of mean inflow, P_0, on parameter d depends on the value of n. For n greater than unity, the value of d increases as the mean inflow increases, and decreases when n is less than unity. The coefficient b has a large range. It depends on the dimensions used for Q and H, the outlet cross-section, river slope and roughness.

(c) *Parameter I_v*. This parameter, sometimes called the index of variability, is the standard deviation of logarithms of the modular coefficients of inflows. It describes the characteristics of inflow. When the average inflow, P_0, and the index of variability, I_v, are known, the distribution of inflows is defined.

CHARACTERISTICS OF OUTFLOWS

1. *Analysis of equation for generating outflow sequences*

The equation for outflow sequences depends partially upon the value of n. Seven values of n: 1/3, 1/2, 3/4, 1, 3/2, 2 and 3, are analyzed as separate cases.

For n equals unity, the most simple case, equation (13) becomes linear

$$Y_{i+1} = [Y_i + d(X_{i+1} + X_i - Y_i)]/(1+d), \qquad (14)$$

in which Y_{i+1} can easily be solved since Y_i, X_{i+1}, X_i and d are known.

For n equals 2, equation (13) becomes

$$Y_{i+1}^2 + \frac{d}{2} Y_{i+1} - Y_i^2 + \frac{d}{2} Y_i - \frac{d}{2}(X_{i+1} + X_i) = 0. \qquad (15)$$

Equation (15) is quadratic, and Y_{i+1} can be immediately found in

$$Y_{i+1} = \frac{-d}{4} + \frac{1}{2}\sqrt{\frac{d^2}{4} + 4 Y_i^2 + 2d(X_{i+1} + X_i - Y_i)} \qquad (16)$$

The positive value of the square root is chosen because the outflows of natural lakes usually are not negative.

2. *Generation of inflows*

When the inflows are log-normally distributed, the modular coefficients of inflows are also log-normal. Hence, a new variable $z_i = \ln X_i$ is normally distributed with mean $E[\ln X] = \mu_z$ and the variance I_v^2. The independent standard normal variable, t_i, is obtained by the transformation

$$t_i = (z_i - \mu_z)/I_v. \tag{17}$$

Since X_i is the modular coefficient, its expected value is unity. By using the relationship between the parameters of the log-normal distribution and corresponding normal distribution, equations (2), (3), (6) and (7) give

$$\bar{X} = \exp(\mu_z + \tfrac{1}{2} \text{ var } z) \text{ and var } X = [\exp(\text{var } z) - 1] \exp(2\mu_z + \text{var } z).$$

As $\bar{X} = 1$ and var $z = I_v^2$, then $\mu_z = -\frac{1}{2} I_v^2$, and substituting in equation (17)

$$X_i = e^{z_i} = \exp(I_v t_i - \tfrac{1}{2} I_v^2) \tag{18}$$

with $z_i = I_v t_i + \mu_z$ where t_i represents the independent numbers of the standard normal variable. Also,

$$\text{var } X = \exp(I_v^2) - 1 \tag{19}$$

or the variance of modular coefficients of inflows can be expressed in terms of I_v only.

The 10,000 random numbers normally distributed with mean zero and variance unity, assuming the initial outflow is equal to the initial inflow, were generated on the CDC 3600 computer and were used as t_i in equation (18) to obtain X_i numbers. Sequences of outflows were obtained for five different values of I_v: 0.15, 0.25, 0.40, 0.60 and 0.90, from equations (13), (14) and (16), for the seven values of n. Six different values of d: 0.3, 1, 3, 10, 30 and 100 were used for generating the outflows. The time series generated, each $N = 10{,}000$ long, represent various combinations of n, I_v and d, or $7 \times 5 \times 6 = 210$ different cases. The statistical parameters of these series were obtained by using the digital computer. The outflows for $n = 1$ are generated by equation (14), for $n = 2$ by equation (16) and for all other n values by iterative solution of equation (13).

3. *Determination of outflow characteristics*

Parameters of the outflow Y_i, computed from 10,000 values of outflow, are

(1) the mean, $\bar{Y} = \frac{1}{N} \sum_{1}^{N} Y_i$;

(2) the variance, $s_y^2 = \frac{1}{N} \sum_{1}^{N} (Y_i - \bar{Y})^2$;

(3) the coefficient of variation $C_v = s_y/\bar{Y}$;

(4) the skewness coefficient, $\gamma_1 = \sum_{1}^{N} (Y_i - \bar{Y})^3 / N s_y^3$;

(5) the excess coefficient, $\gamma_2 = \dfrac{\sum_{1}^{N} (Y_i - \bar{Y})^4}{N s_y^4} - 3$;

and (6) the first ten serial correlation coefficients computed by

$$r_k = \frac{\sum_{1}^{N-k} Y_i Y_{i+k} - \left(\sum_{1}^{N-k} Y_i \sum_{1}^{N-k} Y_{i+k}\right)/(N-k)}{\left[\sum_{1}^{N-k} Y_i^2 - \left(\sum_{1}^{N-k} Y_i\right)^2/(N-k)\right]^{1/2} \left[\sum_{1}^{N-k} Y_{i+k}^2 - \left(\sum_{1}^{N-k} Y_{i+k}\right)^2/(N-k)\right]^{1/2}}$$

with r_k the k-th lag serial correlation coefficient and k the time lag.

4. *Analytical solution for $n = 1$*

For n equals unity, the parameters of the outflow distribution are determined analytically from the equation of outflow sequences when the inflows are mutually independent and independent of previous outflows. For $n = 1$, equation (14) can be written as

$$Y_{i+1} = \frac{1-d}{1+d} Y_i + \frac{d}{1+d} (X_{i+1} + X_i) \tag{20}$$

From this equation the expected value of the modular coefficients of outflows yields unity. Multiplying equation (20) by X_{i+1} and taking the expected values

$$E[X_{i+1} Y_{i+1}] = \frac{1-d}{1+d} E[X_{i+1} Y_i] + \frac{d}{1+d} [E(X_{i+1}^2) + E(X_{i+1} X_i)] = 1 + \frac{d}{1+d} \text{ Var } X \tag{21}$$

because X_{i+1} and Y_i are independent; or $E(X_{i+1} Y_i) = E(X_{i+1}) E(Y_i) = 1$; X_{i+1} and X_i are independent, or $E(X_{i+1} X_i) = 1$; and var $X + 1 = E(X^2)$. From equation (20) then

$$\text{var } Y_{i+1} = \left(\frac{1-d}{1+d}\right)^2 \text{var } Y_i + \left(\frac{d}{1+d}\right)^2 [2 \text{ var } X + 2 \text{ cov}(X_{i+1}, X_i)] +$$

$$+ 2\frac{d(1-d)}{(1+d)^2} \text{cov}(Y_i, X_{i+1} + X_i) = \frac{d}{1+d} \text{var } X, \text{ or } \frac{\text{var } Y}{\text{var } X} = \frac{d}{1+d} \tag{22}$$

because cov $(X_{i+1}, X_i) = 0$, cov $(Y_i, X_{i+1}) = 0$, and

$$\text{cov}(X_i, Y_i) = E(X_i Y_i) - E(X_i) E(Y_i) = \frac{d}{1+d} \text{var } X$$

To find the skewness and excess coefficients, the expected values of X^2, X^3, X^4, Y^2, Y^3, Y^4, XY^2, X^2Y, X^3Y and XY^3 are necessary. By using the moment generation function, $E(X^2)$, $E(X^3)$ and $E(X^4)$ can be found. In a similar way, the remaining expected values were found. The final results give the skewness coefficient of outflows

$$\gamma_1 = \frac{M_3}{s_y^3} = \left[\left(\frac{d}{1+d}\right)(e^{I_v^2} - 1)\right]^{-3/2} [E(Y^3) - 3E(Y^2) + 2] \tag{23}$$

and the excess coefficient of outflows

$$\gamma_2 = \frac{E(Y^4) - 4E(Y^3) + 6E(Y^2) - 3}{s_y^4} - 3 \tag{24}$$

The final solutions for the ratios of skewness or excess coefficients for outflow and inflow, respectively, are given as equations (29) and (30). The expected values computed for Y, Y^2, Y^3 and Y^4 are not given.

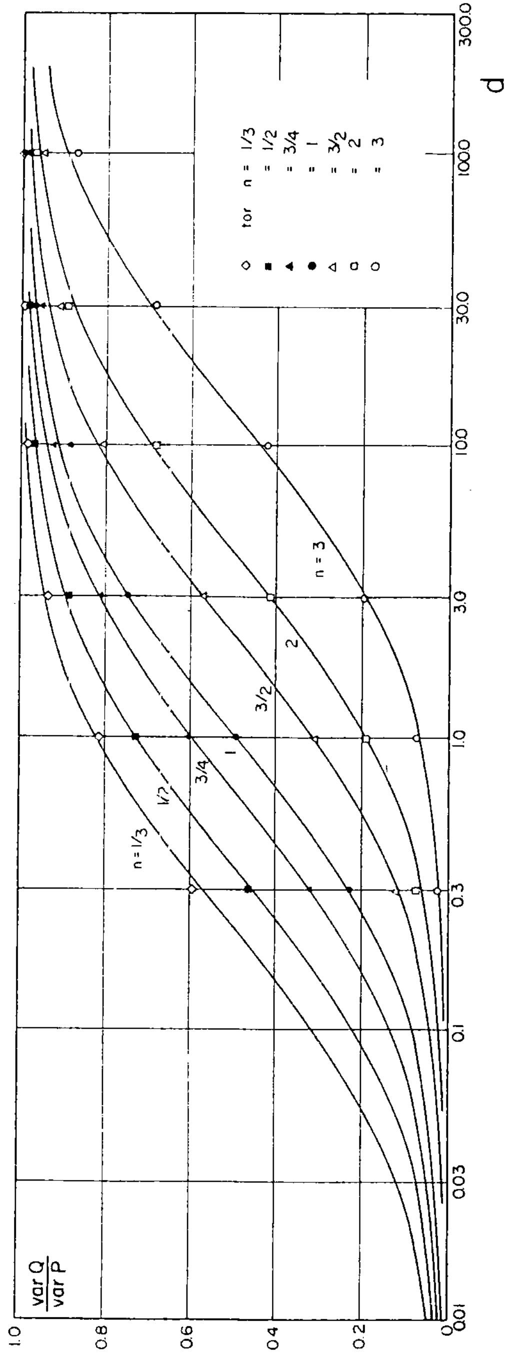

Fig. 1 — Ratio of variances of Lake Outflow and Inflow vs. parameters d and n.

The correlogram o' outflows for $n = 1$ was obtained by using the same method, and the final result was

$$\varrho_k = \frac{(1-d)^{k-1}}{(1+d)^k}, \qquad k \geqslant 1 \tag{25}$$

with $\varrho_k = 1$, for $k = 0$.
Specifically for $k = 1$

$$\varrho_1 = \frac{1}{1+d}. \tag{26}$$

The result given by equation (25) shows that the serial correlation coefficients depend only on the value of d and k and are independent of the value I_v. When d is positive and greater than 1, equation (25) shows that the serial correlation coefficients become negative whenever the value of k is even, and positive whenever k is uneven.

Equations (22) through (25) were used to compute the variance, the skewness coefficient, the excess coefficient and the serial correlation coefficients of outflows, respectively for $n = 1$. As it will be shown, the values obtained by these equations for $n = 1$ agree with the results from the generated outflow sequences of 10,000 numbers. The differences appear to be attributable to sampling errors only. The attempt to analytically solve the cases for other values of n except $n = 1$ did not produce solutions.

5. *Variance of outflow*

Figure 1 shows the relationship of the ratio of variances of outflow and inflow, var Q/var P, versus the dimensionless parameter d, for various values of the parameter n. The 210 points represent the results of the data generation method and the lines represent either the exact relationship, which is the case only for $n = 1$, or the fitted curves, which is the case for other values of n. Equation (22) gives:

$$\text{var } Q/\text{var } P = d/(1+d).$$

It is found that a horizontal translation of the curve of equation (22) fits well the points obtained for the values of n different from unity. However, the position of each line for a given n is obtained by the least square method of fitting the line of equation (22) to the points. For var Q/var $P = 0.5$ and for $n = 1$, the value of d is unity. This value, d_m, is determined for each n different from unity. The logarithms of n and d_m (with arbitrarily added value of 2.00), or $(2+\ln n)$ versus $(2+\ln d_m)$, are plotted in

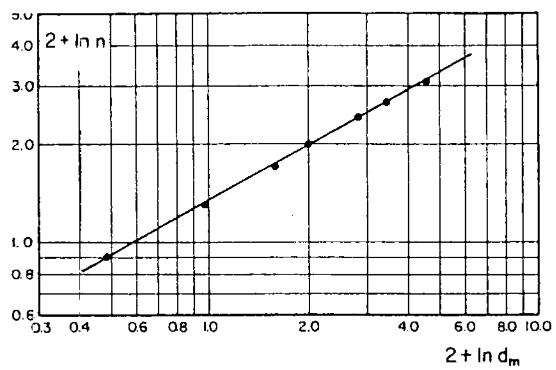

Fig. 2 — Relationships of parameters n and d_m, for the ratio of variances of outflow and inflow of 0.50.

figure 2 in log-log paper (for the double-log relationship). A least square straight line fit in figure 2 gives the relationship of d_m and n which may be considered as an experimental curve which approximates closely the true relationship, or

$$2+\ln d_m = 2(1+\tfrac{1}{2}\ln n)^{1.78}. \qquad (27)$$

The constant 2 is introduced in order to avoid the negative values in taking the logarithms, and the selection of this constant does not change the results, but changes the coefficient before ln n (the coefficient is $\frac{1}{2}$ before ln n, for the constant 2). For $n \geq 0.10$, the constant 2 is sufficient to avoid the negative values. For $n < 0.10$, a larger constant should be used. With this value of d_m, then

$$\frac{\mathrm{var}\ Q}{\mathrm{var}\ P} = \frac{d}{d+0.135\exp[2(1+\frac{1}{2}\ln n)^{1.78}]} \qquad (28)$$

where the variance of $X = P/P_0$ is given by equation (19).

For given values of d and n, equation (28) gives the ratio of variances of outflow and inflow. As expected, the variances of outflows are always smaller than the variances of the corresponding inflows.

6. *Skewness coefficient of outflows*

Figure 3 gives the points which are obtained for ratios of the skewness coefficient of outflow distribution to the skewness co-efficient of inflow distribution, computed by the data generation method, a total of 210 cases, as functions of the parameter d for various values of n.

The analytical approach for $n = 1$ yields the solution of equation (23) and the ratio is

$$\frac{\gamma_1(Q)}{\gamma_1(P)} = \frac{4-d+d^2}{(3+d^2)\sqrt{1+d/d}} \qquad (29)$$

with $\gamma_1(P)$ given as a function of I_v by equation (4).

Figure 3 shows a relatively good agreement for $n = 1$ between the curve of equation (29), plotted by a solid line, and the points for $n = 1$ obtained from 10,000 generated outflows. The dashes connect the computed points for $\gamma_1(Q)/\gamma_1(P)$ versus d for the n values different from unity. No attempt was made to fit a family of curves for these cases because of a much greater sampling fluctuation of the third statistical moment used in estimating γ_1-coefficients for $\gamma_1(P)$ from generated 10,000 independent log-normal inflows, and $\gamma_1(Q)$ from generated 10,000 dependent outflows. The simple horizontal translation of the curve of equation (29) does not seem to apply.

The analysis of the relationship of $\gamma_1(Q)$ and the coefficient of variation of Q, shows that the distributions of outflows do not satisfy equation (4). The distributions of outflows are not log-normally distributed when the inflows are log-normally distributed. The converse is also true.

7. *Excess coefficient of outflows*

The ratio of excess coefficients of outflows and inflows versus the parameter d, for $n = 1$ is obtained by the solution of equation (24), and this ratio is

$$\frac{\gamma_2(Q)}{\gamma_2(P)} = \frac{d(2-d+d^2)}{(1+d)(1+d^2)} \qquad (30)$$

Because the ratio $\gamma_2(Q)/\gamma_2(P)$ was shown not to be independent of I_v, and because of large sampling fluctuations, the computed points are not presented in this paper. The use of the excess coefficient is limited in hydrology and further study was not deemed of any value.

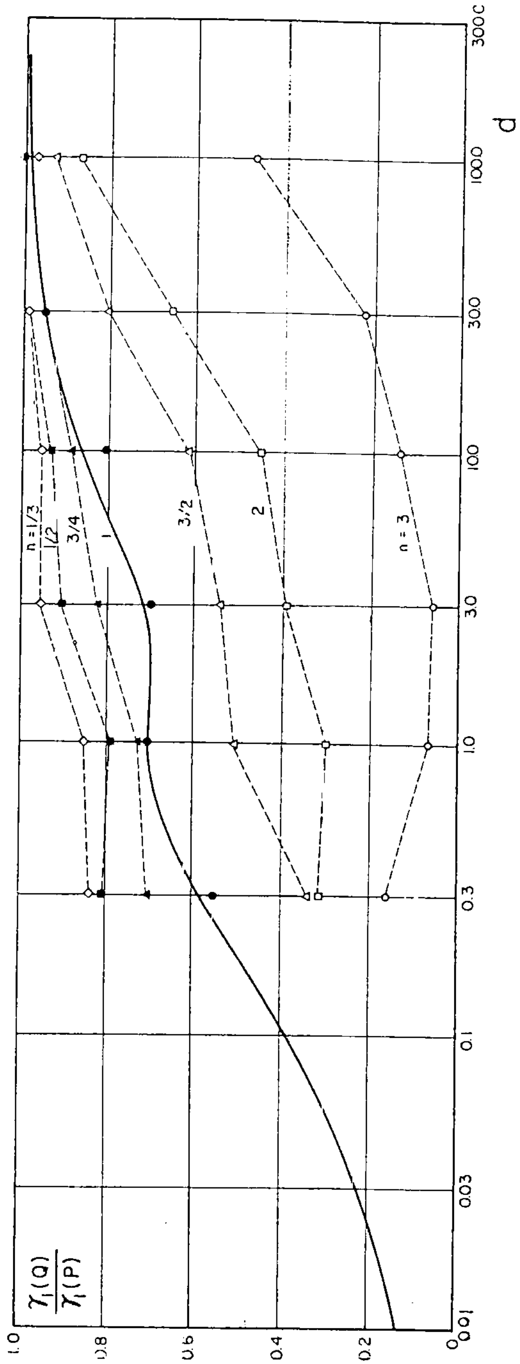

Fig. 3 — Ratio of skewness coefficients of lake outflows and lake inflows vs. parameters *d and n.*

8. *Serial correlation coefficients of outflows*

By using the same procedure for the serial correlation coefficients, ϱ_k, of outflows, as was used for the ratio of variances, the theoretical curve of ϱ_k for outflow for $n = 1$, equation (25), is translated along the d-axis to fit the points (by the least square method) for n different from unity. Figure 4 gives the relationship of $(2+\ln n)$ and $(2+\ln d_m)$, with d_m the value of d for $\varrho_1 = 0.50$. The least square fitted straight line of figure 4 gives the relationship

$$2+\ln d_m = 2(1+\tfrac{1}{2}\ln n)^{1.84} \qquad (31)$$

with the same remarks for the use of the constant 2 as was given above for the ratio of variances. It must be noted that the constant 1.84 is close to 1.78 which is found for the ratio of variances.

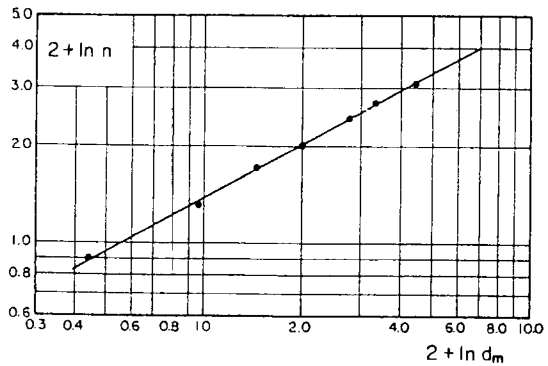

Fig. 4 — Relationships of the parameters n and d_m, for the first serial correlation coefficient $\varrho_1 = 0.50$.

With equation (31), equation (25) for ϱ_k becomes

$$\varrho_k = \frac{(1-d/d_m)^{k-1}}{(1+d/d_m)^k} \qquad (32)$$

with d_m given by equation (31).

For the special case, ϱ_1 (the first serial correlation coefficient), the obtained points by the generation of 10,000 outflows are given in figure 5. The solid line for $n = 1$ gives $\varrho_1 = 1/(1+d)$, and this line is translated horizontally to fit the observed points for n values different from unity. The fit is very good.

Conclusions

The following conclusions are advanced:

(1) By knowing the approximate power function relationships of lake storage and lake outflow rating curve to the water depths above the level of zero outflow, the relationship of outflow to inflow characteristics and vice versa can be established either analytically for some limited cases, or by the data generation method (Monte Carlo method) for all cases. This enables the simple determination of outflow characteristics for given inflow characteristics, or vice versa.

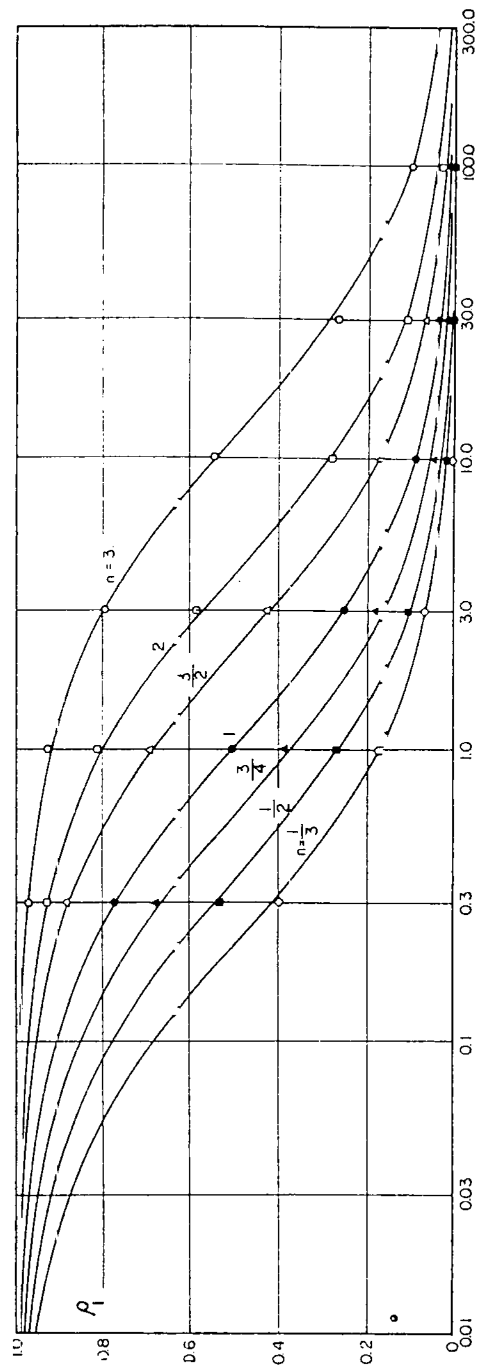

Fig. 5 — First serial correlation coefficient, ϱ_1, of outflow, vs. parameters d and n.

(2) The assumption of independent log-normally distributed inflows is not necessary for establishing the inflow-outflow relations, though it was used in this paper to illustrate the method.

(3) Once the three parameters (n, d and I_w) of the inflow and of the lake characteristics are known, the relationships given in this paper permit a straight forward determination of outflow characteristics, described by the variance, the coefficient of skewness, and eventually the coefficient of excess as well as by the serial correlation coefficients.

(4) Though the parameters of outflows are derived theoretically only for $n = 1$, it may be expected that they could be derived for the other values of n, or for any value of n.

(5) In order to obtain accurate values of the skewness and excess coefficients of outflow, the sample size of 10,000 generated outflows seems to be inadequate mainly because of serial correlation, and larger samples may be inexpensively generated.

(6) Simple statistical tests may be performed to test whether the original generated numbers are normally distributed and independent. These tests were not carried out in this study though they were used in other similar investigations.

LES LACS DE CALDONAZZO ET DE LEVICO

Observations météorologiques, hydrométriques et bathométriques

Livio DORIGO

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Résumé

En se rapportant à des études précédentes : « Les caractères généraux et les conditions morphométriques des lacs de Caldonazzo et de Levico » (1), « Le climat des combes des lacs de Levico et de Caldonazzo » (2) seront élaborées des observations météorologiques faites après la période prise en considération dans les études citées ci-dessus.

En particulier pour le lac de Caldonazzo on répétera les relèvements bathymétriques faits en 1930.

Pour tous les deux lacs on élaborera les observations hydrométriques faites pendant une longue période.

Plusieurs savants se sont occupés des lacs de Caldonazzo et de Levico en étudiant soit le climat des cuvettes des deux lacs, soit les caractéristiques physiques et morphométriques de ces lacs.

Ce mémoire se pose le but d'ajouter de nouveaux éléments aux études déjà faites et en particulier en ce qui concerne une série d'observations pluviométriques et thermométriques plus longue que les séries publiées jusqu'aujourd'hui.

Étant donné qu'en 1930 la «Section pour le Relèvement des Lits» spécialement organisée auprès du Magistrato alle Acque avait effectué le levé topographique et bathométrique des deux lacs, il a été considéré intéressant d'effectuer un deuxième et rapide levé bathométrique du lac de Caldonazzo, afin d'obtenir des éléments indicatifs sur les changements éventuels subis par le lac et cela pour pouvoir prendre des décisions concernant un relevé minutieux.

En effet, les résultats de 12 sections transversales (fig. 1) permettent d'établir que, pour le moment, malgré le grand intervalle de temps, écoulé entre un levé et l'autre (35 ans), un levé plus minutieux n'est pas nécessaire. Les résultats obtenus sont récapitulés dans les tables I et II.

Caractéristiques géo-hydrologiques des deux lacs

L'origine des deux lacs est due au barrage des deux vallées déterminé par les grands cones de déjection, et précisément par ceux de Susà-Pergine et de Caldonazzo pour le lac de Caldonazzo et par celui de Levico pour le lac du même nom.

La surface du bassin tributaire du lac de Caldonazzo est de 50,5 km^2, dont 20,6 km^2, qui représentent le 40% de toute la surface, sont constitués par des roches imperméables.

Sur la rive droite les phyllades et les roches quartzifères prédominent, mais on y trouve aussi des dépôts et des recouvrements de moraines.

L'hydrographie est très pauvre, les divers cours d'eau sont sans eau pour la plus grande partie de l'année, seulement le Rio Mandola, qui se jette près de Calceranica, sur la rive droite, a des eaux pérennes.

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